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On the Periodic Solution of an Oscillator Single Degree of Freedom with a Damping Coefficient that Changes Periodically

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Abstract. In this paper will be studied a simple one degree of freedom system related to the dynamics of cable-stayed bridges. As a result of wind and rain that hits the bridge cable, it will cause vibrations from the cable to become unstable conditions. The presence of water flowing in the cable changes the cable cross-section as experienced by the wind plane. So, this is what causes the instability of cable vibrations. The idea to model this problem is to consider a horizontal cylinder supported by springs in such a way that only one degree of freedom, i.e. vertical vibration is possible. It will be studied a ridge on the surface of the cylinder parallel to the axis of the cylinder. Additionally, let the cylinder with ridge be able to oscillate, with small amplitude, around the axis such that the oscillations are excited by an external force. In this paper it will be studied the possibilities of the damping of the oscillator that changes periodically by using a perturbation method.

1. Introduction

The mathematical model that describes vibration of the cables in vertical direction because of the galloping phenomenon system of differential equation becomes a. The characteristic of cables of cable-stayed bridges usually smooth polyurethane mantle and have a cross section which is nearly circular. In the rain season the water on the cable may induce aerodynamic instability resulting in vibrations with relatively large amplitudes. The Den Hortog's criteria should be considerate for modeling of vibration from a cable bridge. This criterion considers several assumptions that can cause a galloping phenomenon that is vibrations with small frequencies and large amplitude. (see [[1], [2]. [3], [4]]). The instability mechanism for this type of vibrations is known and can be understood on the basis of quasi-steady modeling and analysis. The experimental validation of a new energy harvesting system based on the wake galloping phenomenon has been studied by Jo Jung and Woo Lee [5].

Piccardo, et.al [1], Van Horssen, et.al, [2], and Waluya [3], [4] have studied oscillation of bridge cable because of wind induce vibration, while mechanism and characteristic of rain-induced vibration on high-voltage transmission line have been studied by Zhou et.al [6]. The dynamic of cable stayed bridges with a time periodic damping coefficient have been studied by Van Horssen, et.al [2] and Waluya [3], [4].

Many types of the oscillators have been studied by many researchers. Afzali, et.al [7] have studied Analysis of the Periodic Damping Coefficient Equation Based on Floquet Theory, Bayat, et.al [8] have studied Non-Linear Oscillation by series solution technique. On the separation of fast and slow motions in mechanical systems has been studied by Blekhman and Sorokin [9]. Analytical solution of strongly nonlinear Duffing oscillators has been studied by El-Naggar and Ismail [10]. Mechanism and characteristic of rain-induced vibration on high-voltage transmission line have been studied by Zhou et.al [6]. Von Wantoch et. Al [11] have studied Adaptive phasor control of a Duffing oscillator with unknown parameters. A novel method for the forced vibrations of nonlinear oscillators have been studied by En Du, et.al [12]. Ismail [13] has studied Analytical Technique for Solving Nonlinear Oscillators of the Motion of a Rigid Rod Rocking Bock and Tapered Beams. Many researcher have studied a various perturbation techniques to analyze nonlinear oscillators, such as a modified homotopy analysis method [14], Floquet Theory [7], Multiple-Scales Lindstedt-Poincaré Method [15], a modified homotopy analysis method [14], multiple time scales perturbation [2]. [3], [4], and many others perturbation technique. In this paper will be studied vibration of an oscillator due to small masses which periodically changing with slow and high frequencies. In [3], [4] used the assumption that external forces were not taken into account in the system, while in this paper will be studied that the force energy give significant impact to the vibration.

This paper is organized as follows. In section 2 of this paper it will be shown the mathematical model. The analysis of the solution will be studied in section 3 of this paper. Finally, in section 4 of this paper some conclusions will be drawn and some remarks will be made.

2. Mathematical Model

In this section will be derived a mathematical model which describe the rain-wind induced vibrations of a simple one degree of freedom system related to the dynamics of cable-stayed bridges. The model is an oscillator in one degree of freedom system consisting of a horizontal rigid cable supported by springs with a solid-state ridge moving with small amplitude oscillations. From the point of view, the type of equation of motion will arrive at a second order differential equation with external forcing. The Model Equation for Rain-Wind Induced Vibrations of a Prototype Oscillator is closely related to the quasi-steady approach as given in [2], [7].

Then the equation of motions can be derived by

$$m\ddot{y} + c_y\dot{y} + k_y y = F_y, \tag{1}$$

Where m is the mass of the cylinder, $c_y > 0$ the damping coefficient of the mechanical structure and $k_y > 0$ the constant linear spring. $\dot{y}$ represents the velocity of the system. Set $v = \dot{y}$, and by using aerodynamic force ($F_y = -(D\sin\phi + L\cos\phi)$) then equation (1) becomes.

$$m\ddot{y} + c_y\dot{y} + k_y y = -\frac{1}{2}\rho dl\sqrt{U^2 + \dot{y}^2}(C_D(\alpha)\dot{y} + C_L(\alpha)U) \tag{2}$$

By using the following new variables and parameters. After linearizing the equation (2) (see also [6], [7] for more details reason), we can obtain :

$$\ddot{Z} + (2\beta + KC_{D0} + KC_{L1})\dot{Z} + Z = -KC_{L0} \tag{3}$$

By using some assumptions an according to the galloping phenomena after using transformation we can obtain

$$Z + \ddot{Z} = -Kf(t)[C_{L1} + C_{L3}f(t)^2] + [-2\beta - KC_{D0} - 3KC_{L3}f(t)^2 - KC_{L1}]\dot{Z} \tag{4}$$

Let $f(t) = B\sin(\varphi\tau)$, $\varphi = \frac{\omega}{\omega_y}$, then from equation (4) is obtained :

$$\ddot{Z} + Z = \left[-2\beta - KC_{D0} - \frac{3}{2}KC_{L3}B^2 + \frac{3}{2}KC_{L3}B^2\cos(2\varphi\tau) - KC_{L1}\right]\dot{Z} + \left[-KBC_{L1} - \frac{3}{4}KB^3C_{L3}\right]\sin(\varphi\tau) + \frac{1}{4}KB^3C_{L3}\sin(3\varphi\tau) \tag{5}$$

Set $\varphi = 1 + \varepsilon\eta$, $\varepsilon \ll 1$ and $(1 + \varepsilon\eta)\tau = \phi$, then equation (10) becomes :

$$(1+\varepsilon\eta)\ddot{Z}+Z=(1+\varepsilon\eta)\Big[-2\beta-KC_{D0}-\tfrac{3}{2}KC_{L3}B^2 +\tfrac{3}{2}KC_{L3}B^2\cos(2\phi)-KC_{L1}\Big]\dot{Z} +\Big[-KBC_{L1}-\tfrac{3}{4}KB^3C_{L3}\Big]\sin(\phi)+\tfrac{1}{4}KB^3C_{L3}\sin(3\phi), \quad (6)$$

with $\dot{Z}=\frac{dZ}{d\phi}$. Let K and β are small, that are $K=\beta=O(\varepsilon)$ or $K=\varepsilon K_1, \beta=\varepsilon\beta_1$, and by neglecting $O(\varepsilon^2)$, then it will be obtained from (6) :

$$\ddot{Z}+Z=\varepsilon\big[-(a_0+a_1\sin(2\phi))\dot{Z}-a_2 sin(\phi)-a_3\sin(3\phi)\big], \quad (7)$$

with

$$a_0=2\beta_1+K_1C_{D0}+\frac{3}{2}K_1C_{L3}B^2+K_1C_{L1}\;\; a_1=-\frac{3}{2}K_1C_{L3}B^2,$$
$$a_2=K_1BC_{L1}+\frac{3}{4}K_1B^3C_{L3},\;\; a_3=-\frac{1}{4}K_1B^3C_{L3}$$

3. Solution of The Model

In this section will be studied periodic solution of equation (7). The averaged system of equation (7) can be written as (see how to averaged a system in papaers [6], [7], [8] :

$$\dot{\overline{y_1}}=\varepsilon\Big[-\tfrac{1}{2}a_0y_1+\tfrac{1}{2}a_2-\eta y_2+\tfrac{1}{4}a_1y_1\Big]$$
$$\dot{\overline{y_2}}=\tfrac{\varepsilon}{4}[a_1y_1-2a_0y_2+4\eta y_1], \quad (8)$$

The critical points of equation (8) can be given by :

$$\left(\frac{4a_0a_2}{16\eta^2+4a_0^2-a_1^2},\frac{2a_2(a_1+4\eta)}{16\eta^2+4a_0^2-a_1^2}\right) \quad (9)$$

The existence of the critical point equation (9) is given by $16\eta^2+4a_0^2-a_1^2\neq 0$. If the determinant of the coefficient matrix of equation (8) is not equal to zero (more details see [2] for proof) then system equation (7) has a periodic solution and its stability depends on the eigenvalues of the coefficient matrix. The eigenvalues equation (8) can be given by :

$$\lambda_1=-\tfrac{1}{2}a_0+\tfrac{1}{4}\sqrt{a_1^2-16\eta^2},$$
$$\lambda_2=-\tfrac{1}{2}a_0-\tfrac{1}{4}\sqrt{a_1^2-16\eta^2} \quad (10)$$

The periodic solution can be give by :

$$z=y_1[0]\cos(\phi)+y_2[0]\sin(\phi)$$
$$\dot{z}=-y_1[0]\sin(\phi)+y_2[0]\cos(\phi), \quad (11)$$

with

$$y_1[0]=\frac{4a_0a_2}{16\eta^2+4a_0^2-a_1^2}, y_2[0]=\frac{2a_2(a_1+4\eta)}{16\eta^2+4a_0^2-a_1^2}$$

Plot of the solution approximation and numerical result by using Runge-Kutta method for an initial condition and set of parameters can be given in Figure (1)-Figure (3).

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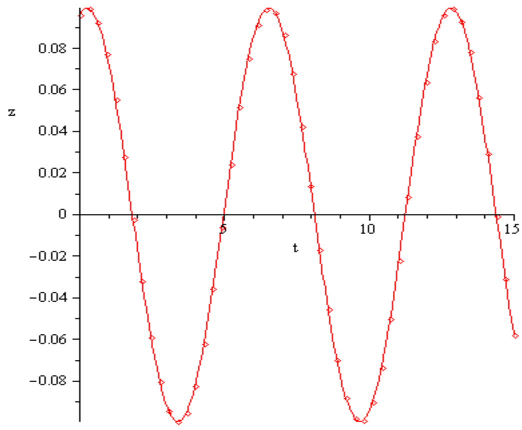

Figure 1. Plot solution approximation versus numeric for $a_0 = 1, a_1 = 0.1, a_2 = 0{,}1. a_3 = 0.1, \varepsilon = 0.01, z(0) = 1, \dot{z}(0) = 0$

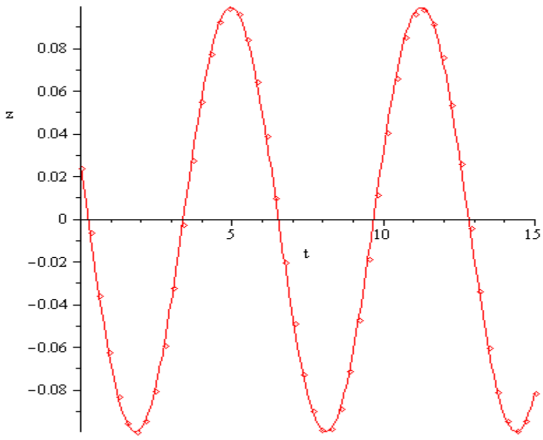

Figure 2. Plot solution approximation versus numeric for $a_0 = 1, a_1 = 0.1, a_2 = 0{,}1. a_3 = 0.1, \varepsilon = 0.01, z(0) = 1, \dot{z}(0) = 0$

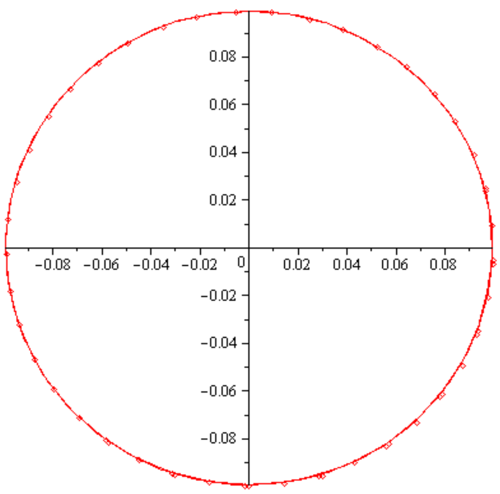

Figure 3. Plot phase portrait approximation versus numeric for $a_0 = 1, a_1 = 0.1, a_2 = 0{,}1. a_3 = 0.1, \varepsilon = 0.01, z(0) = 1, \dot{z}(0) = 0$

From Figure (1) and Figure (3) it can be seen that the approximation of the solution by using perturbation technique also similar with the solution by numerical calculation. From equation (10) can be shown that the stability of critical points (related to periodic solution) if $a_1^2 < \eta^2$, that is the damping coefficient bigger then frequency of the function which respresent of water ridge.

4. Conclusion

In the previous section has been discussed a mathematical model which describe the rain-wind induced vibrations of a simple one degree of freedom system related to the dynamics of cable-stayed bridges. Those vibrations are assumed in the vertical direction due to the galloping phenomena, that is vibrations with small frequencies and large amplitude. The critical points (periodic solutions) can be obtained if the damping coefficient more than the frequency of the function which represent of water ridge. And the periodic solution can be obtained if the damping coefficient bigger then frequency of the function which represent of water ridge.

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