Analyzing the Center of Mass in Collisions Using Video Analysis

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The center of mass (CM) is an abstract quantity but essential as a reference point used to understand the position and movement of an object or a system of objects. Many researchers have used video recordings to analyze and understand the movement of objects in a system.[1,2] In a system with colliding objects, the motion can be obtained precisely from video analysis. Estimating the momentum of each object can allow analysis of the conservation of linear momentum of the collision.[2,3]

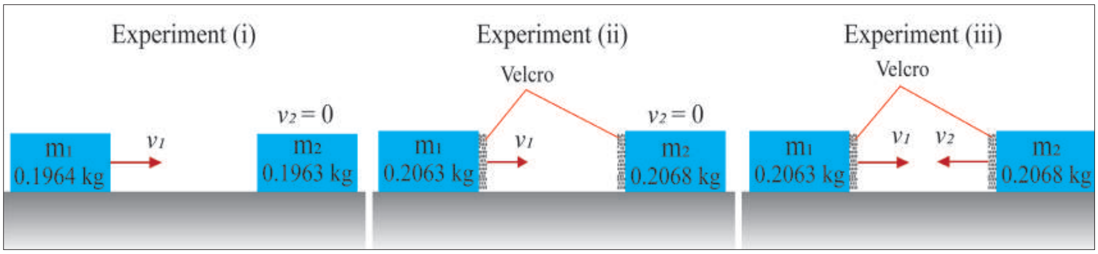

Fig. 1. Three types of collision experiments.

Another essential quantity of motion that can be obtained from the analysis of colliding objects is the velocity of a CM of a system. This quantity is important in studying the basic concepts of collisions. The CM in a collision is rarely studied explicitly experimentally. Most analysis is likely to be done by determining the velocity of each object before and after the collision.[4,5] Video recordings have been widely used in analyzing the movement of colliding objects, facilitating study of the movement of the CM of the system of colliding bodies.[6,7] This quantity helps analyze the linear momentum in a collision. We consider the framework for obtaining the velocity of CM to be indispensable for building a more comprehensive understanding of collisions.

In this work, we present an experiment to estimate the velocity of the CM during collision of two objects. The velocity of the CM is estimated from the video analysis of CM movement during the experiment. The change of the velocity of the CM during the collision is used as the basis for analyzing the conservation of linear momentum during the collision experiment.

Method

The experiment was carried out using an air track device (PMK-145, produced by Pudak Scientific, Indonesia). In this paper, we conducted three types of experiment, as illustrated in Fig. 1. Experiment (i): object m_1 (0.1964 kg) moved at a constant velocity to object m_2 (0.1963 kg) at rest. Experiment (ii): Velcro was attached to both objects, and m_1 (0.2063 kg) traveled with a constant velocity to m_2 (0.2068 kg) at rest. Experiment (iii): Velcro was attached to both objects, and m_1 (0.2063 kg) and m_2 (0.2068 kg) traveled in opposite directions and collided with each other.

A video was recorded for position change analysis using Tracker.[8] The experiments were conducted three times, with consistent results. The reference point was used consistently to determine the position of the object in each experiment as illustrated in Fig. 2. The position of each object was determined three times for each experiment. We used the average value of the object's position to determine the position of the CM of the system during the collision. The position of the CM of a system (x_{CM}) consisting of two objects m_1 and m_2 can be expressed mathematically by Eq. (1):

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}. \tag{1}$$

We can also provide the individual velocity of each object as well as the velocity of the CM before and after the collision by acquiring the slope of the position graph.

Plotting the position of the CM during a collision allows us to estimate the velocity of the CM (v_{CM}), which can be expressed mathematically by Eq. (2):

$$v_{CM} = \frac{\Delta x_{CM}}{\Delta t} = \frac{1}{m_1 + m_2}\left(m_1 \frac{\Delta x_1}{\Delta t} + m_2 \frac{\Delta x_2}{\Delta t} \right). \tag{2}$$

To estimate the velocity of the CM, we conducted the following procedures:

1. The three types of collision as illustrated in Fig. 1 were video recorded.
2. The video was then analyzed using Tracker to obtain the position data of each object before and after the collision.
3. The position of each object during the experiment was graphed vs. time.
4. The CM before and after the collision was estimated using Eq. (1).
5. The position of the CM during the experiment was graphed vs. time.
6. Using Eq. (2), the velocity of the CM was estimated by the slope of the CM position vs. time graph.

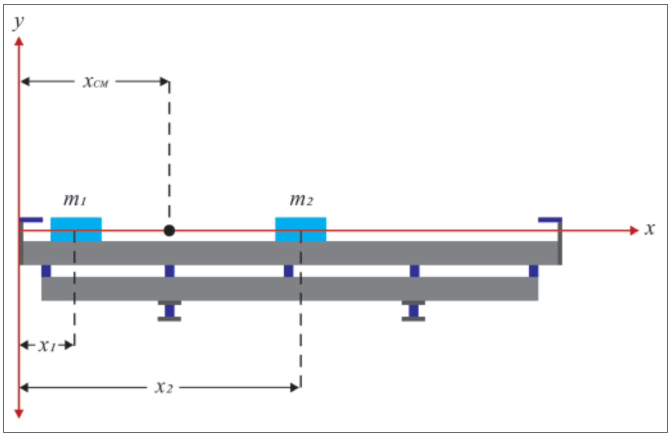

Fig. 2. Illustration of m_1 and m_2 position relative to the reference point.

Conveniently, the velocity of the CM can then be used to analyze total linear momentum in experiments. If the velocity of the CM is constant before and after the collision, the total momentum is conserved. On the other hand, if the velocity of the CM is changed after the collision, the total momentum is not conserved.

Results and discussion

Video analysis showed the position change of m_1 and m_2 before and after the collision in Experiment (i), as shown in Fig. 3(a). According to the analysis, object m_1 moved away from the reference point (positive x direction) with a velocity of $v_1 = 0.781 \pm 0.006$ m/s to object m_2 at rest ($v_2 = 0$). After the collision, object m_1 became stationary ($v_1' = 0$), and object m_2 moved at a speed of $v_2' = 0.336 \pm 0.001$ m/s farther away from the reference point. Using Eq. (1) and position data collected from Tracker analysis, we plotted x_{CM} vs. time during the collision in Experiment (i), as shown in Fig. 3(b). Using the graph plotted in Fig. 3(b), we found the slope of x_{CM} vs. time, which represents v_{CM}. Figure 3(b) shows that v_{CM} is constant before the collision, then changes when the two objects collide with each other, and returns to constant after the collision. The change in velocity of the CM of the two colliding objects is due to the presence of external forces—brief contact between the cars and the track, for example.

We modified both objects by attaching a Velcro adhesive to the collision surface of the objects in Experiments (ii) and (iii). Based on the video analysis, we have the position change of the objects before and after the collision in Experiment (ii) as shown in Fig. 4(a). Object m_1 moved away from the reference point with a velocity of $v_1 = 0.604 \pm 0.006$ m/s to object m_2 at rest ($v_2 = 0$). With the presence of Velcro on the surface of both objects, m_1 and m_2 adhere to each other.

As a result, after colliding, objects m_1 and m_2 move together at almost identical velocities, as $v_1' = 0.293 \pm 0.001$ m/s and $v_2' = 0.295 \pm 0.002$ m/s. Even though the two objects converge and move together after the collision, video analysis was still carried out for each object. However, the velocities as shown in the video analysis were not identical. This shows that motion analysis using video has uncertainty, like other measuring tools. The video analysis that has been carried out on each object can be viewed as repeated measurements. Thus, the mean values of velocity and uncertainty can be obtained as $\bar{v} = 0.294 \pm 0.001$ m/s, with a relative error of 0.3%. This low relative error demonstrates that video analysis shows precise results for an object's velocity measurement.

Using Eq. (1) and position data collected from Tracker analysis, we plotted x_{CM} vs. time before and after the collision in Experiment (ii), as shown in Fig. 4(b). From the graph plotted in Fig. 4(b), we found the slope of x_{CM} vs. time, which represents v_{CM}. According to Fig. 4(b), the slope of the graph is nearly identical before and after the collision. Thus, linear momentum was very nearly conserved, indicating that the external forces during the collision were much smaller in Experiment (ii) than in Experiment (i).

In Experiment (iii), both objects had Velcro attached, as in Experiment (ii), but here m_1 and m_2 traveled in opposite directions and collided with each other. Before the collision, object m_1 moved at a velocity of $v_1 = 0.239 \pm 0.001$ m/s, and object m_2 moved at a velocity of $v_2 = -0.280 \pm 0.001$ m/s. When colliding, objects m_1 and m_2 stuck together. After the collision, both objects united and moved with an average velocity of $\bar{v} = -0.0290 \pm 0.012$ m/s, with a relative error of 4.14%.

The momentum of m_2 before the collision is greater than m_1, so the CM moves in the negative x direction both before and after the collision, as shown in Fig. 5(b) with a negative curve gradient. The slope of the line was slightly changed before

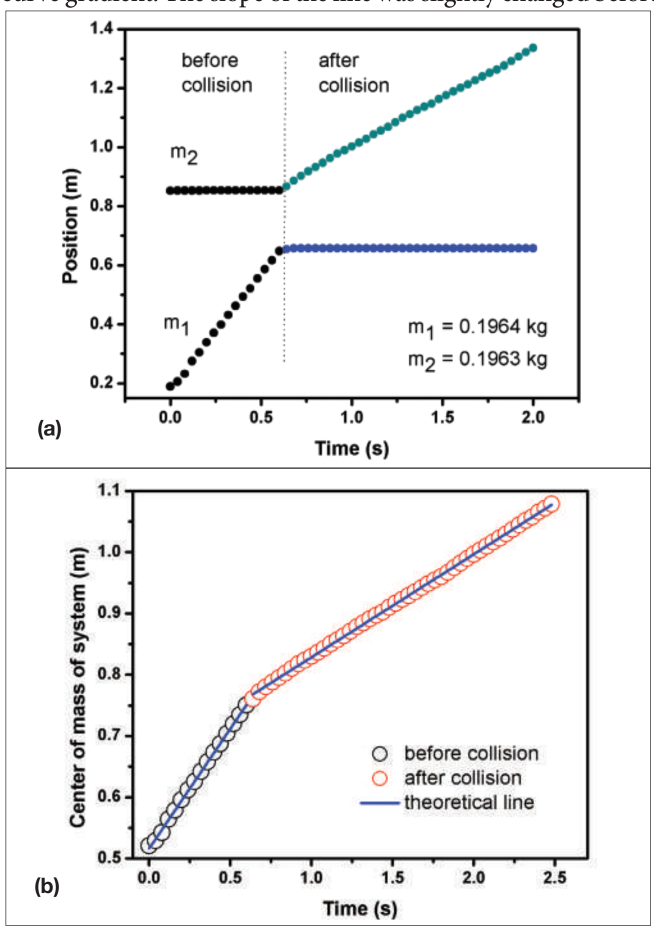

Fig. 3. Experiment (i): m_1 traveling with a constant velocity to m_2 at rest. (a) Position change of m_1 and m_2 before and after the collision. (b) CM position change before and after the collision.

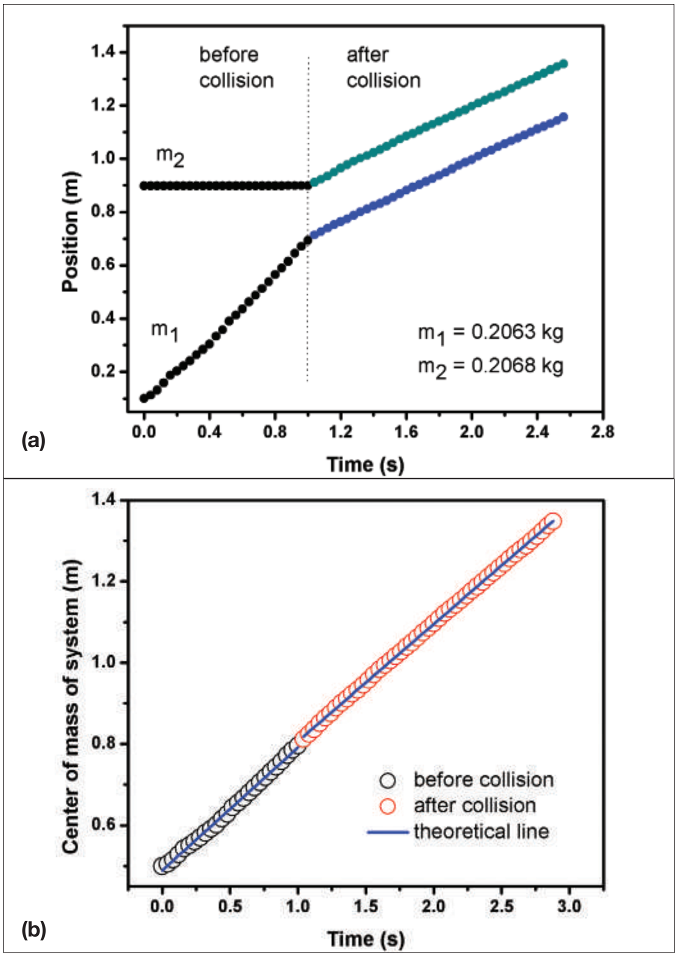

Fig. 4. Experiment (ii): both objects had Velcro attached, and m_1 traveled with a constant velocity to m_2 at rest. (a) Position change of m_1 and m_2 before and after the collision. (b) CM position change before and after the collision.

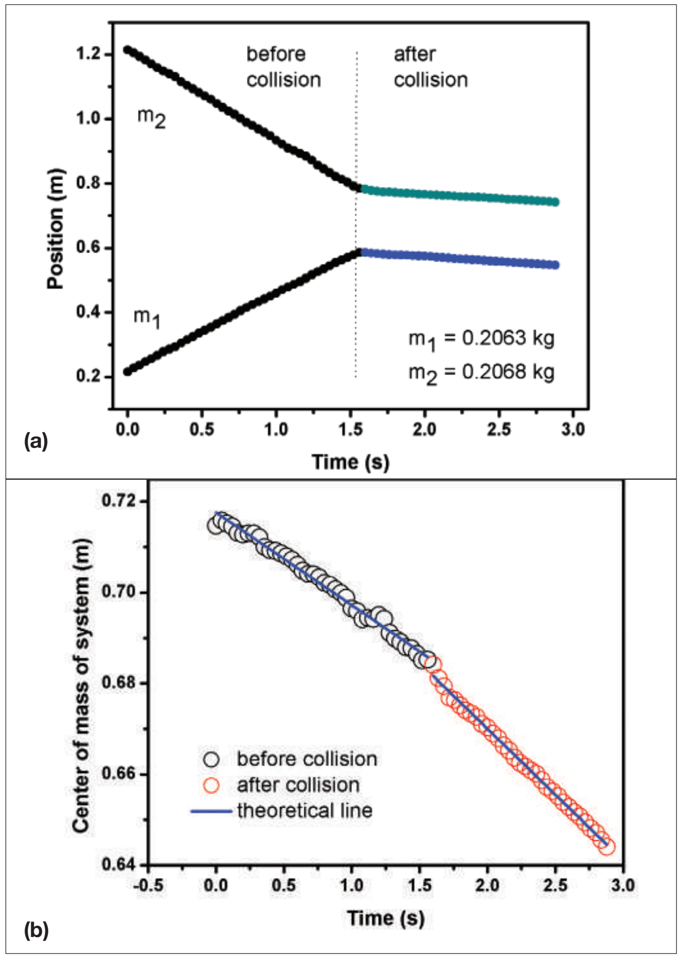

Fig. 5. Experiment (iii): both objects had Velcro attached, and the two objects m_1 and m_2 traveled in opposite directions and collided each other. (a) Position change of m_1 and m_2 before and after the collision. (b) CM position change before and after the collision.

and after the collision, indicating that a significant external force acted on the masses during the collision.

The v_{CM} obtained from Experiments (i)–(iii) is beneficial to simply determine the total linear momentum during the collision. We can determine the total momentum from the CM's velocity with the relationship expressed in Eq. (3):

$$(m_1 + m_2)v_{CM} = \sum p, \tag{3}$$

where Σp is the total linear momentum of the system. v_{CM} and total linear momentum before and after the collision from the three types of experiment are listed in Table I.

In Experiment (i), total linear momentum decreased significantly, by 57%. Table I shows that the velocity of the CM was different before and after the collision, so the total linear momentum was not conserved. In contrast to Experiment (i), the results of Experiment (ii) show only a small percentage change in the total linear momentum (2.6%). Hence, momentum was almost conserved for Experiment (ii). The result of Experiment (iii) has a percentage change in the total linear momentum during the collision of 39.53%. Thus, total linear momentum was not conserved in Experiment (iii); rather, an unexplained boost in the magnitude of momentum is evident.

Furthermore, besides the conservation of linear momentum, we can also analyze the conservation of kinetic energy, which is important to understand the collisions that occurred. The kinetic energy of each object must be calculated and compared before and after the collision. This calculation can be obtained from video analysis without involving the center of mass. The amount of kinetic energy of each object and the release of kinetic energy for each type of experiment are shown in Table II. Experiment (i) lost more total kinetic energy than the other two experiments. A "clink" sound was heard in Experiment (i), corresponding to a release of energy. In Experiments (ii) and (iii), the release of kinetic energy was observed to be lower due to the Velcro applied to the object. The total kinetic energy changed from before to after the collision in all three types of experiments. These results indicated that all three collision experiments were inelastic collisions.[9,10] Thus, the overall results of the analysis of the motion of the CM using video show congruence with the basic concepts of collisions.

The framework in this study can be a supplement to the analytical methods of experiment in the laboratory. Students can perform active observation through video recording followed by analysis using Tracker. Students are also able to present the data in graphical form and analyze it. The important result

Table I. The velocity of the CM and total momentum of two colliding objects before and after the collision.

Experiments	v_{CM}		$\sum p$			Is Momentum Conserved?
	Before Collision (m/s)	After Collision (m/s)	Before Collision (kg · m/s)	After Collision (kg · m/s)	Percent Change (%)	
(i)	0.391 ± 0.003	0.168 ± 0.001	0.153 ± 0.001	0.066 ± 0.001	−57.0	No
(ii)	0.302 ± 0.003	0.288 ± 0.001	0.125 ± 0.001	0.121 ± 0.001	−2.6	No (but almost)
(iii)	−0.0208 ± 0.0004	−0.0290 ± 0.0003	−0.0086 ± 0.0002	−0.0120 ± 0.0001	39.5	No

Table II. Total kinetic energy before and after collision for all type of experiments.

Experiment	Before Collision			After Collision			Total Kinetic Energy Lost (J)	% of Kinetic Energy Lost	Type of Collision
	v_1 (m/s)	v_2 (m/s)	Total Kinetic Energy (J)	v_1' (m/s)	v_2' (m/s)	Total Kinetic Energy (J)			
(i)	0.781 ± 0.006	0	0.060 ± 0.001	0	0.336 ± 0.001	0.0111 ± 0.0001	0.0487 ± 0.0001	81.5	Inelastic
(ii)	0.604 ± 0.006	0	0.0376 ± 0.0008	0.293 ± 0.001	0.295 ± 0.002	0.0178 ± 0.0002	0.0198 ± 0.0092	52.6	Inelastic
(iii)	0.239 ± 0.001	−0.280 ± 0.001	0.0140 ± 0.0001	−0.030 ± 0.001	−0.028 ± 0.001	0.0002 ± 0.00003	0.0138 ± 0.0001	98.8	Inelastic

in this study is a graphical interpretation of the motion of the CM to understand the nature of the collision. This allows students to explore basic concepts of physics related to collisions. We believe this study provides an easy way to determine the conservation of momentum from the graphical analysis of the movement of the CM. The student may already understand that constant velocity of the CM indicates that momentum is conserved. However, the technique demonstrated here also allows students to examine situations in which the velocity of the CM changes, indicating that total linear momentum is not conserved. The possible external force that caused the change of total linear momentum can be a fascinating topic of discussion for students. At a later stage, a similar framework is used to understand energy changes in a collision. Even when the momentum is very nearly conserved as in Experiment (ii), we are able to exhibit that the kinetic energy is still reduced dramatically, helping students to further distinguish these two quantities.

Conclusion

The CM of a system of colliding objects and its motion can be analyzed from a video recording. Analysis of the CM allows us to obtain the velocity of the CM. This parameter can then help analyze whether linear momentum is conserved or not and determine the total kinetic energy before and after the collision.

References

1. James Lincoln, "Enhancing physics demos using iPhone slow motion," *Phys. Teach.* **55**, 588–589 (2017).
2. Douglas Brown and Anne J. Cox, "Innovative uses of video analysis," *Phys. Teach.* **47**, 145–150 (2009).
3. Taylor Kaar, Linda B. Pollack, Michael E. Lerner, and Robert J. Engels, "Vocabulary and experiences to develop a center of mass model," *Phys. Teach.* **55**, 409–412 (2017).
4. Liya Kholida and Fourier Dzar Eljabbar Latief, "Investigation of linear momentum and impulse using video analysis," *Adv. Social Sci. Educ. Humanities Res.* **173**, 176–179 (2018).
5. Bambang Supriadi et al., "Study of central and non-central collisions in billiard games," *IOP Conf. Ser. Earth Environ. Sci.* **243**, 012022 (2019).
6. W. Klein and G. Nimtz, "Inelastic collision and the motion of the center of mass," *Am. J. Phys.* **57**, 182 (1989).
7. Zoe M. Leinhardt and Sarah T. Stewart, "Collisions between gravity-dominated bodies. I. outcome regimes and scaling laws," *Astrophys. J.* **745**, 1–27 (2012).
8. https://physlets.org/tracker/.
9. Lane Seeley and Eun-Hee Shin, "Colliding without touching: Using magnets and copper pipe fittings to explore the energetics of a completely inelastic collision," *Am. J. Phys.* **86**, 712–717 (2018).
10. Akihiro Ogura, "Analyzing collisions in classical mechanics using mass–momentum diagrams," *Eur. J. Phys.* **38**, 055001 (2017).

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