Mathematical Error Patterns to Facilitate Solving Math Problems for Junior High School Students

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Mathematical understanding is important in problem-solving. This study aimed to find patterns of mathematical understanding errors in solving mathematics problems for junior high school students. The method used in this study is qualitative. Data collection uses triangulation. Interviews were conducted on three subjects during one month at Islamic Junior High School. Data collection techniques are carried out through triangulation, namely test results, interviews, and observations. The intrusion used is a math problem-solving test. This study found patterns of errors in solving mathematical problems in the field of geometry. From the results, it was found that the biggest error pattern was in calculating the area and perimeter of rectangles and parallelograms. Stakeholders can also use the results of this research in education to encourage understanding of geometric concepts in education at the secondary level. This study offers educational authorities policy options regarding the importance of understanding mathematics in general and geometry in particular for students.

Keywords: mathematical error, error patterns, problem solving, math education

INTRODUCTION

Mathematical understanding is one of the basics of practicing physics. The development of mathematical understanding involves mathematical structures and actions (Kaba & Şengül, 2018). An understanding of mathematical learning characterizes and articulates the growth of mathematical understanding and mathematical ways of thinking (Gulkilik et al., 2020). The interaction between learning strategies and critical thinking skills influences a student's mathematical understanding (Andayani et al., 2020). Mathematical understanding forms the initial conception for the meaning of a mathematical concept or initial idea in solving a particular mathematical problem (Yao & Manouchehri, 2020). A person's

mathematical ability can facilitate problem-solving. Problem-solving is one of the most important human cognitive activities (Gr. Voskoglou, 2021). One of the mathematical abilities is to master mathematical concepts, which include interrelated mathematical objects (Simon et al., 2018). Basic mathematical concepts that cannot be avoided include algebra and geometry. There are mathematical skills that can be used to understand concepts, reason, solve problems, communicate mathematical ideas, and make mathematical connections (Hafiz et al., 2017). The basic concept of geometry can solve everyday life problems. Geometry is a study of space and shape that requires a high-level of visualization skills (Liburd & Jen, 2021).

However, in reality, the basic concepts of mathematics, including geometry, are not understood by many students (Maričić & Stamatović, 2018). According to Baye et al. (2021), the inclusion of a mathematic learning curriculum can improve a student's understanding of mathematical concepts, including the basic concepts of geometry that are necessary for solving everyday problems. The concept of geometry is used for problem-solving and critical thinking skills that can be used in the coding environment of students (Kim et al., 2021). Understanding of matter is not balanced with prerequisite material as the primary link. Group prerequisite material requires logic to recognize mathematical structures (Durand-Guerrier & Hausberger, 2015; Junarti et al., 2020). Errors in solving mathematical problems result when prerequisite material is not mastered beforehand. The misconception of the prerequisite material is also found in the function material (Kamid et al., 2020). The conceptual error that often occurs is the inability to identify relevant information. This research aims to identify common error patterns in understanding mathematical problems in order to improve mathematical problem solving. This research will assist teachers in improving math learning and highlight the significance and importance of understanding geometry for high school students in the policy to the education authorities.

RESEARCH METHOD

The method used in this study is qualitative. Three students from class VII from every semester of 2021/2022, participated in this study. The three subjects are called Student 1 (S1), Student 2 (S2), and Student 3 (S3). The students were selected based on specific considerations; the student was cooperative, interactive and once interviewed, information regarding the mathematical understanding process of the student could be obtained. The assessments used included the results from math problem-solving tests on geometry subjects as well as information obtained from the interviews.

The mathematical problem-solving tests were first validated in geometry students, before presenting them to S1, S2 and S3. Data was obtained through test results, interviews, and observations. Stages in solving mathematical problems include reading (R), comprehension (C), transformations (T), processing (P) and encoding (E). Data analysis techniques used source triangulation models, namely data collection, data tabulation and presentation, and inference.

RESULT AND DISCUSSION

TABLE 1
STUDENT ACHIEVEMENTS FOR PROBLEM NUMBER 1: THE ABILITY TO MASTER BASIC MATHEMATICAL CONCEPTS

Student	**Question 1**				
	R	C	T	P	E
S1	√	√	X	X	X
S2	√	√	√	X	X
S3	√	√	√	X	X

R=Reading; C=Comprehension; T=Transformation; P=Process Skill; E=Encoding

At stages R and C, all students have answered correctly (Table 1). For S1 at stage T, writing the formula to solve mathematical problems is incorrect, resulting in the subsequent stages being incorrect. S2 and S3 were unable to perform algebraic operations of the known rectangle in the problem, therefore, stage P is incorrect. The results from the student interviews indicate that the students do not understand algebraic operations. The pattern of the students' mathematical misrepresentations in solving mathematical problems on rectangular and cross-aligned materials involves four error patterns. First, the students do not understand error patterns in algebraic operation if the data in the problem is given in algebraic form. However, the students can logically determine the length in algebraic form because the area of the rectangle is 200 m and the width is 8 m, therefore the length can be determined by a simple division of these values. Third, the area of the road installed ceramics cannot be determined because of the previous errors made by the students. The student does not understand that if each side of the pool is bigger than 2 m, the size of the pool and the road to be made needs to be 29 m long and 12 m wide. The student solves the problem by trial and error to get the length asked, but the next step cannot be performed. Therefore, the students did not understand basic mathematics concepts, including basic geometry concepts. Errors in algebra learning are largely teacher-centered and procedurally oriented, limiting students' opportunities to understand algebra (Litke, 2020). Students have difficulty distinguishing between numbers and signs of operations involved in algebra (Pratiwi et al., 2019). Student reasoning in representing algebra have various performance levels (Lepak et al., 2018).

TABLE 2
STUDENT ACHIEVEMENTS FOR PROBLEM NUMBER 2: CONVERTING QUESTIONS INTO MATHEMATICAL SENTENCES

Student	**Question 2**				
	R	C	T	P	E
S1	√	√	√	X	X
S2	√	√	√	X	X
S3	√	√	√	X	X

R=Reading; C=Comprehension; T=Transformation; P=Process Skill; E=Encoding

At stages R, C, and T, all the students answered correctly (Table 2). However, it appears that the three students cannot convert the problem into a mathematical equation. The students could not write the length of the garden twice the width top = 21. The students experimented in determining the width of the cassava garden by halving the known circumference.

Furthermore, the width of the garden determined is divided by three to get a garden width of 8 m. The interview results indicate that the students did not understand how to convert the problem into a mathematical equation. These results are in line with previous research that found that students had difficulty understanding the meaning of the questions and had difficulty translating questions into math sentences (Sidik et al., 2021).

TABLE 3
STUDENT ACHIEVEMENTS FOR PROBLEM NUMBER 3: CHANGING THE UNIT OF WEIGHT

Student	**Question 3**				
	R	C	T	P	E
S1	√	√	√	X	X
S2	√	√	√	√	√
S3	√	√	√	√	√

R=Reading; C=Comprehension; T=Transformation; P=Process Skill; E=Encoding

The students did not understand the unit of weight, resulting in a miscalculation of the problem. At phase R, C, and T, all students answered correctly (Table 3). However, S1 could not convert ounce to kilogram. In addition to not being able to convert weight units, S1 could not perform functions properly at the process skill stage. The interview results confirm that S1 could not convert ounce to grams because they did not understand how to do so. On the other hand, S2 and S3 work stage by step correctly. Supported by the interview information, these students understand that 1 kg equal 10 ounces. A middle school students' understanding of using units of measure relates to the students' understanding of numbers and measurements (Sidik et al., 2021). In contrast, units of measurement are necessary for conceptual understanding of size and scale (Delgado, 2010). The basic concept of geometry can be used to calculate the amount of ceramic cassava products to be installed in plantations, the amount of income gained from harvesting fish in a pond, tomato harvest income, and others as previous research has stated that understanding units of measurement is essential for search engines (Heidari et al., 2021).

TABLE 4
STUDENT ACHIEVEMENT FOR PROBLEM NUMBER 4: IDENTIFYING RELEVANT INFORMATION WITHIN A MATHEMATICAL PROBLEM

Student	Question 4				
	R	C	T	P	E
S1	√	√	√	X	X
S2	√	√	√	X	X
S3	√	√	√	X	X

R=Reading; C=Comprehension; T=Transformation; P=Process Skill; E=Encoding

The students could write down the problem but did not understand how to identify information within the sentence that was needed to solve the problem. All students answered stages R, C, and T correctly (Table 4). All students could write down what is known or asked, as well as the formula used, but could not determine the length of side AB. S1 could not complete anything at the skill process stage. S2 used the triangle formula by incorrectly inserting known quantities. S3 used the Pythagoras formula to determine the length of the AB side. The students could not determine the base and height of the line during the interview. The students were confused with the different plinths and heights of the same line in determining its breadth. The students demonstrated an inability to identify the relevant information that is beneficial in solving problems. According to Setiani et al. (2018), students with low self-efficacy cannot structure problem-solving in math literacy problems. However, according to Taufik & Zainab (2021), dependent field students can identify information by using sentences in questions and using verbal, symbolic, and representative mathematical models. Identifying relevant information in mathematical problems and arguments affects the different levels of students' reasoning ability to solve problems (Wulandari & Wutsqa, 2019). Computation skills determine a student's ability to identify relevant information to complete calculations (Hegener, 2021). In addition, the ability to identify information in different contexts, learning and developing students assist academic work information literacy skills in managing their well-being (Martzoukou et al., 2021).

Understanding of the material is not balanced with the prerequisite material as the main material link. Prerequisite material is supportive in the breakdown of rectangular and aligned materials. However, student's ability to understand mathematics is still low because students believe that mathematics is not important (Nurapriani et al., 2020). Prerequisite materials use algebraic operations in calculating the size to be sought. Therefore, it is important to emphasize the prerequisite materials students must understand.

CONCLUSION

Mathematical error patterns can facilitate mathematical problem solving of high school students. Patterns of mathematical comprehension errors can be applied in the teaching and learning process to

facilitate mathematical problem solving. Teachers can use the study results to illustrate how important the concept of geometry is for junior high school students. Applying pattern concepts requires the supervision in the implementation of learning. The pattern presented in this study is expected to be an important contribution to parties invested in improving students' understanding of the concept of mathematical geometry both locally, nationally, and internationally.

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