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# Student's Mathematical Reasoning Ability in Junior High School in Indonesia

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## Abstract

This study was aimed at investigating students' Mathematical Reasoning Ability (MRA) and their failure to achieve the indicators of MRA. It deployed descriptive qualitative and took place at one of the Junior High Schools in Sumedang Regency, West Java province. There were 32 students of class VIII participating in this study. The data were collected through a test, document analysis, and interview. The instrument of the test was based on five indicators adopted from the work of Bjuland (2010). The results showed that students' MRA was still low with an average score of 42.40 compared to the minimum criteria of achievement (MCA) score of 65. This happened because some students failed to achieve the five indicators of MRA. They felt that they did not need to write down problem identification. Besides, they were unable to determine problem-solving strategies, implement, evaluate, and draw logical conclusions from the mathematical problem assigned. This study suggests that teachers should spend their efforts to innovate in teaching mathematics to improve students' MRA.

**Keywords**: Bjuland Framework, Geometry, Mathematical Reasoning Ability, Polya's.

## 1. Introduction

Mathematics is one of the compulsory subjects taught in schools. It receives great financial support from the government in many countries **(Wright, 2020)**. It is taught to develop students' logical, analytical, systematic, critical, creative thinking skills. Those skills are closely related to mathematics reasoning ability (MRA) and they play a central part in the subject **(Kollosche, 2021)**. They serve as a building block for higher-order thinking which is an important objective of education **(Bronkhorst et al., 2020)**.

National Council of Teachers of Mathematics (NCTM) has set the curriculum standard for teachers to achieve. **NCTM (2000)** outlines that there are five key areas in mathematics learning: 1) representation, 2) reasoning and proof, 3) Communication, 4) Problem Solving and 5) Connection **(NCTM, 2000)**. In addition, several guidelines concerning the preparation for mathematics teachers have been released by **NCTM (2003)**. They were designed to prepare mathematics teachers that demonstrate abilities in terms of content and pedagogy **(NCTM, 2003)**. Each country has also developed its curriculum for mathematics which to some extent is in line with the program from NCTM. In Indonesia, Depdiknas (the Ministry of National Education) develops its curriculum for Mathematics. One of its general objectives of mathematics education in junior high school is to use MRA concerning attributes and patterns in geometry to manipulate in making generalizations or explaining mathematical ideas and statements.

MRA should be the priority for mathematics teachers to train their students since it is pivotal for them to face complex issues facing society **(Khan & Krell, 2019)**. This ability requires ample opportunities in the classroom for students to practice and demonstrate their reasoning ability (**Stacey, 2012)**. MRA can be achieved by designing Mathematics as reasoning exercise not a subject to memorize **(Brodie, 2010)**. There are many ways that teachers can achieve students' MRA by applying some methods like Creative and Imitative Reasoning **(Lithner, 2008)** and Mind Mapping strategy **(Ayal et al., 2016)**.

One of the most notable works of MRA was conducted by **Bjuland (2007)** which was based on **Polya's (1985)** well-known masterpiece that is three problem-solving models. According to Bjuland, reasoning is five interrelated processes of mathematical thinking activities. The five processes are categorized as follows: sense-making, conjecturing, convincing, reflecting, and generalizing **(Bjuland, 2007)**.

In Indonesia, students struggle to achieve MRA and many students have the low ability in their mathematics achievement. Two reputable institutions have released their report recording low performance in mathematics for Indonesian students. Trends in International Mathematics and Science Study (TIMSS) recorded with an average math score of 397, ranked 45th out of 50 countries for Indonesia **(Mullis et al., 2015)**. Another report released by OECD organization called Program for International Student Assessment (PISA which recorded Indonesian students with an average math score of 386, or ranking 63 out of 70 countries **(OECD, 2016)**.

The problem related to MRA has also been confirmed by the results of interviews with several mathematics teachers when teaching geometry material at one of the junior high schools in one of the regencies in Indonesia. They stated that students' MRA were still low, especially in understanding space, shape, and completion in working on story problems related to geometric shapes. This ability has not been honed because schools do not hold structured and periodic tests of MRA.

Several studies on MRA have been conducted by researchers. Their studies include students' MRA of seventh-grade **(Erdem & GÜRBÜZ, 2015)**; teachers' inadequate knowledge of MRA **(Bozkuş & Ayvaz, 2018)**; and Challenges in assessing MRA **(Herbert, 2019)**, the influence of learning styles on students' MRA in solving trigonometric problems **(Setiawan et al., 2020)**, the relationship between mathematical reasoning MRA and students' attainment in mathematics **(Adegoke, 2013)** and the influence of gender toward MRA **(Kadarisma et al., 2019)**.

In Indonesia, there is little information concerning studies investigating students' MRA and the causes of students' low MRA. Given the importance of the issue, this study was guided by the following research questions:

(1) How did students achieve MRA on each indicator in a mathematical problem?
(2) Why did students fail to achieve the indicators of MRA?

## 2. Methodology

### 2.1. Research Design

This research is descriptive qualitative research **(Edmonds & Kennedy, 2017)** and it was sought to investigate the subject in their natural environment **(Sari & Nayır, 2020)**. It was not designed to connect this educational research with the phenomenological philosophy **(Sohn et al., 2017)** since it is educational practice and viewed in a particular framework. The findings will be interpreted to answer the research **questions (Safari & Razmjoo, 2016)**.

### 2.2. Participans

This research was conducted at one junior high school in Sumedang Regency, West Java, Indonesia. There were 32 students of second-grade junior high school (class VIII). They were grouped based on the results of their MRA test. Based on this test, students are grouped into three groups, namely, the low achiever group (LAG), the moderate achiever group (MAG), and the high achiever group (HAG).

### 2.3. Data Collection

The data were collected through a test and interviews **(Nowakowska & Pisula, 2021)**. The former was aimed at describing students’ achievement on five indicators of MRA on a mathematical problem and the latter was applied to investigate the cause of their failure to achieve the indicators of MRA. This research consists of several stages. First, it prepared test questions (making test questions for MRA based on the theoretical indicators of **Bjuland (2007)** and referring to the Minimum Criteria of Accomplishment (MCA) to construct items of questions. The score set for MCA is = 65. Second, it administered the test (selecting research subjects and assigning test questions). Third, it analyzed students' answers and conclude them. Fourth, it applied interview to students to investigate the cause of their failure to achieve the indicators of MRA and finally it analyzed the result of the interview.

The test was designed to measure students’ achievement on five indicators of MRA. The test instrument to measure students’ MRA is shown in Figure 1 below:

**Figure 1.** Showing the mathematical problem to test students’ achievement on five indicators of MRA.

A group of teenagers held a camp in a mountain. They set up a standard of Indonesian military tent. However, on the first day they set up their tent, a natural disaster occurred when a tornado caused the tent to collapse. The torn covering material also flew around so that it could not be used again.

Fortunately, the iron supports stood still. To rebuild the tent, they took the initiative to rebuild the platoon tents by using small cloth tents as coverings which they brought as reserves. The standard military tent has a height of 9 meters, a length of 16 meters, and a width of 8 meters. It is shaped as shown below. The ratio of the wall height to the roof is 2: 1. If the area of the cloth used for small tents is 59 m2. How many small tent fabrics were needed to cover the entire surface of the military tent?

**Table 1.** The five indicators of (Bjuland, 2007) to achieve MRA in solving mathematical problems.

| Five processes of Bjuland's framework | Indicators |
|---|---|
| Sense-making | Students should be able to build problem schemes and represent their knowledge |
| Conjecturing | Students discover incomplete facts (settlement strategy) and predict a conclusion |
| Convincing | students conduct or implement a settlement strategy based on the previous stage |
| Reflecting | Students re-evaluate the three previous processes to see how they relate to theories that are considered relevant |
| Generalizing | students conclude the entire process, and identify and generalized a solution to the mathematical problem |

To investigate the cause of students low MRA, the interview was conducted with students who failed to achieve the indicators of MRA. They were asked the causes of their failure to achieve the indicators in solving a mathematical problem.

### 2.4. Data Analysis

The data analysis included descriptions of the results of students' answers to the MRA test. Bjuland (2007) framework of five process or indicators were applied to analyze students' responses. Based on the students' work that failed to achieve the five indicators, the interview was conducted to investigate their failure. The results of student interviews were analyzed to explain the causes of their failure to achieve the indicators of MRA. Excerpt of interview was transcribed and used to explain their failure to achieve the indicators of MRA. Then the researchers applied data reduction, data selection, and interpretation were applied to describe students' MRA and the causes of their failure.

## 3. Findings

The first section deals with students' achievement on five indicators of MRA. The data from the test including their score and examples of students' work that achieved and failed the indicators will be described.

### 3.1. Students' Achievement on the Five Indicators

Related to Research question 1, this study administered the test and the results of student responses were observed and analyzed based on the Bjuland (2007) indicators or five processes of MRA. The results of observations and analysis of student responses (Subject N = 32) are shown in table 2 as follows:

**Table 2**. The results of the students’ achievement on the five indicators.

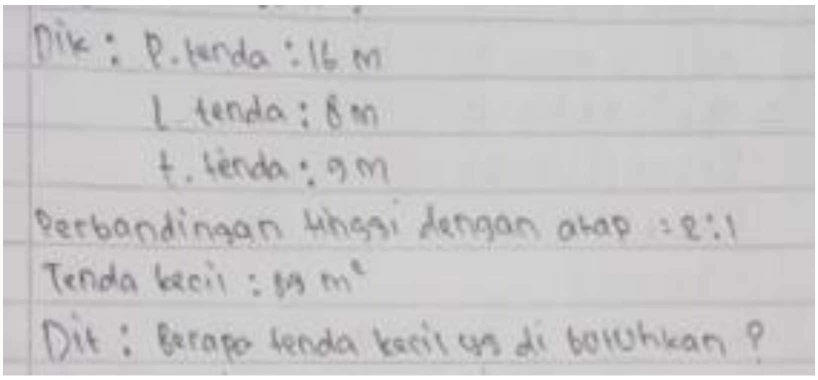
| **Indicator** | $x_{min}$ | $x_{max}$ | **LAG (N= 11)** | | **MAG (N = 10)** | | **HAG (N = 11)** | | **Score $\bar{x}$** | **%** |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | **Skor $\bar{x}$** | **%** | **Skor $\bar{x}$** | **%** | **Skor $\bar{x}$** | **%** | | |
| *Sense making* | 0 | 3 | 2,91 | 96,96 | 3 | 100 | 1,63 | 54,55 | 2,51 | 83,33 |
| *Conjecturing* | 0 | 3 | 0,64 | 21,21 | 1,1 | 36,67 | 1,27 | 42,42 | 1,00 | 33,33 |
| *Convincing* | 0 | 3 | 0 | 0 | 0,27 | 9,09 | 2,36 | 78,78 | 0,91 | 31,25 |
| *Reflecting* | 0 | 3 | 0,09 | 3,03 | 0,5 | 16,67 | 2,36 | 78,78 | 0,97 | 32,29 |
| *Generalizing* | 0 | 3 | 0 | 0 | 0,7 | 23,33 | 2,09 | 69,70 | 0,97 | 32,29 |
| **Score Total** | 0 | 15 | 3,64 | | 5,57 | | 9,73 | | 6,36 | |
| **Average Values** | | | 24,24 | | 37,15 | | 64,86 | | 42,40 | |

The above table consists of students’ achievement related to the five indicators of Bjuland’s (2007) and features three groups classification on students’ performance to achieve the indicators of MRA namely low achiever group (LAG), moderate achiever group (MAG) and high achiever group (HAG).

### 3.1.1. Sense-making

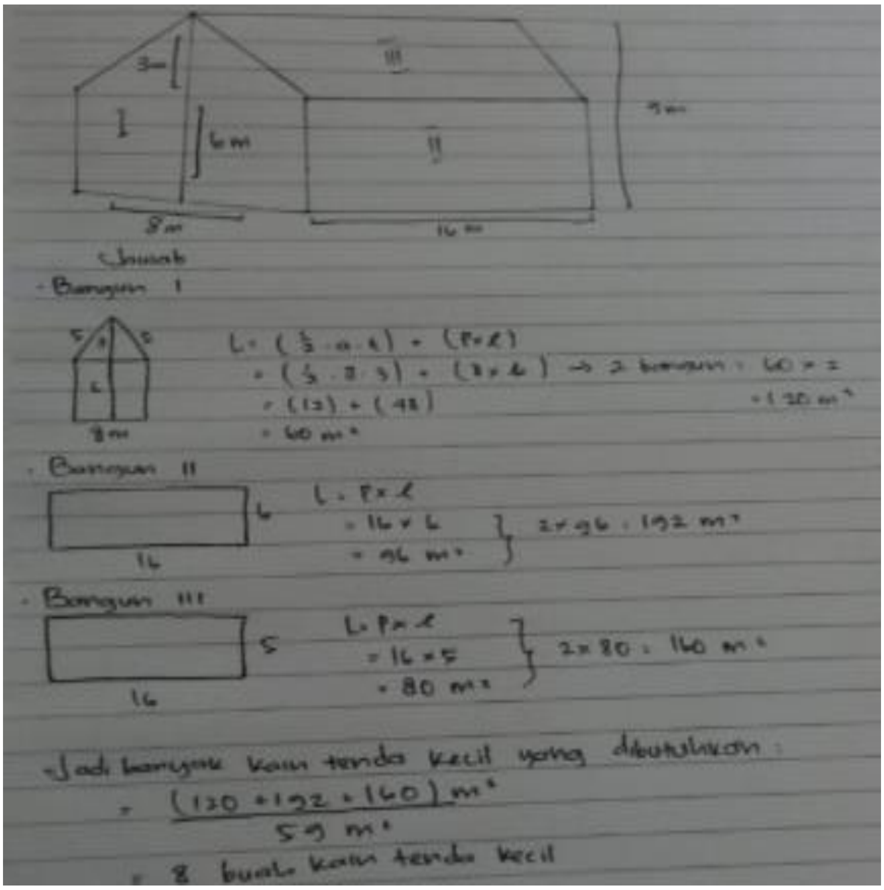
In this indicator, all students from three groups could perform well as can be seen in table 2. Their average achievement was 83.33%. This means that their MRA is above the Minimum Criteria of Achievement that is 65. Students from LAG achieved 96.9%, MAG 100%, and HAG 54.55%. They were able to understand the problem presented by analyzing the situation then try to communicate it into symbols or mathematical language by identifying known elements. To exemplify, in figure 1 below, it can be seen how students applied the indicator to a mathematical problem presented in the test.

**Figure 2.** Showing the student achieved sense-making indicator.



In figure 2, several known elements were written to demonstrate student's achievement of first indicator. The student could see and build the link between mathematical ideas or concepts, mathematics and objects, and mathematics and everyday life. They were able to discover what sort of spaces are available for a particular military tent in the mathematical problem presented. They can identify the elements being asked, namely how many small tents are needed to cover the entire surface of the large tent if the area of the small tent is 59 m2. On the other hand, some students failed to achieve this indicator. It can be seen in figure 3.

**Figure 3**. Showing the student did not achieve sense-making indicator.



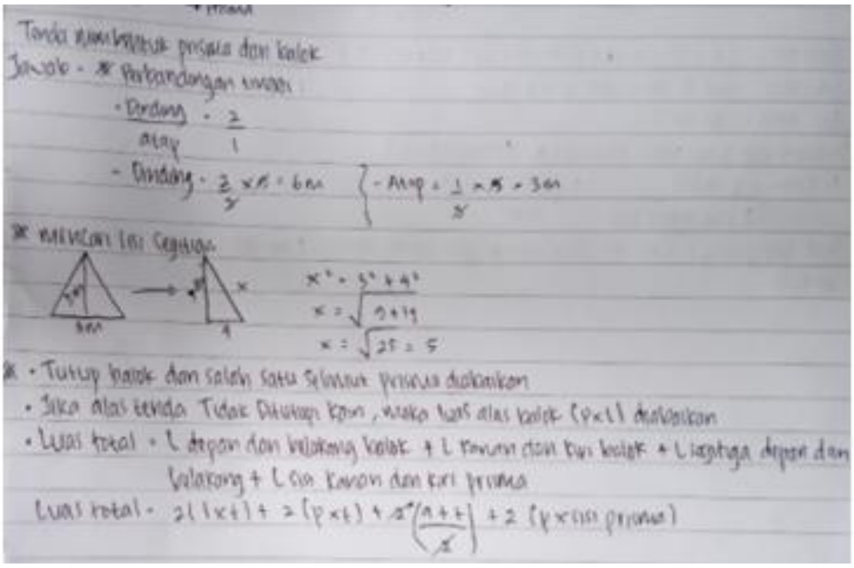

The figure shows that the student had not performed the ability to make sense of the mathematical problem presented. In this figure, students missed essential information that should be taken from the mathematical problem presented.

### 3.1.2. Conjecturing Indicator

Table 1 shows that students' achievements for this indicator reached 33.33%, with each group achievement as follows: students from LAG 21.21%, MAG 36.67%, and HAG 42.42%. This shows that student exhibited their ability in identifying incomplete facts and concepts then predict accomplishment strategy. By looking at the military tent image, students could identify and write abstractly by illustrating in the form of a combination of blocks and prisms. Next, they wrote down the formula for the area of a bar without a base and a roof and also write down the formula for the area of a triangular prism without a base because it is part of the tent space.

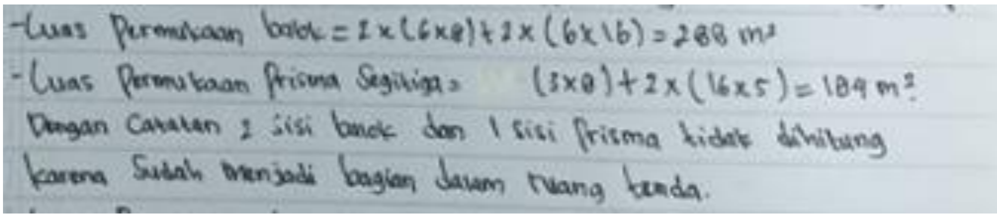
What students should do is write down the formula for the surface area of the block A=2lh+2lh = and the formula for the area of the prism = L = (2 × areas of the triangle) + (2 × the perpendicular plane of the prism) = (2 × 1/2 × a × h ) + (2 × w × l). From this formula, it can be seen that in an upright prism there is an unknown element, namely the width, so to find the width of the prism, the relationship of the three sides of a right triangle can be used. Pythagorean triple is 5 as shown in Figure 4 below:

**Figure 4**. Showing the student achieved conjecturing indicator.



In figure 4, the student achieved the indicator of conjecturing. The work has demonstrated the formula needed to accomplish the process and solve the mathematical problem. On the other hand, some students failed to achieve the indicator. Figure 5 shows the example from HAG student.

**Figure 5**. Showing the student did not achieve the conjecturing indicator.

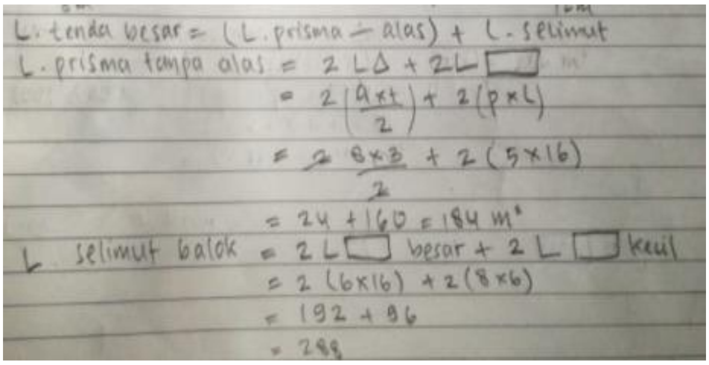



Their work did not identify the problem concerning incomplete facts and concepts, namely the surface area of the rectangular prism.

### 3.1.3. Convincing Indicator

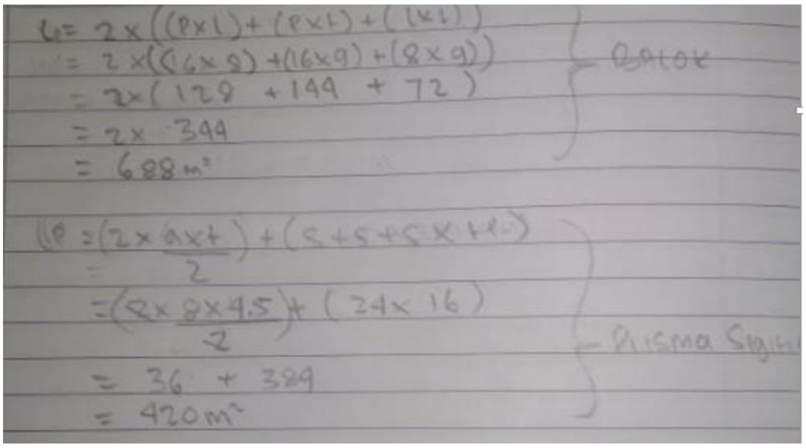
In table 2, student achievement on the indicator was 31.25% with the achievement of LAG 0%, MAG 9.09%, and HAG 78.78%. Some students had achieved the indicator of convincing when dealing with the steps of MRA to solve the problem. In general, students from HAG could use the information obtained to determine the next pattern in solving the problem. They accompanied this process by relevant reasons so that they were able to implement the settlement strategy following the previous stage by applying the formula: A= 2 lh + 2 wh and the formula for prism area = L = (2 × areas of the triangle) + (2 × perpendicular prisms) = (2 × 1/2 × a × h) + (2 × w × l) then substitute the known elements into the formula.

**Figure 6.** Showing the student achieved convincing indicator




This figure shows that the work represents the formula needed to accomplish the problem and the formula demonstrated students' knowledge to accomplish settlement strategy. On the other hand, some students did not achieve the convincing indicator. One of the examples can be seen in figure 7 below.

**Figure 7**. Showing the student did not achieve the convincing indicator

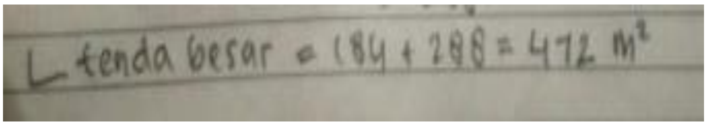



This figure shows that the student failed to achieve the indicator of convincing since the formula of accomplishment was not demonstrated in this work. Students from LAG and MAG experienced difficulties and even they were not able to accomplish this process. They failed in this step because in the previous process they could not plan strategies or plan solutions in understanding the problems given. Furthermore, they did not know the formula to solve this problem. Even though they know the formula, they still got it wrong in that respective stage. Their response was automatically wrong.

### 3.1.4. Reflecting Indicator

In table 2, the average students' achievement on the indicator is 32.29% with students from LAG= 3.03%, MAG= 16.67%, and HAG= 78.78%. In figure 7, a student achieved the indicator.

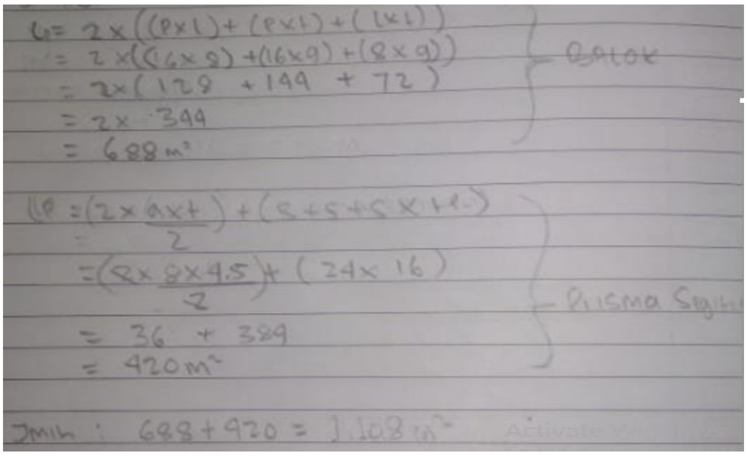
**Figure 8.** Showing the student achieved reflecting indicator



This figure shows that student had achieved the indicator of reflecting since the work has shown the ability to re-evaluate the three previous stages and to relate the problem with relevant theories to solve the mathematical problem. Most students from HAG could re-evaluate the three processes that had been carried out in the previous stages. They looked back at the relationship with theories that are considered relevant, namely seeing the military tent which is a rectangular prism in terms of its space. Given this knowledge, they calculated the surface area of the large tent which is a combined surface area of the block with the surface area of the prism whose calculations have been carried out in the previous step.

Some student did not achieve a reflecting indicator. An example can be seen in figure 9.

**Figure 9**. Showing the student did not achieve reflecting indicator



This figure shows that the student did not achieve reflecting indicator because the work failed to demonstrate the knowledge of the three previous steps to identify unknown elements, formulate the settlement strategy and solve the mathematical problem.

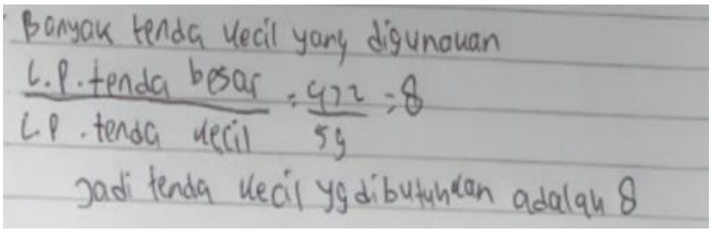
### 3.1.5. Generalizing Indicator

In table 2, students generally achieved 32.29%. LAG students achieved 0%, MAG 23.33% and HAG 69.7%. Most HAG students could carry out activities in this step to draw logical conclusions. They obtained the result from the whole process. They identified the problem and followed the steps to generalize the result. In this step, students should determine the amount of small tent canvas material needed to cover the military large tent. This should be expressed in the formula as follows:

small tents canvas needed $= \frac{\text{the surface area of the military tent}}{\text{the surface area of the small tents}} = \frac{472}{59} = 8.$

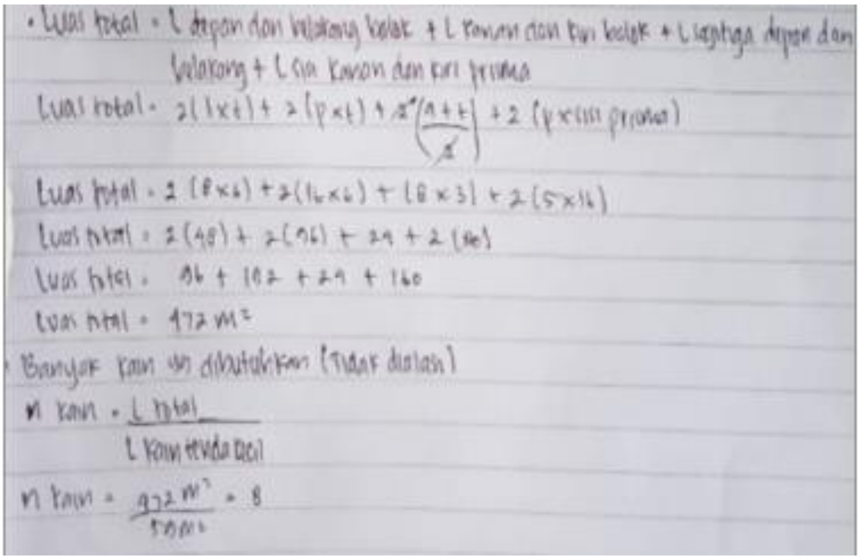
To this point, the step carried out by students was not sufficient to end the answer. Students must be able to logically mention the conclusion as a series of the whole process. This process can be concluded using the statement: to cover the military large tent, it takes 8 pieces of small tent canvas material. An example of a student's achievement of this generalizing indicator can be seen in figure 10 below.

**Figure 10**. Showing the student achieved generalizing indicator




This figure shows that the student had achieved the indicator of generalizing since the work demonstrates the ability to conclude the previous stages and identify and generate the solution to the mathematical problem. On the other hand, some students didn't achieve the generalizing indicator. It can be seen in figure 11.

**Figure 11.** Showing the student did not achieve generalizing indicator



The figure has shown that the work had not achieved the generalizing indicator. It failed to demonstrate the previous stages consisting of identification of unknown elements, formula creation, and steps needed to solve the problem.

### 3.2. Cause of Failure to Achieve Indicators of MRA

In the previous section, the examples of students' work that did not achieve the indicators have been described. In this section, the causes of their failure to achieve the indicators will be presented. The data were taken from the interview.

### 3.2.2. The Cause for not Achieving a Conjecturing Indicator

In figure 3, a student from HAG failed to achieve a sense-making indicator. The researchers interviewed student 12 to find out the cause.

*"I do not really understand how to identify the known elements and questionable elements in this matter. I found the numbers written from the problem. The elements that are known and asked in this question are the length of the tent = 16 m, the width of the tent = 8 m, and the height of the tent = 9 m with the*

*ratio of the height of the tent to the height of the roof = 2: 1 and it is asked how many small tents are needed to cover the entire surface of the large military tent if the area of the small tent is 59 m2, but I didn't write it down. I don't think I should write it down, but it's enough to just write down the answer.”*

Based on the excerpt of the interview above, it is clear that the student's failure to achieve this indicator does not refer to his inability to identify or analyze the problem. He knew the essential information but he skipped this stage because he thought that it was not necessary. After the interview, he became aware that the process of analyzing and identifying problems needs to be done and accomplished.

### 3.2.2. The Cause for not Achieving a Conjecturing Indicator

Students from LAG and MAG had difficulty achieving the conjecturing indicator. They failed to create an abstract of the picture and to illustrate it. Given this situation, they were not able to solve the problem. An interview with student 27 explains the cause.

*“I don’t feel that I have to write down the formula to solve the problem, hmmm I think I can immediately solve the problem. I answer that problem only from the problem given. I try to explain the reason I answered like that, namely: the tent building consists of a block without two sides and a triangular prism without 1 prism side). Because it is part of the space of the tent, I wrote in my answer i a description of the formula for the surface area of the block rectangular prism A=2lh+2wh and the formula of the surface area of the prism = A (2xsurface area of a triangle) + (2xsurface area of prism= (2X surface area of the prism)=(2 x 1/2 x a x t)+(2 x p x l)*

The student continued to explain that he knows the formula but he skipped the process to write the formula of the essential information. He stated in the excerpt of the interview as follows:

*“now from the formula, I gained the answer that I described in my answer sheet, ma'am. I didn't know that writing down the formula was part of the problem-solving process. I also didn't write down the cube height and prism height. But I know where the cube height and prism height are obtained from the tent height = 9 m with the ratio of the cube height to the prism height is 2: 1 so that the cube height = 2/3 × 9 = 6 m and the prism height = 1/3 × 9 = 3 m. Oh yes ma'am, I also forgot to write down the width of the prism, but I know the width of the prism from the height of the prism = 3m, length = 1/2 x 8 = 4 m so that the width of the prism which is the hypotenuse is obtained from the Pythagorean triple 3, 4, 5 so the width the prism is 5. "*

The failure to achieve the conjecturing indicator illustrates their low ability in identifying unknown elements from the mathematical problem given. Since they had a low ability to analyze a reading text, students failed to predict, formulate answers, develop strategies for solving problems.

### 3.2.3. The Cause for not Achieving a Convincing Indicator

In figure 6, the student failed to achieve a convincing indicator. The cause for the failure to achieve the indicator can be seen in the excerpt of an interview with student 14 as follows:

*“I know the formula mam. I think I don’t need to write the steps that’s why I immediately write the description. I know it is important to follow the steps. I should write known elements of information and identify an unknown element of information from a particular mathematical problem. I can do it in the future mam.”*

Based on the interview, the cause relates to the inability to identify things that are not yet known. He did not understand the basic concepts of matter related to the flat side space. For this reason, in completing the calculation, a basic concept or formula is needed so that students will be able to perform calculations properly, correctly, and in a structured manner

### 3.2.4. The Cause for not Achieving a Reflecting Indicator

In figure 8, a student failed to achieve a reflecting indicator. The cause for not achieving the indicator is explained in the following excerpt of the interview with student 6.

*"I know maam, I had made a mistake in the previous steps. That is why I could not accomplish to solve the problem. I can revise the fault from the previous step so I can calculate correctly to solve the problem"*

LAG and MAG students could not complete this step because they did not understand the previous steps. However, there are some MAG students who could re-evaluate the three previous processes but in the previous step, they could not answer correctly. This means that they knew the concept and knew how to calculate the surface area of the tent is to add the area of the block to the area of the prism, but they don't know the formula so it's wrong in planning and implementing the solution strategy.

### 3.2.5. The Cause for not Achieving a Generalizing Indicator

In figure 11, a student failed to achieve the generalizing indicator. The cause for this failure was explained by student 15 in the interview.

*"I have double-checked ma'am, the answer is 8. This means that 8 pieces of tent cover are needed to cover the entire large military tent. And I didn't know that I should write this explanation as to the final answer. "*

Based on the interview, the student did not conclude the previous steps and they directly wrote the answer 8 small tent cover needed.

## 4. Discussion

In the previous section, all the findings to answer research questions one and two have been presented. This section will discuss the findings of this study with the relevant literature and previous studies to position the present study and state what it contributes to the body of knowledge.

This study found that most students from LAG, MAG, and HAG achieved the sense-making indicator. A student from HAG did not achieve this indicator. His failure to achieve this indicator does not refer to his inability to identify or analyze the problem. He skipped this stage because he thinks this is not necessary. The process of analyzing and identifying problems is something that needs to be done and is one of the reasoning processes that must be accomplished **(B. Y. . Putra et al., 2020)**. Other studies reported that students are expected to be able to analyze and identify mathematical problems in junior high school **(Neneng Tita Rosita et al., 2020)** and in vocational school **(Aisyah et al., 2016)**. Students should follow and accomplish each step of **MRA (C. D. Rosita, 2014)**.

Concerning the conjecturing indicator, this study found that some students did not achieve the indicator. This occurs because students could not understand the problem given in the reading text.

They were not able to analyze a reading text so that students could not predict, formulate answers, develop strategies for solving problems. Students should evaluate the statements in math problems to allow them to proceed and solve the problem **(Payadnya, 2019; Supriadi et al., 2021)**. In this stage, students are required to harness their ability to analyze, information, understand concepts, regulation, rules, obtain scientific findings and draw a conclusion. If they fail to accomplish this step, they will suffer from difficulty to solve the mathematical problem **(Arivina & Prabowo, 2017).**

When it comes to the convincing indicator, this study found that some students could not understand the contents of the reading questions given. Students need a basic concept or formula so that they will be able to perform calculations properly and in a structured manner. In this phase, students are required to possess high reasoning skills to demonstrate their logical, critical, and creative thinking skill. Creativity, basic knowledge, critical thinking, communication skills can be a source for students to achieve MRA (**Tisngati & Genarsih, 2021)**.

Regarding reflecting and generalizing indicators, since students failed to achieve the indicators in the previous steps, they were not able to solve the mathematical problem. Students need to identify the amount required to solve the problem and determine the relationship among elements in the problem. They should be able to infer, study, and exploit the laws, axioms, and rules of symbol manipulation to solve the problem. Teachers are in a position to train them to read the formulation of the problem and understand what the mathematical problem is asking from students in the context of the storyline **(Lepak et al., 2018)**. Having ample exercises, students could achieve MRA which is a core skill in human intelligence **(Saxton et al., 2019)**.

In this study, many students from LAG and MAG performed poorly to achieve the indicators of MRA. When it comes to correcting students' mistakes, the teachers should manage themselves to spend time asking students to elicit students' thinking when they made a mistake. This effort may open up the opportunity and enhance the pedagogical response to allow the student to recognize their mistakes and revise their work **(Shaughnessy et al., 2020).**

This study used the framework of **Bjuland (2007)** proposing five steps to demonstrate students MRA. The five indicators of MRA demonstrate complete thinking activity before reaching a logical conclusion to solve a mathematical problem **(Minarni, 2010)**. Teachers may apply some methods to help students achieve MRA in dealing with geometry topics. One of the approaches is problem-solving **(Rott et al., 2021)**. Other approaches to teach mathematics include the Inductive approach for students in junior high school **(Rochmad, 2010)**, and commognitive point of view **(Zayyadi et al., 2019).** Students need also to learn algebraic reasoning to achieve good competence in mathematics **(N. T Rosita, 2018)**.

Teachers need strong evidence when students produce correct answers. Without an explanation of the solution from students, teachers should seek evidence of their MRA **(Hughes et al., 2020)**. Mathematics learning teaches people to solve problems by paying attention to a procedure or process that prioritizes pedagogical aspects through a scientific approach so that students can understand more meaningfully through the process of observing, asking, trying, reasoning, presenting and creating **(Quigley, 2011); (Hidayat et al., 2018a)**.

This study also shares one thing in common with the report released by TIMSS and PISA concerning students' low performance in mathematics. In line with the findings of this study, several researchers reported students' poor performance in MRA for junior high school **(Rizqi, N.R., & Surya, 2017)**, in senior high school **(Anshori et al., 2018)**. This study found some causes for students' failure to achieve the indicators of MRA. In line with these findings, some studies reported several problems in learning mathematics including artificial reasoning, which means students tend to use routine procedures when dealing with reasoning **(Sukirwan et al., 2018)**. In an inclusive classroom, the challenge will be higher to teach mathematics **(Griffin et al., 2013).**

Considering the problem related to learning mathematics, teachers should pay attention to the way students learn **(R. W. . Putra, 2017)**, the way it is delivered like using adversity quotient and argument-driven inquiry learning **(Hidayat et al., 2018b)**, and creative and imitative reasoning **(Lithner, 2008)**. Teachers may seize educational technology to assist mathematics instruction **(DEBBAG et al., 2021)**, to assist low achiever students **(Baccaglini-Frank, 2021**), and to achieve one of its goals that is MRA **(Saal et al., 2019)**.

## 5. Conclusion

Based on the results of the research conducted, it can be concluded that in general, there are still many students who experienced problems achieving the five indicators of MRA. In this study, HAG students exhibited good performance to achieve the indicators of MRA but LAG and MAG students had a low level of performance to achieve the indicators of MRA developed by **Bjuland (2007).** Some students were still unable to identify the unknown elements in question, understand a problem from the reading that was presented, perform numeracy skills, plan and implement the formula to solve the mathematical problem. They experienced those conditions because they failed to attain the indicators in each stage. It was difficult to solve the mathematical problem as the students failed to accomplish it in each stage. The causes of their failure to achieve the five indicators varied. According to students, they rarely practice solving a mathematical problem, applying formulas, and demonstrating numeracy skills. The important thing that students must remember and apply is that reasoning is a thought process or activity whose achievement must go through the stages of identifying, strategizing, implementing, evaluating to drawing logical conclusions, all of which are complete processes or stage.

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