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Fuzzy shortest path approach for determining public bus route (Case study : Route planning for "Trans Bantul bus" in Yogyakarta, Indonesia)

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Abstract

Fuzzy graph became a nice tool for modeling and designing a network that contains indeterminacy. Public transportation has become a necessity in Bantul Regency because it is one of the regions that became popular in Yogyakarta Special Region Province (YSR), Indonesia. There are many tourism sectors in Bantul that became favourite destinations in recent years. However, we cannot obtain convenient public transportation that connects the favourite destinations in Bantul. Therefore, it is needed a bus rapid transit (BRT) in Bantul which will be called as "Trans Bantul". The problem is how to determine optimal routes that connect public facilities in Bantul. In the real problem cases, a route between two places contains indeterminate parameters, such as distance, time, and cost. Hence, it is suitable to represent the bus stops and all possible roads in a fuzzy network, especially fuzzy weighted network. In this research, we use fuzzy shortest path approach to find optimal routes for

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public bus. We design an algorithm to determine optimal routes based on fuzzy shortest path algorithm. We construct a Matlab code according to a combination of Chuang-Kung and Yadav-Biswas algorithms. The weights on the network are the distances between two bus stops which are represented in triangular fuzzy numbers and we use the code to find the optimal routes. We have implemented the algorithm for route planning of the BRT "Trans Bantul" in Yogyakarta, Indonesia. The experimental results show that we can use three routes where the shortest distance in route 1 is 30.8 Km; the shortest distance in route 2 is 26.8 Km; and the shortest distance in route 3 is 20.8 Km. Further, the information of the bus routes is displayed in the Matlab GUI.

Subject Classification: *Primary 05C38, Secondary 05C90.*

Keywords: *Shortest path, Chuang-Kung algorithm, Yadav-Biswas algorithm, Fuzzy network, Triangular fuzzy number.*

1. Introduction

The growth of the Yogyakarta Special Region Province (YSR) in terms of education, economy, tourism, and others is not only concentrated in the city of Yogyakarta but also in several districts in the province. Specifically, Bantul Regency is currently one of the districts that grows rapidly. The fields of economy, education, and tourism are growing there, and are predicted to thrive rapidly in the coming years. There are many tourism sectors in Bantul, such as beaches, pinery, art markets, souvenirs, etc [22].

The development of the Yogyakarta Special Region Province (YSR) must be balanced with the development of public transportation facilities. One of the public transportations in Yogyakarta is the Bus Rapid Transit (BRT) "Trans Jogja" that gave a positive impact on transportation problems in Yogyakarta [7]. Trans Jogja which was first operated in 2008, has increased its services both in terms of quantity and quality [5]. Since Bantul Regency becomes a favorite destination for tourists not only from Java island but also from other islands in Indonesia, an effective public transportation becomes a necessity in Bantul. Hence, the government needs to plan a better system for the public transportation. Following the BRT "Trans-Jogja" ([5], [7]), we propose a convenient public transportation in Bantul which we call BRT "Trans-Bantul".

A route between two places involves fuzzy parameters, such as distance, time, and cost. Therefore, it is suitable to represent the transport route in a fuzzy network, especially a fuzzy weighted network (FWN). One of the problems arising in an FWN is finding the shortest path with fuzzy length, we call it the fuzzy shortest path. The concept of shortest path in the FWN has been proposed by many researchers. Deng et al. [4] developed

Dijkstra algorithm in fuzzy network. Recently, Pešić et al. [14] proposed Fuzzy Floyd's algorithm to find the shortest path in the FWN. Meanwhile, Chuang and Kung [3] and also Yadav and Biswas [20] gave algorithms to find the fuzzy shortest path length. For more relevant results on shortest path and fuzzy networks, readers may refer to [1,8,10,13,14,16-19].

Several applications of shortest path in transport routes have been studied. Jariyasunant et al. proposed an application of optimal path in public transit network [9]. Bozyiğit et. al. have modified Dijkstra algorithm for determining a public transport route [2]. An application of shortest path for public transportation using GPS and Map Service was proposed in Setiawan et al. [20]. Further, an application of fuzzy optimal path algorithm for bus route expansion was investigated by Duong et al. [6]. Moreover, an optimum public transportation route for tourism using shortest path in the FWN has been proposed in [12]. Hence, there are not many papers that discussed fuzzy shortest path for determining the optimal route of public bus.

Based on the above literatures and due to the indeterminate phenomena in the real problem cases, we get the fact that one of the effective ways of determining a public bus route is by finding a fuzzy shortest path. Therefore, we focus on developing a method of computation of finding an optimal route for BRT "Trans-Bantul" using fuzzy shortest paths in the FWN. Firstly, we construct an algorithm to find the routes based on a combination of Chuang-Kung algorithm and Yadav and Biswas algorithm. Secondly, we design an application via Matlab GUI for presenting information related to the routes of the buses. We hope that the result could be a recommendation for realizing the BRT "Trans-Bantul" in Yogyakarta, Indonesia.

2. Materials and Methods

We recall some materials used to solve the problems, i.e triangular fuzzy numbers, graded mean value of fuzzy numbers, fuzzy shortest path length (FSPL) procedure based on Chuang-Kung and Yadav-Biswas algorithms (Table 1 and Table 2). Further, the methods used in this research are described in this section.

2.1 *Triangular Fuzzy Number*

A fuzzy set $\tilde{A}$ on X is defined as a set $\tilde{A} = \{(a, \mu(a)) \mid a \in X\}$ with a membership function $\mu : X \to [0,1]$. Further, the classical sets are named as crisp sets [17]. The support of $\tilde{A}$, denoted by $S(\tilde{A})$, is the crisp set given by $S(\tilde{A}) = \{x \in X \mid \mu(x) > 0\}$. Let $\alpha \in (0,1]$, the α-cut of the fuzzy set $\tilde{A}$ is a crisp set $A_\alpha = \{x \in X \mid \mu(x) \geq \alpha\}$. The height of $\tilde{A}$, denoted by $h(\tilde{A})$, is a set $h(\tilde{A}) = \sup\{\mu(a) \mid a \in X\}$. If $h(\tilde{A}) = 1$, then $\tilde{A}$ is called a normal fuzzy set, otherwise it is called a sub normal one [3].

A fuzzy set $\tilde{A}$ on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number if the following conditions are satisfied: " $\tilde{A}$ is normal, i.e., there is $a \in R$ such that $\mu(a) = 1$, A_α is a closed interval for every $\alpha \in (0,1]$, the support of $\tilde{A}$ is bounded". A triangular fuzzy number α is a triplet (a_1, a_2, a_3) with a membership function as follows [3]:

$$\mu_\alpha(x) = \begin{cases} 0 & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & a_1 < x < a_2 \\ \dfrac{x - a_3}{a_2 - a_3} & a_2 < x < a_3 \\ 0 & a_3 < x. \end{cases}$$

Let $\alpha = (a, b, c)$ and $\beta = (p, q, r)$ be two triangular fuzzy numbers. The sum α and β is a set $\alpha + \beta = (a + p,\ b + q,\ c + r)$. Meanwhile, a graded mean integration representation of α is as follows [14]:

$$P(\alpha) = \frac{a + 2b + c}{6}.$$

2.2 *Some Basic Concepts in Graph Theory*

Let $G(V, E)$ be a graph with a nonempty vertex set $V = V(G)$ and an edge set $E = E(G)$. An edge e in G is an unordered pair $e = (u, v) = uv$ of vertices of G. A path P from v_0 to v_n in G is finite sequence of vertices and edges in G in the form of v_0, $e_1, v_1, e_2, v_2, \ldots, v_{n-1}, e_n, v_n$ such that $e_1 = v_0 v_1$, $e_2 = v_1 v_2, \ldots, e_n = v_{n-1} v_n$ and no vertices in P are repeated [16].

If each edge in a graph G is assigned a numerical weight (which is usually taken to be positive integer), then G is called a weighted graph. A shortest path in a weighted graph is a path where the sum of the weights of edges is minimum [15]. A fuzzy weighted graph (network) in this research is graph (network) with fuzzy weights which are triangular fuzzy numbers. A fuzzy shortest path (FSP) is a shortest path in a fuzzy weighted graph with a minimum weight. Meanwhile, the fuzzy shortest path length (FSPL) is the weight of a fuzzy shortest path ([3],[21]).

2.3 *Chuang-Kung Algorithm to Calculate the FSPL*

The Chuang-Kung algorithm [3] to compute the FSPL is displayed in Table 1.

Table 1

Chuang-Kung Algorithm

Input	Length $\boldsymbol{L_i} = (\boldsymbol{a_i}, \boldsymbol{b_i}, \boldsymbol{c_i}),\ i = 1,2,\ldots,m$
Output	$\boldsymbol{L_{min} = (a,b,c)}$
Steps	1. **Form** the set $Q = \{Q_1, Q_2, \ldots, Q_m\}$ based on ordering $\boldsymbol{L_i}$ in ascending order of b_i where $Q_i = (a_i', b_i', c_i')$ for $i = 1,2,\ldots,m$. 2. **Set** $\boldsymbol{L_{min} = (a,b,c)} = Q_1 = (a_1', b_1', c_1')$. 3. **Set** i=2. 4. **Count** $new\_a = \min(a, a_i')$; $new\_c = \min(c, b_i')$. 5. if $b \le a_i'$ 6. $new\_b = b$ 7. else 8. $new\_b = \dfrac{(b \times b_i') - (a \times a_i')}{(b + b_i') - (a + a_i')}$ 9. endif 10. Set $a = new\_a, b = new\_b, c = new\_c,$ 11. **Set** FSPL $\boldsymbol{L_{min} = (a,b,c)}$. 12. Set $i = i + 1$. 13. **While** $i < m + 1$, repeat steps 4 until 12.

2.4 *Yadav-Biswas Algorithm to Find the Shortest Path*

To find an optimal route for the public bus, we use the shortest path algorithm from Yadav and Biswas [21] as described in Table 2.

Table 2

Yadav-Biswas Algorithm

Input	Initial_node=I and target_node=T
Output	The shortest path
Steps	1. Find all possible path P_i $(i = 1,2,\ldots,m)$ from I to T and compute the distance L_i by using sum of triangular fuzzy numbers. 2. Calculate the FSPL $\boldsymbol{L_{min}}$ by using the algorithm in Table 1. 3. Count the Euclidean distance d_i between the distance $\boldsymbol{L_i} = (\boldsymbol{a_i}, \boldsymbol{b_i}, \boldsymbol{c_i}),\ i = 1,2,\ldots,m,$ and $\boldsymbol{L_{\mathbf{min}} = (a,b,c)}$ where $d_i = \sqrt{(a_i - a)^2 + (b_i - b)^2 + (c_i - c)^2}$ 4. Choose the shortest path with a smallest value d_i

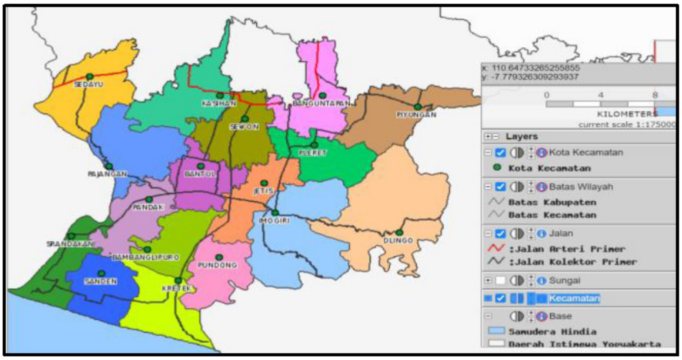

Figure 1
The map of Bantul Regency [22]

2.5 *General Information of Bantul Regency*

The Bantul Regency is located on 07° 44′ 04″-08° 00′ 27″ South latitude and 110° 12′ 34″-110° 31′ 08″ East longitude. The capital of Bantul Regency is located in Bantul sub-district. In 2019, the population of Bantul Regency is 1.006.692 inhabitants. The area of Bantul Regency is 508.85 Km2 (15,905 percentage of the area of YSR) and it is divided into 17 sub-districts, i.e., "Srandakan, Sanden, Kretek, Pundong, Bambanglipuro, Pandak, Bantul, Jetis, Imogiri, Dlingo, Pleret, Piyungan, Banguntapan, Sewon, Kasihan, Pajangan, and Sedayu". The map of Bantul Regency is given in Figure 1.

2.6 *The Method Used*

The steps used in this research are as follows.

1. Select priority locations that will be passed by the bus based on observation via Google-maps. In the study case, we select priority locations in Bantul Regency, Yogyakarta, Indonesia.
2. Select locations for bus stops in each route.
3. Set bus stops as nodes and all possible roads between two bus stops as edges in a graph $G(V,E)$.
4. Set initial and target nodes, symbolized as I and T. In this research, the initial node is I= "Giwangan" bus stop. The target nodes are "Baru" beach, "Parangtritis" bus stop, and "Dlingo" sub-district.

5. Assume that there are no one direction ways which means that a route from X to Y is equal to a route from Y to X. Find out roads between two bus stops and the associated distances (weights).
6. Fuzzify data of weights.

   The weights are represented as triangular fuzzy numbers in the following way:

   a. Find out all possible roads between two locations and consider the distances between the locations.

   b. Represent a distance between two locations into triangular fuzzy number (m_1, m_2, m_3) where m_2 has the degree 1, i.e., is the most probable shortest distance; m_1 is the smallest likely value of the distance; and m_3 is the largest possible value of the distance between the locations.
7. Get a fuzzy weighted network.
8. Determine the Fuzzy Shortest Path Length (FSPL) in the network and the shortest path between initial node I and target node T by combining algorithms in Table 1 and Table 2, respectively.
9. Transform the fuzzy distance of the shortest path into crisp number using the graded mean value (defuzzification).
10. Design a Matlab GUI application for presenting the fuzzy weighted network and the shortest route.

3. Results and Discussions

The results obtained in this research are presented in this section. We construct an algorithm to find optimal route (route with shortest distance) of the public bus as shown in Table 3.

3.1 *An Algorithm for Finding Optimal Routes of the Public Bus*

Table 3

Algorithm for Determining Shortest Route of Public Bus

Input	Initial_node=I and target_node=T
	V={v_1,v_2,..., v_{nv}} % The set of bus stops (nodes) between I and T, nv is the number of bus stops
	E={e_1,e_2,...,e_{ne} | e=(v_i,v_j),i≠j,i,j=1,...,nv} % The set of connected bus stops, ne is the number of connected bus stops
	m_1={w_{11},w_{12},…,w_{1ne}} % the set of smallest likely value of the distance between two bus stops
	m_2={w_{21},w_{22},…,w_{2ne}} % the set of the most probable shortest distance between two bus stops
	m_3={w_{31},w_{32},…,w_{3ne}} % the set of the largest possible value of the distance between two bus stops
Output	The shortest path SP with the distance *L*
Steps	1. Count nE=length(E); nm2=length(m2) 2. Count n=length(V) 3. Fuzzify the distance by using the function *weight* (m1,m2,m3,nm2) in Figure 2 4. for i=1:nE 5. s(i)=E(i,1) 6. t(i)=E(i,2) 7. endfor 8. Set G=graph(s,t) and A=adjacency (G) % define a graph with *s* vertices and *t* edges and adjacent matrix of G 9. Set function Path=AllPath (A,I,T) % determine all possible path $P_i\,(i=1,2,\dots,m)$ in *G* by using the function *AllPath* [11] 10. Count length L_i of each path P_i in *G* by using the function *sum_tfn* in Figure 3 11. Count The FSPL $\boldsymbol{L_{min}}$ by using the algorithm in Table 1 12. Count The Euclidean distance d_i by using the algorithm in Table 2. 13. Choose a shortest path SP from I to T that is a path with a smallest distance d_i by using the function check_SP in Figure 4. 14. Convert the fuzzy distance *L* = (*a*, *b*, *c*) of the shortest path SP into crisp number by using graded mean value $L=\frac{a+2b+c}{6}$ 15. Present the shortest route of the bus via Matlab GUI.

We create a Matlab program file for the algorithm in Table 3. The function "*weight*" in Step 3 is given in Figure 2.

```
function [W] = weight(m1,m2,m3,nm2)
for i = 1:nm2
    W(i,1:3)=[m1(i) m2(i) m3(i)]
end
end
```

Figure 2

A function "*weight*" for fuzzifying the data of distances

Further, we create a function *sum_tfn* (in Figure 3) to calculate the length of each path based on the sum of triangular fuzzy numbers in Step 10.

```
sum_tfn.m
function [L,L11,L12,L13] = sum_tfn(A)
    L11=sum(A(:,1));
    L12=sum(A(:,2));
    L13=sum(A(:,3));
    L=[L11 L12 L13];
end
```

Figure 3

A function "*sum_tfn*" for calculating the length L_i of each path P_i

```
function [shortest_dist,d,shortestpath] = cek_SP(Min_dist,L_min1,dist_L_min1_LLMIN,dist_P1_LLMIN,LI,LII,LIII,Y1,Y2,Y3);
if (isequal(Min_dist,dist_L_min1_LLMIN)==1)
        shortest_dist=L_min1;
        d=(shortest_dist(1)+4*shortest_dist(2)+shortest_dist(3))/6;
        if L_min1==LI
        shortestpath=Y1;
        elseif L_min1==LII
         shortestpath=Y2;
        else
           shortestpath=Y3;
        end
elseif (isequal(Min_dist,dist_P1_LLMIN)==1)
        shortest_dist=P1;
        d=(shortest_dist(1)+4*shortest_dist(2)+shortest_dist(3))/6 ;
        if L_min1==LI | L_min1==LII
             shortestpath=Y3;
        elseif L_min1==LII | L_min1==LIII
             shortestpath=Y1;
        else
            shortestpath=Y2;
        end
else
        shortest_dist=P2
        d=(shortest_dist(1)+4*shortest_dist(2)+shortest_dist(3))/6;
        if L_min1==LI & P1==LII
             shortestpath=Y3;
        elseif L_min1==LI & P1==LII
             shortestpath=Y2;
        else
            shortestpath=Y1;
        end
end
```

Figure 4

A function "*check_SP*" for selecting the shortest path.

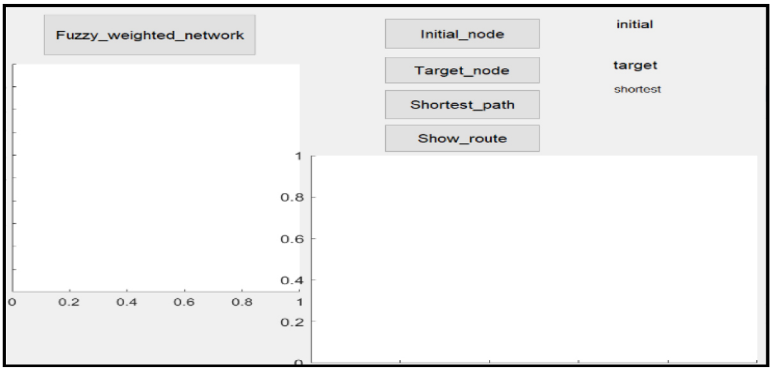

Figure 5

Main menu in the Matlab GUI.

Moreover, We also construct a function *check_SP* for selecting the shortest path (*SP*) in Step 13 as presented in Figure 4.

3.2 *A Matlab GUI for Presenting the Bus Route*

An application based on Matlab GUI has been designed for presenting the shortest route for the BRT "Trans Bantul". The information displayed in the GUI consists of the fuzzy weighted network, an initial node, a target node, a shortest path, and the shortest route in Google Maps. The main menu in the GUI is displayed in Figure 5.

3.3 *Experimental Results*

Firstly, we select the priority locations in the map of the Bantul Regency (in Figure 1) that will be passed by the bus. A node in the graph model represents a bus stop and there is an edge between two nodes if there is a road which connects the two bus stops. Meanwhile, an edge-weight represents a distance between two bus stops. Sometimes there is an indeterminacy of the distance data so that we convert the data into triangular fuzzy numbers.

The initial node in all routes of the BRT "Trans Bantul" bus is "Giwangan" bus station, that is the biggest bus stop in Bantul. We decide three destination nodes for the BRT "Trans Bantul" because the routes can cover many public facilities as follows:

1. Route 1: from "Giwangan" bus station to "Baru" beach (round trip).
2. Route 2: from "Giwangan" bus station to "Parangtritis" bus stop (round trip).

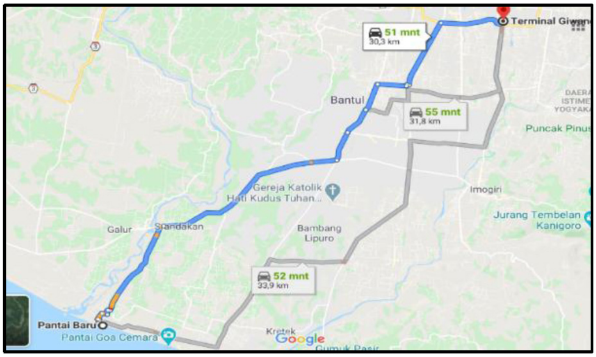

Figure 6

Possible roads from Giwangan to Baru beach.

3. Route 3:from "Giwangan" bus station to Dlingo sub-district (round trip).

3.3.1. The shortest path in route 1

"Baru" beach is one of the popular tourist destinations in Bantul Regency. Therefore, it is needed a public bus to reach the beach from "Giwangan" bus station. All possible roads in route 1 are provided in Figure 6.

Based on observation via Google-maps, we choose some locations for bus stops in route 1 as presented in Table 4. The selection of the bus stop is based on the topography of the locations, closeness to the public facilities, and traffic densities around the locations which are analyzed from Google-maps.

We describe the way to convert the data of distances into triangular fuzzy numbers. In Figure 7, we present some locations of bus stops in route 1. The steps to fuzzify the data of distances are described below.

1. Determine the smallest likely value of the distance (m1). For example, the most probable shortest distance of (1-2) is 3.7 Km and and the largest possible distance of (1-2) is 4.9 Km. Hence. we set the m1 of (1-2) is 2.5 Km (see Figure 7, A). Further the most probable shortest distance of (2-4) is 4.5 Km and the largest possible distance of (2-4) is 5.2 Km, therefore the m1 of (2-4) is 3.8 Km (see Figure 7,B). By using the same way, the m1 of (4-6): 3 Km (see Figure 7,C).
2. Determine the most probable shortest distance (m_2). For example, the distance m_2 of (1-2) is 3.7 Km; the value m2 of (2-4) is 4.5 Km; and distance m_2 of (4-6) is 3.3 Km (see blue lines in Figure 7).

Table 4

The Bus Stops in Route 1

Code	Location of Bus Stop
1	Giwangan bus station
2	Alfamart minimarket, Jl. Parangtritis Km 4.5, Sewon
3	Pleret 1 Vocational School
4	Post office, Jl. Cepit Tembi
5	Jetis 3 Middle School
6	Bantul market
7	Bambanglipuro Public health center
8	Respira hospital
9	TPR Samas beach, Sanden
10	Srandakan 1 High School
11	Baru beach bus stop

3. Determine the largest possible value of the distance (m_3). For example, distance m_3 of (1-2) is 4.9 Km; the value m_3 of (2-4) is 5.2 Km; and distance m_3 of (4-6) is 3.6 Km (see red lines in Figure 7).

4. Use the function weight (W) in Step 3 to fuzzify the distances (1-2), (2-4), and (4-6) in triangular fuzzy numbers as follows:

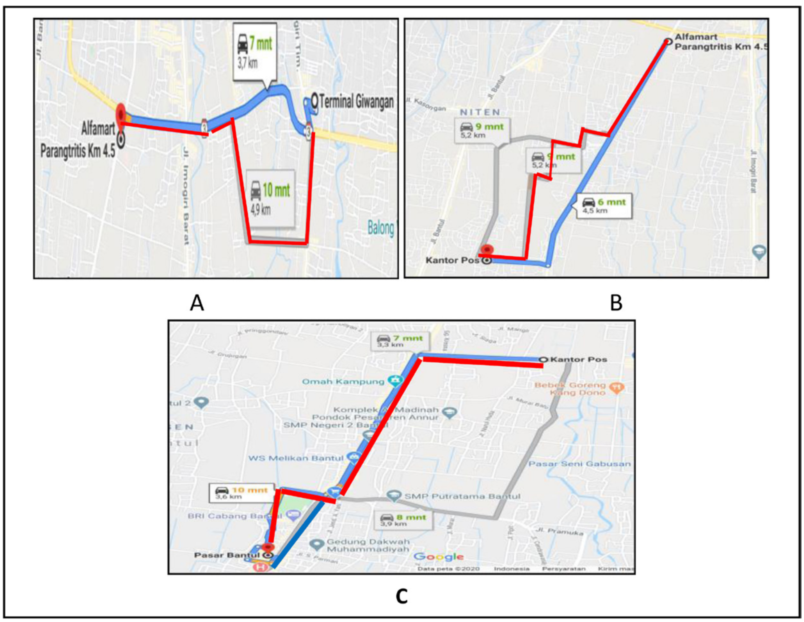

Figure 7

Possible roads between nodes 1-2 (A); 2-4 (B); 4-6 (C).

W(1-2) = (2.5 3.7 4.9), W(2-4) = (3.8 4.5 5.2), and
W(4-6) = (3.0 3.3 3.6).

According to the algorithm, we get all possible paths from "Giwangan" bus station to "Baru beach", their fuzzy distances and the corresponding fuzzy shorthest Path lengths (FSPL), L_{min}. All possible paths and the corresponding distances which are computational results of the Steps 9-11 (in Table 3) are shown in Table 5 below.

Table 5

All Possible Paths and the Related Distances in Route 1

Paths	Distances			L_min		
1-3-5-7-9-11	32.7	34	35.3	25.8	30.8	35.8
1-2-4-6-8-10-11	25.8	30.8	35.8	25.8	29.716	32.3
1-3-4-6-8-10-11	29	32.3	35.6	25.8	29.716	32.3

Step 12-14 give the FSPL L_{min} = (25.8000 29.7157 32.3000) and the Euclidean distances between each path and the L_{min} are as follows:

$$d(P_1, L_{\min}) = 5.2734;\ d(P_2, L_{\min}) = 3.6641;\ d(P_3, L_{\min}) = 8.6583.$$

It is shown that the path P_2:1-2-4-6-8-10-11 is the shortest path to reach the beach from Giwangan bus station with the shortest distance around (25.8000 30.8000 35.8000) Km. By using graded mean value, we get the crisp value of the distance:

$$d = \frac{(25.8 + 4 \times 30.8 + 35.8)}{6} = 30.8 \text{ Km}.$$

Conversely, we can use route P_1: 11-9-7-5-3-1 to go back to Giwangan from Baru beach with the shortest distance of 34 Km. By using the assumption that the average speed of bus is 30 Km per hour, the travel time for path P_2 is around 1.03 hours (Giwangan bus station-Baru beach) and the travel time for path P_1 is around 1.13 hours. The total travel times for the round-trip meet the standard travel time for public buses, i.e., 2 hours. Both routes can be seen in Google maps through the following links: https://goo.gl/maps/ZH8TgBGJjetFquPM6 and https://goo.gl/maps/rvJYgchz6vKo72HQ6.

Information related to shortest path in route 1 is displayed in Figure 8.

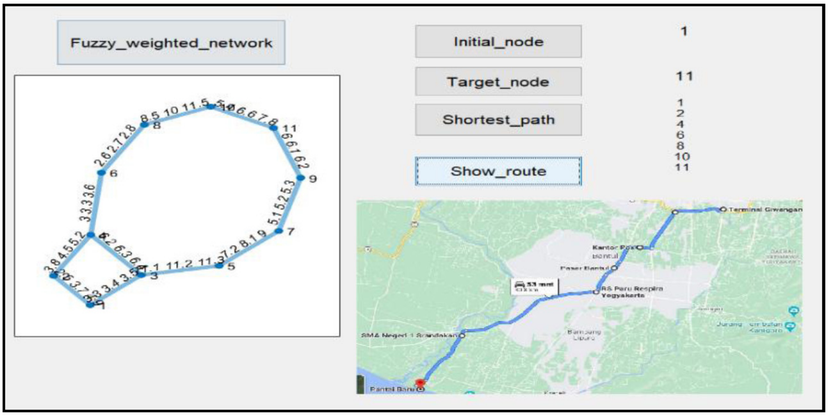

Figure 8

The shortest path in route 1 displayed in the Matlab GUI.

3.3.2 The shortest path in route 2

"Parangtritis" is one of the famoust destinations in Bantul Regency. It is located around 27 Km to the south from the downtown of Yogyakarta. The beach can be achieved in two ways, i.e., via Kretek Village or via Imogiri and Siluk Villages. The location of the beach is shown in Figure 9 that is taken from Google Maps.

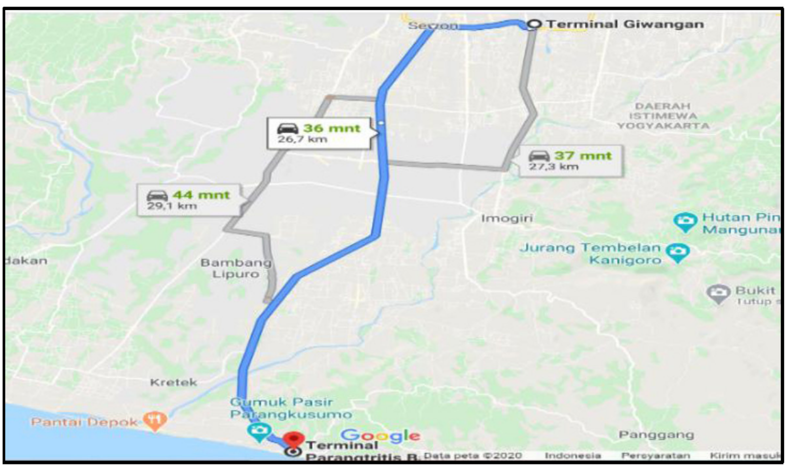

Figure 9

Possible roads from Giwangan to Parangtritis beach.

Table 6

The Bus Stop in Route 2

Code	Location of Bus Stop
1	Giwangan bus station
2	Sewon 1 High school
3	Pleret 1 Vocational School (SMK)
4	Post office, Jl. Cepit Tembi, Sewon
5	BPD bank, Gabusan Office
6	Jetis 1 Public health center
7	Muhammadiyah-Serut primary school (SD)
8	Pariwisata Vocational School
9	Parangtritis bus stop

The selected locations for bus stops in route 2 are given in Table 6.

The network in Figure 9 is transformed into the fuzzy weighted network which is presented in the Matlab GUI in Figure 10. Further, Steps 9-11 (in Table 3) give all possible paths from "Giwangan" bus station to "Parangtritis" bus stop and the associated distances as presented in Table 7.

Table 7

All Possible Paths and the Related Distances in Route 2

Paths	Distances	L_min
1-2-5-8-9	24.2 26.8 29.4	24.2 26.8 29.4
1-3-6-8-9	26.5 27.4 28.3	24.2 26.577 27.4
1-2-4-7-8-9	27.7 29.2 30.8	24.2 26.577 27.4

By executing Steps 12-14, we obtain: FSPL L_{min} = L_min3 = (24.2000 26.5771 27.4000) and the Euclidean distances in route 2 are as follows:

$$d(P_1, L_{\min}) = 2.0124;\ d(P_2, L_{\min}) = 2.6033;\ d(P_3, L_{\min}) = 5.5398.$$

We can see that the path P_1: 1-2-5-8-9 is the shortest path to reach Parangtritis beach from Giwangan bus station with the shortest distance around (24.2000 26.8000 29.4000) Km or $d = \frac{(24.2000+4\times 26.8000+29.4000)}{6} = 26.8$ Km (by using graded mean value). Otherwise, we can use the route P_2: 9-8-6-3-1 to go back to Giwangan with shortest distance of 27.4 Km. We assume that the average speed of bus is 30 Km per hour, we get the travel time for

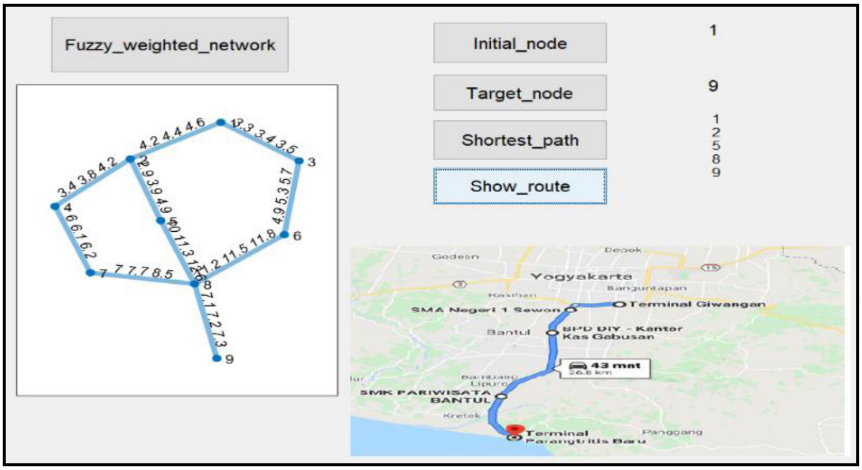

Figure 10

The shortest path in route 2 displayed via Matlab GUI.

path P_1 is 0.89 hour (Giwangan bus station-Parangtritis bus stop) and the travel time for path P_2 (Parangtritis bus stop-Giwangan bus station) is 0.91 hour. The total travel times for the round-trip meet the standard travel time for public buses, i.e., 2 hours. The two routes can be viewed via the links: https://goo.gl/maps/7QKEaE91b71pRGRDA and https://goo.gl/maps/GkZ59LU12wkZtxMn8.

Information related to the shortest path in route 2 is displayed in Figure 10.

3.3.3. The shortest path in route 3

Route 3 passes the regions of Imogiri, Dlingo and Banguntapan. This route is selected based on a number of tourism sites spreads along the route. There some tourism destinations in Imogiri, such as pine forests, caves, handicraft industries, etc. Dlingo sub-district also has some famoust tourism destinations, such as "Mangunan, Asri, Sendangsari" pine forests, "Lintang sewu" hill, and some orchards. Banguntapan is the largest and most populous sub-district. The population is 120,123 people with a density of 4,218 people / km2. The locations of the bus stops in route 3 are provided in Figure 11.

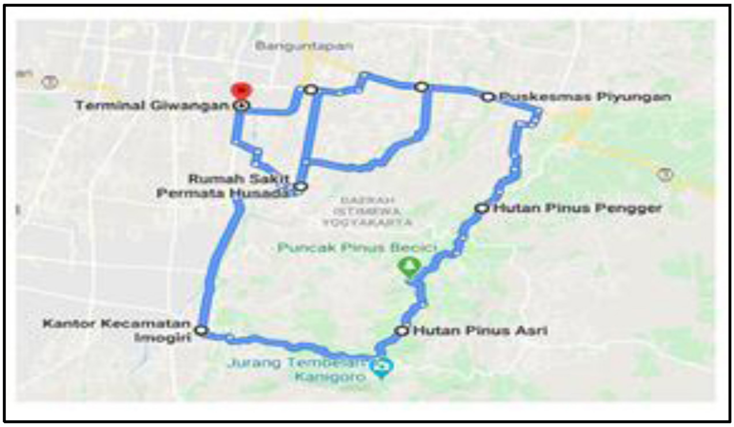

Figure 11

Possible roads that connect bus stops in route 3.

The locations of bus stops in the third route are presented in Table 8.

Table 8

The Bus Stop in Route 3

Code	Location of Bus Stop
1	Giwangan bus station
2	Permata Husada hospital
3	The office of Imogiri sub-district
4	The office of Banguntapan sub-district
5	Piyungan Public health center
6	Pengger pine forest
7	Asri pine forest

Steps 9-11 (in Table 3) give all possible paths in route 3 and the associated distances that is shown in Table 9.

Table 9

All Possible Paths and the Related Distances in Route 3

Paths	Distances			L_min		
1-2-5-7	19.3	20.8	22.3	19.3	20.8	22.3
1-3-4-6-7	21	23.6	26.2	19.3	20.8	22.3
1-2-4-6-7	25.5	27.7	29.9	19.3	20.8	22.3

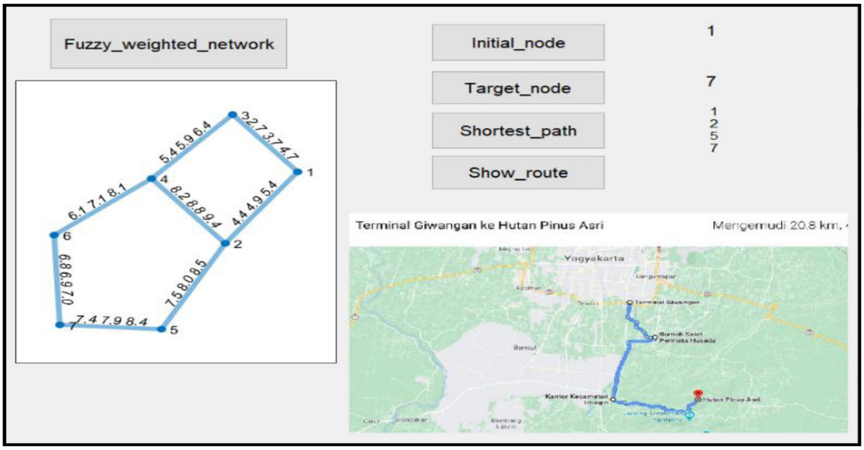

Figure 12

The shortest path in route 3 displayed via Matlab GUI

Further, by executing Steps 12-14, we get L_{min} = L_min3 = (19.3000 20.8000 22.3000). Moreover, we obtain the Euclidean distance between all paths and the L_{min} in Route 3 as follows:

$$d(P_1, L_{\min}) = 0; d(P_2, L_{\min}) = 5.09; d(P_3, L_{\min}) = 11.99.$$

The shortest path in Route 3 is P_1: 1-2-5-7 with the shortest distance around 20.8 Km. By assuming that the average speed of bus is 30 Km per hour, travel time for path P_1 (Giwangan to "Asri" pine forest) is around 0.69 hour. In the opposite direction, we can go through the different route P_2: 7-6-4-3-1 ("Asri" pine forest to Giwangan bus station) with the distance around 23.6 Km and the travel time around 0.79 hour. The two routes can be found in Google maps via the links: https://goo.gl/maps/xnYC6wFk1UViCjg7A and https://goo.gl/maps/xnYC6wFk1UViCjg7A.

The shortest path in route 3 is presented in Figure 12.

3. Conclusion

In this paper, we have proposed an algorithm for route planning of a public bus based on fuzzy shortest path algorithm. Further, an application that displays the fuzzy weighted network and the routes has been created through Matlab GUI. We have evaluated the algorithm through a case study for route planning of the BRT "Trans-Bantul" in Yogyakarta, Indonesia. Experimental results show that the shortest path in route 1 (from "Giwangan" bus station to "Baru-Beach") has a distance of 30.8 Km;

the shortest path in route 2 (from “Giwangan” to “Parangtritis” bus stop) has the shortest distance of 26.8 Km. Further, the shortest path in route 3 (from “Giwangan” to Dlingo) has the distance of 20.8 Km. The application designed in this research could be an alternative tool for route planning of a public bus. In future research, we will extend to compute the fuzzy shortest path under interval-valued fuzzy weighted network and design an online application for presenting the routes of the BRT.

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