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Students' difficulties in solving geometry problems

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Abstract

This study is based on research findings that examines students' difficulties utilizing Polya's problem to solve geometry problems - solving stages. Thirty-one students were assigned to work on questions about the material in order to calculate the distance in space. Researchers examined each student's work to determine which stage of Polya they struggled with. In-depth interviews were also conducted to clarify answers and investigate students' difficulties in solving math problems. The results showed that students who were able to answer completely and correctly, based on Polya's stages in understanding the problem, compiling or thinking about a solution plan, solving the problem according to the plan and re- examining the procedure and the results of solving the answer, had a very low success rate. Scaffolding is given to students who have difficulty and recognition of the accuracy of correct answers for those who are successful.

Keywords: Geometry, Polya's problem-solving stages, students difficulties.

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1. Introduction

1.1. Conceptual and theoretical framework

Problem-solving is a critical effort formed to eliminate the complex process of potential challenges in order to achieve a specific goal. Individuals engage in problem-solving as a cognitive behavioural process to find practical answers to the challenges that are experienced in everyday life. Tall (2002) states that problem-solving is a more creative activity that includes the formulation of possible conjectures, a series of testing activities, modification and refinement until a formal proof of a well- defined theorem can be produced. According to Zhu (2007), The method for resolving mathematical issues requires a complex level of mental effort. The ability to build logical reasoning skills that can be utilized in a variety of circumstances in everyday life, both now and in the future, can be gained through solving mathematical questions (Mullis et al., 2012). The process of solving mathematical problems is one that is difficult, abstract, and needs the thinking and reasoning of a human being. Mathematical education places a significant emphasis on the development of problem-solving abilities (Chimmalee & Anupan, 2022).

When a task encounters some obstacles, mathematical problem-solving occurs. The application of problem-solving strategies in mathematics calls for inventive thought as well as a methodical approach (Blanco et al., 2013). This condition refers to the ability to create contextual problem- solving programmes using patterns in order to solve problems. Students require specific methods in order to find solutions to the challenges they are facing. (algorithms). When students are unable to solve a problem simply by following the work method in the form of an algorithm, other work methods must beused to facilitate the solution (heuristic). Students will channel their thoughts through heuristics so that they do not work by simply experimenting without direction. Concept maps which are created by drawing pictures, patterns and schemes are a type of heuristic for solving mathematical problems. Students' ability to find algorithms and heuristics in solving mathematical problems are components of metacognitive abilities.

Problem-solving ability, according to Trilling and Fadel (2009), Students need to have a process skill throughout the mathematics learning process. Problem-solving ability is a

dynamic process that develops in response to the type, characteristics, and framework of the issue at hand. Students are able to think about unique situations, decide what to do and how to do it, and make use of chances that exist to obtain solutions when they have problem-solving skills. These skills lay somewhere between learning skills and renewal skills. Kazemi et al. (2010) It should be stated that the ability to solve issues is intricately tied to both cognition and metacognition. According to Booker et al. (2014), problem- solving components include manipulating the concept, understanding the concept, noting similarities, differences and analogies, identifying critical issues and selecting appropriate procedures, analysing incorrect details, interpreting the relationship between facts and makinggeneralisations.

Students learning mathematics need problem-solving, critical thinking and analytical thinking skills. The ability to solve problems is a critical requirement that mustbe met in order for a teacher to survive. Individuals' problem-solving abilities can vary depending on the type of problems they face (Anilan & Berber, 2019). The mathematical problem-solving abilities of students are characterized by their capacity to solve story problems and non-routine tasks; apply the use of mathematics in regular life or other situations; prove, develop, or test conjectures; and apply the use of mathematics in regular life or other situations.

Lessons in mathematics are generally seen as being challenging and difficult to understand for students. (Kadirbayeva et al., 2022). Students are allowed to build their capacity to alter and modify their approach to a variety of new problem scenarios by solving mathematical problems (Chimmalee & Anupan, 2022). Students are able to learn and apply a variety of mathematical concepts and procedures, as well as cultivate and value adequate mathematical competences, through the development and acquisition of problem-solving capabilities in their educational experiences (Antunovic & Baranovic, 2022).

According to Charles and O'Daffer (1997), The objectives of problem-solving learning in mathematics are to develop students' thinking skillfulness, develop students' capability to select and use problem-solving strategies, develop problem-solving beliefs and attitudes, develop students' capacities to use interrelated knowledge, develop students' capability to observe and evaluate their thinking and the results of their career while solving problems, and develop students' capabilities to use interrelated knowledge.

Siregar (2017) contends that when using problem-based learning, students must motivate students in group study so that problem-solving activities have an impact on improving mathematical problem-solving skills. Scaffolding is provided to students in problem-based learning so that they can solve problems without the assistance of others. Critical thinking, in the form of bright ideas, is required during the problem-solving process in order to find effective and accurate solutions. Critical thinking is a well-known capacity that aids in the development of both a person and a system by fostering the production of insights about important problems (Amhar et al., 2022). Students' scholastic achievement, as well as their capacity to comprehend scientific principles and the very basis of scientific inquiry, is directly correlated to their level of logical reasoning and reasoning abilities (Aiym et al., 2022).

Internal student factors such as initial knowledge ability, mathematics appreciation and mathematical logical intelligence are particularly important in influencing mathematical problem-solving abilities (Eka et al., 2016). According to Fauzi et al. (2019), prior knowledge of what students have missed or forgotten is important in developing students' creative mathematical abilities to learn new information. Students with a low ability to solve problems are characterized by having a low ability to analyze problems, a low ability to build problem-solving plans, and a low ability to carry out computations, particularly those connected to sensitive material that supports the problem-solving process.

According to Polya (1973) (as cited in Misa'adah & Mariani, 2021; Nalurita, 2019;Siregar, 2017; Sukoriyanto et al., 2016), The following are the steps involved in problem-solving: first and foremost, having a comprehension of the issue; Activities at this level involve determining what (data) is known, determining what (questions) is unknown, determining if the knowledge is adequate, and determining what requirements (conditions) must be satisfied in order to solve the original problem in a more operational form. Second, compiling or considering a solution plan; activities that can be undertaken at this stage include attempting to locate or recall previously solved problems that have similarities with the problem to be solved, searching for patterns or rules and compiling procedures for problem-solving (making conjectures). Third, resolving the problem in accordance with the plan; activities that can be undertaken at this stage include carrying out the procedures devised in the previous stage in order to obtain a solution. Fourth, eexamine the procedure and the consequences of the settlement; Activities that can be performed at this stage include analyzing and evaluating whether the processes employed and the findings gained are accurate, perhaps there are other methods that are more efficient, whether the procedures created can be utilized to solve problems, and whether or not the method may be used for a wider population.

Students who take geometry classes are better able to develop various skills, including the ability to conjecture, reason deductively, argues logically, and prove their conclusions (Zhang, 2017). One of the factors that contributes to pupils having difficulty understanding mathematics is the fact that geometry is a topic that requires students to comprehend abstract concepts (Andika et al., 2020).

1.2. *Purpose of the study*

This study aimed to investigate the students' attitudes toward the difficulties in solving geometric problems involving determining the distance in space using Polya's problem-solving stages.

2. Method

2.1. Research design

The methodology for the research used descriptive-analytical, quantitative techniques, and correlational research design were utilized in this study's research design. Techniques used to collect data are documentation, interviews and the problem- solving ability assessment instrument. The problem-solving ability assessment instrument sheet is designed in an answer column format that is sorted by the Polya stage.

2.2. Participants

This study was conducted on 31 students from year XII IPA 3 at SMA Negeri 1 Salem, Indonesia, with 11 (35.48%) males and 20 (64.52%) females assigned to work on questions about the material in order to calculate the distance in space. In-depth interviews were also conducted on six students to clarify answers and investigate students' difficulties in solving math problems.

2.3. Data collection process

Students were given a geometric problem to solve in order to determine the distance in space using the Polya-based problem-solving stages. The researcher corrected each student's answer to determine where they struggled at the Polya stage. In-depth interviews were conducted after analysing the results of students' answers in order to clarify answers and explore the difficulties students encountered when solving mathematical problems.

2.4. Validity and reliability of the instrument

The problem-solving capability evaluation tool sheet was explicitly created in an answer column sorted by the Polya stage. This instrument has been validated by educational experts and practitioners with an average score (Va) of 4.65, which is in the range $4 \leq Va < 5$, with a very valid category, and the percentage of validator perceptions is 93.06, which is in the range of $84\% \leq x \leq 100\%$, with very good criteria.

The following illustrates the assessment rubric that will be used to establish the requirements for acquiring mathematical problem-solving abilities, as shown in Table 1.

Table 1

Criteria for Achievement Mathematical Problem-Solving Abilities

Average score	Achievements	Criteria
$4.2 \leq Va < 5$	$84\% \leq x \leq 100\%$	Very good/very high
$3.4 \leq Va < 4.2$	$68\% \leq x < 83\%$	Good/high
$2.6 \leq Va < 3.4$	$52\% \leq x < 67\%$	Fairly good/medium
$1.8 \leq Va < 2.6$	$36\% \leq x < 51\%$	Not good/low
$1 \leq Va < 1.8$	$20\% \leq x < 35\%$	Poor/very low

Va is average score, *x* is achievement mathematical problem-solving abilities.

Table 2 shows the description of results on students' answers in geometry problem-solving.

Table 2

Description of Results on Students' Answers in Geometry Problem-Solving

Students' answers	Descriptions of students in geometry problem-solving

Complete	Understand the facts presented in the problem accurately and the problem itself and can simplify it by employing the language most familiar to them. Able to apply ideas correctly completely and in detail and can combine ideas so as to produce novelty. Complete verification, able to find and implement ideas appropriately
Incomplete	Understand the information contained in the problem correctly and be able to state theessence of the problem in their own language. Understand the problem well, be able to apply the solution ideas correctly and in detailand can combine ideas but are not complete. Complete verification but have not implemented ideas because they are not complete.
Incorrect	Understand the information contained in the problem but still write it down in theproblem and do not understand the problem presented. Doing work on the questions but still wrong because they don't understand the problems presented in the questions. Not verifying.
No response	Not working on the questions at all or just copying the questions, there is no completion process and no verification process.

2.5. *Data analysis*

In the data analysis section, statistical data were analyzed in the Statistics programme by using percentage (%) and linear regression analysis.

3. Findings and discussion

Table 3 displays the percentage of results obtained from the analysis of the responses provided by the 31 pupils by Polya's stages of problem-solving.

Table 3

Percentage of Answer Analysis Results Based on Polya's Stages

Polya's stages	Activities performed	Answer and percentage											
		Complete			Incomplete			Incorrect			No response		
		JM	PA	PR	JM	PA	PR	JM	PA	PR	JM	PA	PR
Understanding the problem	Identifying the known (data)	4	12.9	6.5	12	38.7	63.2	1	3.23	3.23	14	45.2	27.1
	Recognizing what is unknown (asked)	2	6.45		21	67.7		2	6.45		6	19.4	
	Determining whether the information is adequate	2	6.45		13	42.0		0	0		16	51.6	
	What prerequisites must be met?	1	3.23		25	80.7		1	3.23		4	12.9	
	Restating the initial issue in an operational (solvable) manner	1	3.23		27	87.1		1	3.23		2	6.45	
Devising a solution plan	Attempting to locate or recall a previously solved problem that has similarities to the problem at hand.	1	3.23	3.2	18	58.1	62.4	9	29.0	20,4	3	9.68	14.0
	Looking for patterns or rules	1	3.23		19	61.3		7	22.6		4	12.9	
	Developing completion procedures (making conjectures)	1	3.23		21	67.8		3	9.68		6	19.4	
Carrying out the plan	Carrying out the procedures that have been made in the previous stage to get a solution	2	6.45	6.5	19	61.3	61.3	3	9.68	9,68	7	22.6	22.58
Looking back	Analyzing and evaluating the correctness of the techniques used and the results obtained	1	3.23	1.6	23	74.2	73.4	1	3.23	8,06	6	19.4	16.94
	Is there another procedure that is more effective	1	3.23		22	71.0		2	6.45		6	19.4	
	Can the procedures created be used to solve similar problems?	0	0		24	77.4		3	9.68		4	12.9	
	Can the procedure be generalised?	0	0		22	71.0		4	12.90		5	16.1	
	KPM (%)		4.44			65.07			10.35			20.15	

JM = Number of students, PA = Percentage of each activity carried out, PR = Average percentage of each stage Polya, KPM = Problem-solving skills.

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Based on Table 3, the average percentage of problem-solving ability test answers shows that 4.44% of the responses given by the pupils were exhaustive and correctly. This demonstrates that students have a shallow capability to solve geometric problems based on the Polya stages. The average percentage of students who can answer completely and correctly based on Polya's stages in understanding the problem is 6.45%; students compile or think of a solution plan is 3.23%; students solve problems according to plan is 6.45%; and students re-examine the procedures and results of completion of answers is 1.61%. In Table 2, an average percentage of 65.07 answered incompletely, 10.35% answered incorrectly and 20.15% did not respond.

Based on Polya's stages of problem-solving, examples of displaying student work in a complete and correct manner are provided. The following questions are being worked on by students: A building in the form of a block has a floor area of 4 m × 8 m and a height of 10 m. Suppose the shape of the block is called ABCD.EFGH. Find the length of the distance from point G to the plane of the diagonal CDEF.

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The following are the Polya-based problem-solving stages performed by students.

Polya's stages	Activities performed	Student answers	Transliteration
Understanding theProblem	Identifying what (data) is known	Sebuah balok ABCD. EFGH dengan panjang AB = 4cm, BC = 8cm dan BF = 10 cm.	A block ABCD.EFGH with length AB = 4 cm, BC = 8 cm and BF = 10 cm
	Identifying what is not known (asked)	Jarak antara titik G dengan bidang CDEF	The distance between point Gand the diagonal plane CDEF
	Identifying whether the information is sufficient	Belum, karena harus menentukan sebuah garis CF pada bidang CDEF. Selanjutnya menarik sebuah garis dari titik G yang tegak lurus ke bidang CDEF melalui garis CF	Not yet, because we have to define a line CF on the CDEF diagonal plane. Next draw a line perpendicular from point G to the diagonal plane CDEF through line CF.
	What conditions must bemet	Dalam segitiga siku-siku CGF harus dicari panjang GI.	In a right triangle CGF mustfind the length of the line GI
	Restating the initial issue in an operational (solvable) manner	Untuk menentukan jarak antara titik G dan bidang CDEF kita harus menentukan sebuah garis CF pada bidang CDEF kemudian menarik sebuah garis dari titik G yang tegak lurus	To determine the distance between point G and the CDEF diagonal plane, you must make a line CF on the CDEF diagonal plane, then make a perpendicular line from point G

The students understood the problem in stage 1 by identifying known and asked data, informationon questions, requirements that must be satisfied, as well as rephrasing the issue in an operational manner.

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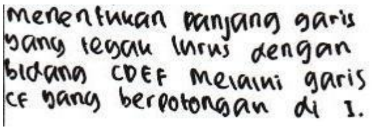
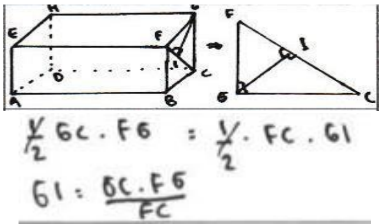
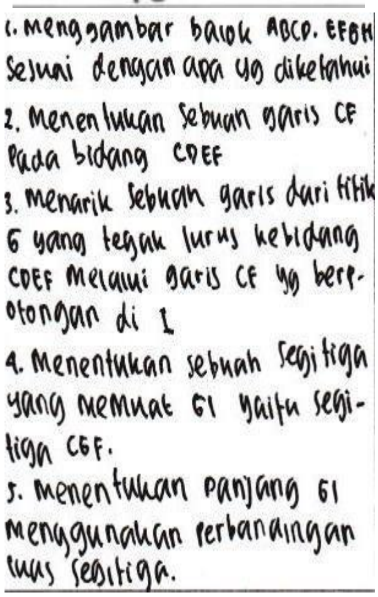
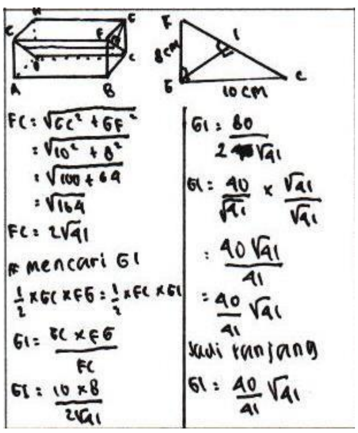
Polya's stages	Activities performed	Student answers	Transliteration
Devising a solution plan	Attempting to locate or recall a previously solved problem that has similarities to the problem at hand.		Determine the length of the line perpendicular to the plane of the diagonal CDEF through the line CF that intersects at point I
	Looking for patterns or rules		$\frac{1}{2} GC.FG = \frac{1}{2} FC.GI$ $GI = \frac{GC.FG}{FC}$
	Developing completion procedures (making conjectures)		Draw blocks ABCD.EFGH according to what you know Determine a line CF on the diagonal plane CDEF Draw a line from point G that is perpendicular to the diagonal plane CDEF through the line CF that intersects at point I Make a triangle containing the line GI, i.e. triangle CGF Determine the length of the line GI using the ratio of the area of the triangle

In stage 2, the students have compiled or thought of a solution plan by recalling previous problemsthat have similarities with the problems to be solved.

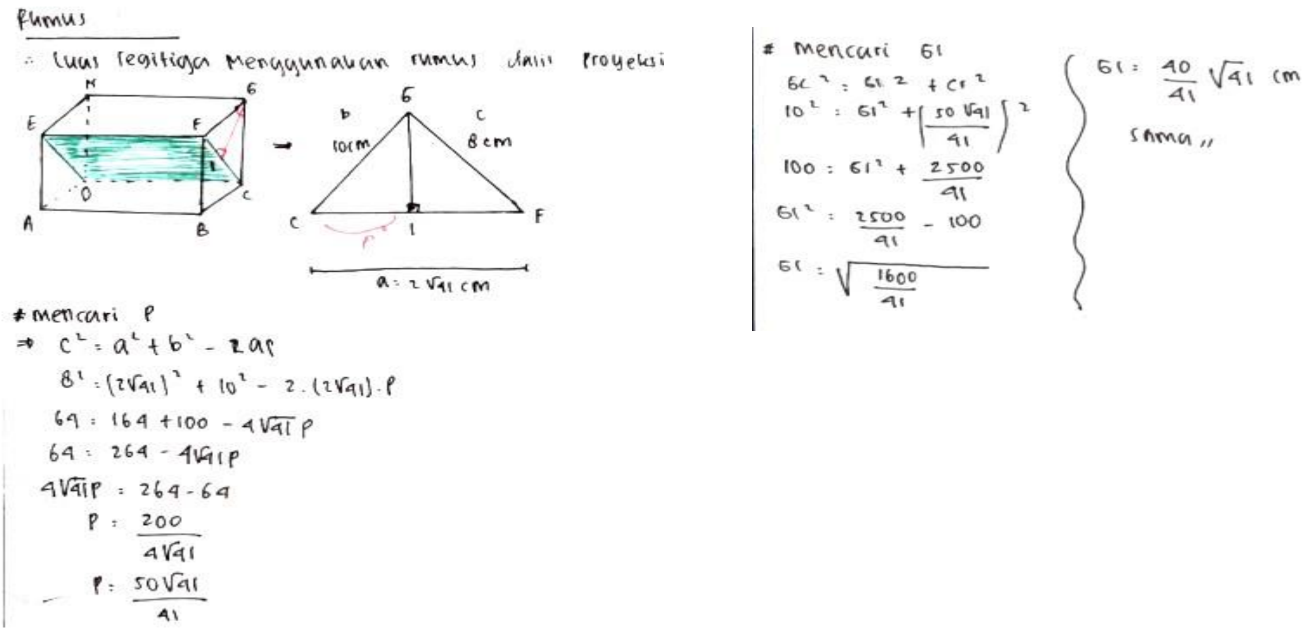
Polya's stages	Activities performed	Student answers	Transliteration
Carrying out the plan	Carrying out the procedures that have been made in the previous stage to get a solution		Find the length of GI (usingthe ratio of the area of the triangle) So the length of the line GI (concluded that the distance of point G to the diagonal plane CDEF = $\frac{40}{41}\sqrt{41}$

In stage 3, the students solved the problem using the procedural plan devised in stage 2 to arrive ata solution.

Polya's stages	Activities performed	Student answers	Transliteration
Looking back	Analyzing and evaluating the correctness of the employed procedures and the obtained results	Mencari panjang FG menggunakan nilai GI FG = FC × GI / GC = 2√41 · 40/41 √41 / 10 FG = 8 cm {benar}	Find the length of FG using the value of GI $FG = \frac{FC \times GI}{GC}$ $= \frac{2\sqrt{41} \times \frac{40}{41}\sqrt{41}}{10}$ FG = 8 cm Then the procedure used andthe result is correct
	Is there another procedure that is more effective	Bisa, menggunakan rumus dari proyeksi	Yes, by using the projection theorem formula
	Can the processes developed be utilized to find comparable solutions to issues?	Ya	Yes
	Can the procedure be generalised?	Ya	Yes

In stage 4, the students re-examined the procedure and its completion results by performing an investigation and evaluation to determine whether the methods that were applied and the results that were achieved were accurate.

The next step is for students to make assumptions in the form of new ideas about whether there are other more effective procedures that can be applied to the resolution of similar challenges and generalise. In this activity, students recognise the answers obtained through other procedures that provide accuracy that the answers obtained are correct and make generalisations through reinforcement. In this geometry problem, students continue to solve the problem by using the projection theorem formula with the triangle area approach.



Transliteration

1) Area of a triangle using the projection formula

2) Find length P(CI)
3) Find the of GI.
4) Find the length of the GI with other procedures obtained the same and correct results.

This study found, based on an analysis of student work outcomes, that the characteristics of students who had difficulties answering issues were characterized by the outcomes of the work done by pupils who answered in incomplete detail, incorrectly and did not answer, so that students tended to be unable to understand the problem, were unable to formulate or think of solution plans, and were not successful in resolving the issue by the plan. In addition, students are unable to perform re-examine of the analysis of the process as well as the final findings.. This finding supports the findings of Vinner's (1997) research, which discovered facts about teachers and students in problem-solving situationsin which students frequently do not exercise control when solving a problem. Students who cannot solve mathematical problems in a structured and accurate manner are given scaffolding to work through the challenges, and students who have been directed to do recognition in the form of strengthening the answers obtained through other procedures that provide accuracy that the answers obtained are correct and make generalisations.

In-depth interviews were conducted after analysing students' answers in order to clarify answers and explore students' talents in finding solutions to mathematical issues using Polya's stages as a guide. The interviews were conducted with six students, two of whom represented the low group (RD and HL), two of whom represented the medium group (TJ and AG) and two of whom represented the high group (RF and LF). Table 4 it contains a summary of what was said in the interviews about the questions on the test of problem-solving.

Table 4
Summary of the Interview Results

Participants	Interview summary
RD	Students cannot distinguish between what they know (data) and what they do not know (questions), so they do not understand the questions well and tend to only rewrite the parts written on the subject matter (stem). Students have been unable to determine whether the information on the subject is adequate. Students have been unable to identify what conditions must be met in order to solve theproblem.
HL	Students attempt to comprehend the problem and restate it in a more operational (solvable)form. Students attempt to devise or consider a problem-solving strategy.
TJ	Students have a thorough understanding of the problem and are capable of devising problem-solving procedures (making conjectures). Students are able to solve problems according to plan
AG	Students have a good understanding of the problem, can devise problem-solving procedures and can solve problems according to the plan. Students re-examined the procedure and the completion results.
RF	Students have a good understanding of the problem, can devise problem-solving procedures and can solve problems according to the plan. Students re-examined the procedure and the completion results.
LF	Students have a good understanding of the problem, can devise problem-solving procedures, and can solve problems according to the plan. Students re-examined the procedure and the completion results.

According to the data presented in Table 4, it is clear that students' difficulties Utilizing Polya's stages of problem-solving in the process of solving geometry problems is due to a lack of student's ability to understand the context of problems involving geometry material.

4. Conclusion

The ability of students to solve geometric problems based on Polya's stages suggests that most students still have difficulty in solving mathematical problems, as marked by the outcomes of the work done by pupils who answered in incomplete detail, incorrectly, or did not answer so that students tended to be unable to understand the problem, formulate or think the solution plans, and were unable to solve the problem according to the plan. In addition, students could not re-examine the procedure and the complete results. Scaffolding is provided for students who cannot solve mathematical problems in a structured and correct manner and for students who have been directed to do recognition in the form of strengthening the answers obtained through other procedures that provide accuracy that the answers obtained are correct and make generalisations.

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