Effect of Two-Dimensional Geometry Learning on Geometric Thinking of Undergraduate Students During the COVID-19 Pandemic

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This study aimed to analyze the effect of two-dimensional geometry learning on the geometric thinking of undergraduate students during the COVID-19 pandemic. This qualitative research involved students in the sixth semester. Data were collected using documentation, test descriptions, and interviews. The results showed there are some reasons why the students faced difficulty in mastering a higher level of geometric thinking: the lack of understanding of the concepts in geometry, the lack of knowledge of the definitions of terms and statements, and how to use them to prove. This study concludes that the variation in the geometric thinking level of undergraduate students shows that there are various students' abilities in two-dimensional geometry, starting from mastering concepts, definitions, and theorems to their use in proof. The suggestion is that several strategies are needed to serve these variations, starting from learning assistance and using diverse learning media. Stakeholders, teachers, and prospective teachers can use the results of this study to improve their understanding of geometry and how to teach it in schools.

Keywords: COVID-19 pandemic, geometric thinking, two-dimensional geometry, undergraduate students

INTRODUCTION

Geometry has great pedagogical value as it can cultivate students' cognitive skills and directly connect mathematics with the real world (Voskoglou, 2017) as well as other fields of science. In addition, learning geometry can train students' logical, systematic, and creative thinking skills (Fitriyani et al., 2018). These skills are indispensable for studying other branches of mathematics and solving everyday life's problems. This makes geometry a subject for mathematics taught from primary, secondary to college levels. Numerous studies on the importance of geometry have been carried out in various countries around the world, such as in America (Bergstrom & Zhang, 2016; Yi et al., 2020), Africa (Armah et al., 2018; Usman et al., 2020), and Asia (Decano, 2017; Mdyunus & Hock, 2019; Pasani, 2019). However, the achievement of learning geometry remains low compared to other areas of mathematics (Mammarella et al., 2017). Hohol (2020) elucidated that the reason behind the low accomplishment is its abstract nature and that it is carried out in stages as stated in van Hiele's theory.

Learning geometry is a burden for many students. Two-dimensional geometry learning during the COVID-19 pandemic was carried out online using the Learning Management System (LMS) called "elena" which was used at the Universitas Negeri Semarang. Students often fail to understand geometric concepts and acquire geometry problem-solving skills (Armah & Kissi, 2019). Basic geometry concepts are important, especially in the process of solving problems (Siskawati et al., 2022). This also happened in the Elementary Teacher Education (PGSD) Degree Program, Universitas Negeri Semarang, in which students who scored above 61 (maximum score of 100) were only 29.97% or 104 people out of 347 students in total. Theoretically, the geometry contents are abstract for elementary school students. However, with an inductive approach, it is expected to facilitate the distance between abstract materials and the cognitive development stage of elementary school students who are in the concrete operational stage. The inductive approach used requires a concrete model representing abstract geometry objects. This causes the need to study concrete objects accurately and specifically for each object of study. To this extent, the purpose of learning geometry is to find out the relationship between geometric objects and proofs of either two- or three-dimensional geometry (Kurtulus, 2013).

Students' comprehension and familiarity with the shapes as well as objects' properties, and chances for discussion must be ensured in learning geometry (Decano, 2017). Van Hiele introduced levels in geometric thinking consisting of visualization, analysis, deductive informal, deductive, and rigor. These levels have nothing to do with age, although geometry for undergraduate students is expected to reach the deductive stage which means that they have completed the previous levels; visualization, analysis, and informal deductive. The formal deductive level is characterized by the ability to understand and use the ideas of formal geometry. Students should understand the importance of deduction and use it to construct geometric theories based on axioms and proofs (Ngirishi & Bansilal, 2019). This is supported by several studies explaining that the undergraduate students' achievement of geometric thinking is at level 3 or formal deductive thinking formal (Bulut & Bulut, 2012; Fitriyani et al., 2018) and infrequently happens to reach the highest level.

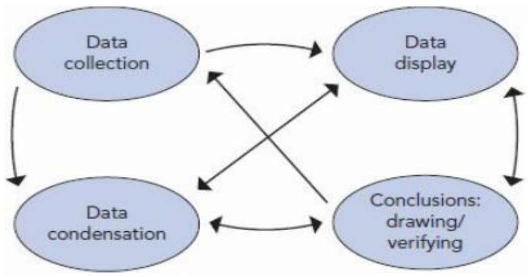
Systematically, van Hiele's geometric thinking level is explained as follows. The first level of visualization is characterized by students perceiving geometric shapes as entities according to their appearance without paying attention to the properties explicitly. The second level is analysis in which the students determine the properties of geometric shapes through informal analysis starting from the components and begin to recognize the properties. The third level is an abstraction or informal deduction where students can relate the properties of geometric shapes, distinguish sufficient and necessary conditions in the relationship between these properties, determine concepts, and form abstract definitions. The fourth level, formal deductive, and the fifth level, rigor, have similar characters where the basics of geometry have been mastered, and students can compare different geometric systems (Subbotin & Voskoglou, 2017). The accomplishment of these levels is sequential and impossible for one to acquire the highest level without mastering the previous ones.

On the other hand, PGSD Degree Program has denser 2-dimensional geometry contents than 3-dimensional geometry as the 2-dimensional study is taught first. With this in mind, this study aims to analyze the geometric thinking level of undergraduate prospective elementary school teachers, so that the geometric thinking level and student response patterns after two-dimensional geometry learning can be identified. Teachers, and prospective teachers, can use the results of this study to improve their understanding of geometry and how to teach it in schools.

RESEARCH METHODS

This qualitative research involved eight 6th semester PGSD students who had taken and passed two courses with material on two-dimensional geometry, three-dimensional geometry, transformation geometry, and geometry learning for elementary schools. The research subjects were taken by purposive sampling. The researchers collected the data using documentation, test questions, description, and interviews. The test questions in the form of 14 description questions are made based on each level of geometric thinking indicators but are limited to two-dimensional geometric material. The qualitative data obtained were analyzed using an interactive model (Miles et al., 2014), where there are simultaneous flows of activity including condensation data, data presentation, and conclusions or verification (figure 1).

FIGURE 1
QUALITATIVE DATA ANALYSIS



RESULTS

Documentation data shows that of the eight students who were the subject of the study, four students were graduates of the high school science major and four students of the high school social studies major. The grades for the two courses taken for geometry were Basic Mathematics and Geometry and Measurement Learning in Elementary Schools. Eight students graduated with a minimum grade of B, except M3 and M4 students who graduated with BC and C. The data from the essay test with 14 questions were analyzed and confirmed by in-depth interviews with eight students. After data reduction has been made, the data is presented in table 1.

TABLE 1
ANALYSIS OF STUDENT ANSWERS ACCORDING TO INDICATORS OF GEOMETRIC THINKING LEVEL QUESTIONS

No	Geometric Thinking	Indicator	Students' Answer Analysis
1	Level 0	Grouping two dimensional figures into square and non-square	Students recognized square shape with various positions and sizes; 7 out of 8 answered correctly, only one student (M2) did not determine a square image because, at first glance, the third-order image was not a square.
2	Level 0	Drawing two parallel lines	2 out of 8 students could draw two parallel lines very well, where M2 and M8 drew complete lines with line names and sign parallel directions on the two drawn lines. Meanwhile, the other six students had correctly drawn two parallel lines but had not given the name of the line and the sign that the two lines are parallel.
3	Level 0	Drawing an obtuse corner	2 out of 8 students (M2 and M3) could draw an obtuse angle from two-line rays with the same starting point and the obtuse angle formed and

No	Geometric Thinking	Indicator	Students' Answer Analysis
			equipped with a name on the starting point and two-line rays. Meanwhile, for the other six students, two of them drew angles instead of two line rays, and four of them drew obtuse angles of two-line rays with the same starting point but did not give names to the starting point and line rays.
4	Level 0	Forming a new shape of two congruent shapes of a congruent isosceles right triangle	Eight students answered incorrectly because of the three two-dimensional shapes (isosceles triangle, parallelogram, square) only two students (M1 and M6) were able to name the three shapes and added a rhombus shape which was depicted from a square with the corner position on the upper left side and right. M2, M3, M5, and M6 only mention a triangular shape in their answers.
5	Level 1	Mentioning the nature of the shape	Two students (M1 and M2) mentioned the properties of two-dimensional shapes in terms of sides and angles, while other students mentioned the properties of two-dimensional shapes in terms of sides and angles and diagonals, folding symmetry, and rotating symmetry.
6	Level 1	Comparing the properties of shapes	All students could compare two shapes' properties by looking at the similarities and differences in the two shapes' properties. Only one student (M1) incorrectly mentioned one trait in a parallelogram with the same diagonal length.
7	Level 1	Grouping shapes based on their properties	Six students had been able to group rectangular shapes based on the number of parallel sides and angles. However, there were two students (M4 and M5) who did not classify the kite as a two-dimensional shape that did not have parallel sides.
8a	Level 2	Explaining the definition of a shape	Four students had explained the definition of a rhombus correctly (M1, M2, M3, M6), while the other four students incorrectly explained the definition of a rhombus because they stated that a rhombus was formed from two congruent isosceles triangles (M4, M5, M8). One student mentioned the definition of a rhombus by saying that it had four ribs (it should have sides) that were the same length and were parallel.
8b, 8c	Level 2	Explaining the reasons for grouping shapes based on definitions and properties	Four students had correctly explained the reasons for grouping two-dimensional shapes (M1, M2, M3, M6). Meanwhile, four other students gave the wrong reasons.
9	Level 2	Proving the implication statement regarding the angle	The four students proved the implication statement in a deductive way, using theorem (M2, M3, M7, M8). Four other students provided empirical examples of one case regarding the truth of the implication statement.

No	Geometric Thinking	Indicator	Students' Answer Analysis
10	Level 2	Explaining sufficient terms and conditions for grouping two-dimensional shapes	Three students could explain the requirements to build a square, including the kite correctly (M1, M2, M3). Meanwhile, the other four students answered incorrectly.
11	Level 3	Proving the theorem about the number of angles in a triangle	Four students (M1, M4, M6, M8) proved the theorem about angles by using the theorems about the alignment of the lines and the angles formed. However, two students (M2 and M3) proved the same thing but the guidelines drawn were not parallel to one of the triangle sides. Moreover, two other students (M5 and M7) used the empirical example of a right triangle to show the theorem's truth.
12	Level 3	Proving the Pythagorean theorem	Two students (M1 and M6) proved the Pythagorean theorem by approximating the congruence on right triangles. One student (M8) proved this by approximating the area of a square. Meanwhile, one student did not provide answer, and three other students showed the truth of the Pythagorean theorem empirically in a right triangle with known side lengths.

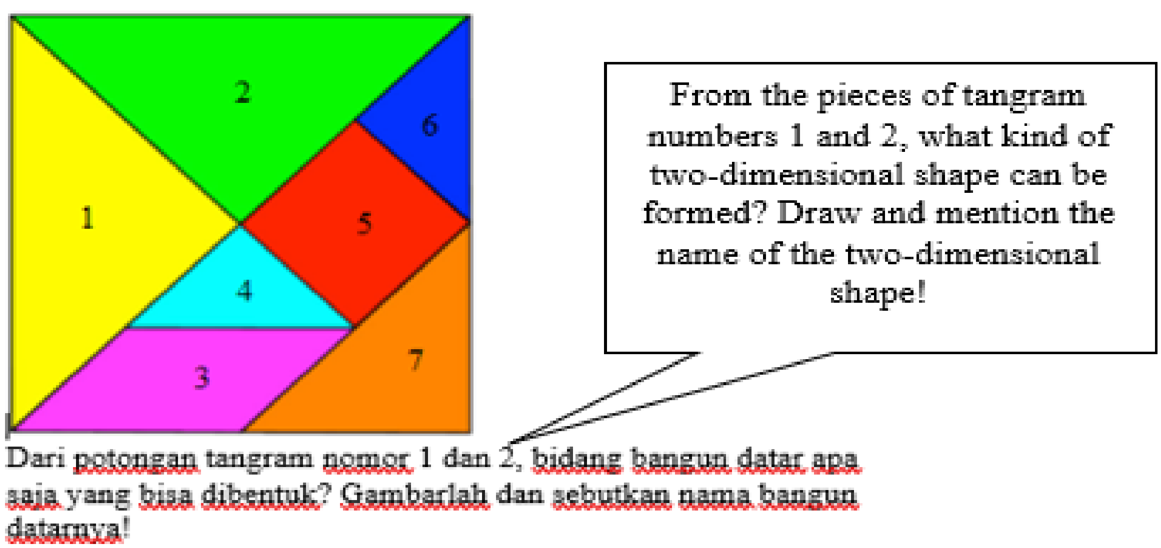
After the students worked on the questions, they were interviewed to explain the process of working on the questions and the possible difficulties they faced. The results are as follows:

- The two-dimensional shapes were recognized by students easily, although they shared positions and sizes.
- All students can draw two parallel lines. However, not everyone knows how to name lines and signs of two parallel lines.
- All students are familiar with obtuse angles from the size of the angle, but the definition of angles and their names are not well understood.
- Few students can build a new two-dimensional shape from a two-dimensional cut shape. Students do not think of seeing these two congruent triangular shapes separately and manipulating them to form another shape.
- However, two students used the term "edge" when mentioning the shape's side because they forgot the correct term.
- There were two students who used the term "edge" when mentioning the side of the shape.
- Some students were able to group the rectangular shapes based on the number of parallel sides they had, even though two students did not mention the kite as a quadrilateral that does not have parallel sides.
- Half of the students can explain the definition of two-dimensional shape well, although some are still inaccurate.
- Students' answers have given reasons for grouping the shapes based on the relationship between the definition of the shape as well as the similarities and differences in the properties of the shape.
- Some students still think with inductive reasoning to answer questions about proof, namely by providing examples of cases. However, the example given is only one, so it cannot be said that students have proven the implication statement.

- Students categorize the grouping of two-dimensional shapes by mentioning the sufficient condition for a square shape called a kite shape by explaining the kite's definition and the kite's properties, which is also owned by a square.
- Most students can use their deductive reasoning to prove this theorem, even though two of them are not suitable because they make the wrong picture. After being confirmed by interviews, they stated that they did not remember the theorem which was formed based on the proof.
- Deductive proof of Pythagoras's theorem by using another theorem related to the congruence of the triangle and the area of a square is often used. However, some students still use intuition, where the Pythagorean theorem is proven true for one example of a right triangle with known side lengths because students have found no other way.
- No student has yet stated why the definition of a square is not singular because it can be related to other two-dimensional shapes such as parallelograms, rectangles, rhombuses, or kites. However, a square's properties must be the same, whether in terms of sides, angles, diagonals, rotational symmetry, and folding symmetry.
- Students conclude by paying attention to premise I, and premise II that was given to obtain correct conclusions. However, most of the students forget to use inference related to mathematical logic.

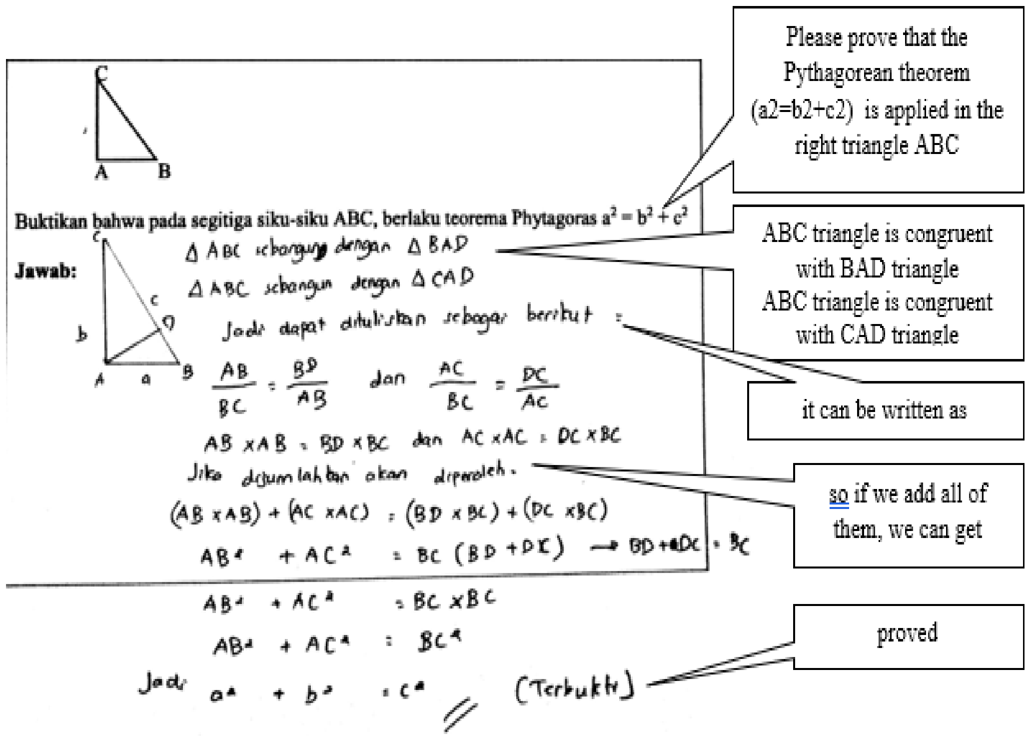
Some of the interesting findings that will be discussed are related to students' answers to numbers 4 and 12. In question level 0, number 4, it is shown that there are still a few students who can build a new two-dimensional shape from a piece of a two-dimensional shape (figure 2). Students M2, M3, M5, and M6 only mention the triangle shape as the answer. It turns out that they only look at the tangram image presented in the problem and do not try to see shape 1 and shape 2 separately, manipulate (rotate the triangle) and assemble them again to form another two-dimensional shape in order to obtain a new two-dimensional shape, namely a square and a line. This ability can be related to Piaget's theory of the conservation of two-dimensional areas (Trimurtini et al., 2018), which is important to master and useful for finding the formula for the of a two-dimensional area. Four students have studied geometry using tangram.

FIGURE 2
QUESTION NUMBER 4 (LEVEL 0)



Question number 12 for geometric thinking level 3 shows variations in student answers that lead to deductive reasoning and intuition. A student (M1) who uses deductive reasoning can prove the Pythagorean theorem by using another theorem associated with congruence in triangles, as in Figure 3.

FIGURE 3
DEDUCTIVE REASONING USED BY M1 STUDENT TO PROVE PHYTAGORAS THEOREM



In addition, there is also a student (M8) who proved the Pythagorean theorem using the square area approach as shown in Figure 4.

Meanwhile, some students also still use their intuitions to prove the Pythagorean theorem, namely by using an example of a right triangle with a known length of all three sides. This method cannot prove a theorem in mathematics because mathematical proof is a general statement of truth about the nature and relationships between mathematical entities (Pier et al., 2019). The interviews with students (M2, M3, M4, M7) reveal that they had difficulty proving the theorem for a variety of reasons, including forgetting how to prove the theorem, so they decided to make an example of a right triangle with side lengths. Perpendicular and hypotenuse are known, so the Pythagorean theorem can be used to prove it. Some students struggle with mathematical proof due to a lack of knowledge about the definitions of terms and statements and how to use them in proof, a lack of understanding of concepts, and a lack of generating and using students' examples of evidence-based statements (Belin & Akar, 2020; Moore, 1994). The description of the student's answers' results is then coded to identify the achievement of the student's geometric thinking level. Table 2 shows the results of the identification about the achievement of students' geometric thinking levels.

FIGURE 4
HOW TO PROVE THE PYTHAGOREAN THEOREM BY APPROXIMATING THE AREA OF A SQUARE

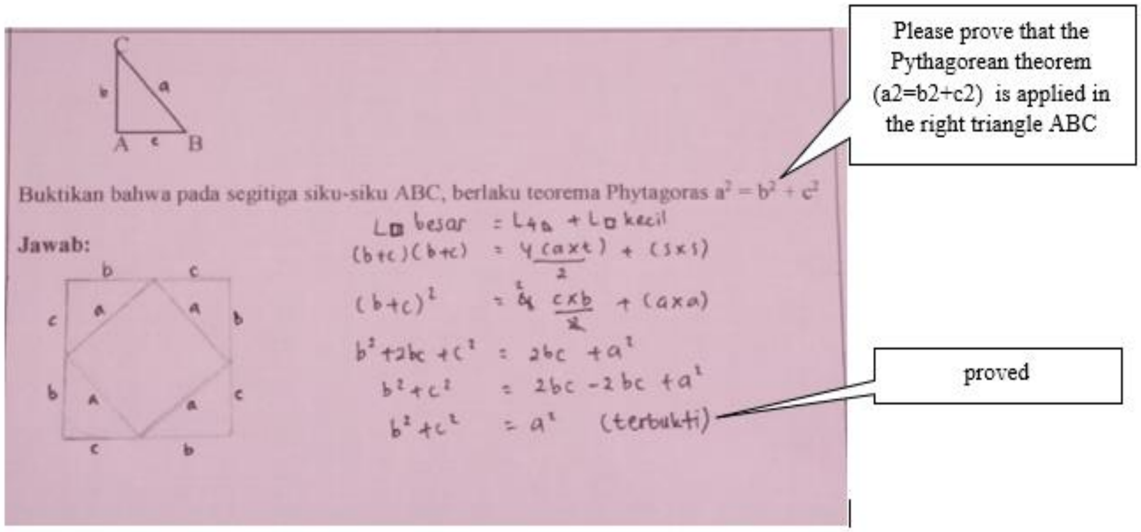


TABLE 2
THE ACCOMPLISHMENT LEVELS OF STUDENTS' GEOMETRIC THINKING

	M1	M2	M3	M4	M5	M6	M7	M8
level 0	v	v	v	v	v	v	v	v
level 1	v	v	v		v	v		v
level 2	v	v	v			v		v
level 3	v					v		v
level 4								

DISCUSSION

From eight students who became the subjects of this study, three students were at level 3, two students were at level 2, one student at level 1, and two students at level 0. Seeing the distribution of geometric thinking levels, students were from level 0 to level 3. However, as in previous studies, reaching level 4 is still uncommon at the student level, if not impossible (Bulut & Bulut, 2012; Fitriyani et al., 2018). Students struggle to reach level 3 (deduction) because they have difficulty thinking and logically organizing evidence. They have difficulty constructing theorem proofs, comprehending the role of axioms, and understanding the definition and meaning of necessary and sufficient conditions. Evidence must be built rather than memorized so that it is not forgotten but can be reconstructed (Fuys et al., 1988). Furthermore, the difficulty in reaching level 4 (rigor or accuracy) is caused by students' difficulty comparing different geometries based on different axioms and learning them without a concrete model. There are very few research studies on attaining the highest level of geometric thinking. However, theoretically, one cannot master each level without first experiencing it at the previous one, forcing one to think intuitively. Furthermore, if the language used in learning is higher than students' geometric thinking level, they will experience difficulties (Mayberry, 1983).

Variations in geometric thinking level in research subjects show that students' varying abilities in geometry are needed. Several strategies are needed to serve these variations. These strategies start from learning assistance according to Van Hiele's steps (Hassan et al., 2020; Škrbec & Čadež, 2015; Yi et al., 2020) to using various appropriate learning media (Cahyono et al., 2020; Primasatya & Jatmiko, 2018) so that the geometric thinking level of students can reach the optimal level for each student.

Because undergraduate students in the PGSD Degree Program have varying levels of geometric thinking, it is clear that learning assistance and appropriate media are critical in providing a geometry learning experience that corresponds to the thinking levels while also improving level achievement (Pasani, 2019). Geometric thinking can be fostered by physically manipulating geometric objects or using dynamic geometric tools (Margaret et al., 2012). As a result, students can learn the same materials but with different learning aids based on their level of geometric thinking. By meeting these learning needs, it is hoped that child-friendly education principles can be consistently applied at various stages of education. As proclaimed by UNICEF, child-friendly education must meet the criteria of quality learner, content, learning process, learning environment, and results.

CONCLUSION

This study concluded that during the COVID-19 pandemic, learning two-dimensional geometry had an effect on the distribution of students' geometric thinking levels from level 0 (visualization) to level 3 (deduction), but no one had yet reached level 4 (rigor). The variation in geometric thinking level of PGSD students demonstrates the range of students' abilities in two-dimensional geometry, from mastery of concepts, definitions, and theorems to their application in proof. One of the reasons students struggle to master higher levels of geometric thinking is a lack of understanding of concepts in two-dimensional geometry, as well as a lack of knowledge of term and statement definitions and how to use them in proof.

This qualitative study is limited to the study of 2-dimensional geometry and employs sixth-semester students to investigate the level of geometric thinking and related issues.

It is proposed for future research that several strategies are required to serve these variations, beginning with learning assistance based on learning needs and using a variety of learning media to ensure that each student achieves the highest geometric thinking level possible.

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