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Students' learning difficulties in integral calculus based on critical thinking skills

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Abstract. This study aimed to describe students' learning difficulties in integral calculus based on critical thinking skills. This study used descriptive exploratory methods with instruments: test, interview, documentation, and observation. Data was taken from 20 undergraduate students from the study program of mathematics education at Universitas Negeri Semarang. The results showed that: (1) there were 24 types of student difficulties identified in calculus lectures; these difficulties consist of 20 types of difficulties related to calculus material (types of difficulties number 1 to 20) and four types of difficulties that were not related to material (types of difficulties number 21 to 24), and (2) these types of difficulties were graded according to the level of student critical thinking skills. A higher level of critical thinking skills had fewer types of difficulties; on the other hand, a lower level of critical thinking skills had more types of difficulties and more fundamental. The fundamental types of difficulties based on the critical thinking skills aspect include understanding integral concepts and drawing graphics.

1. Introduction

The development of critical thinking skills in the industrial revolution 4.0 is a challenge in education. This challenge must be faced so that the benefits of critical thinking skills can be obtained, including helping in (1) differentiating between facts and opinions, (2) giving consideration in making decisions, (3) problem-solving, and (4) getting calmness in solving complex problems [1]. The challenge in developing critical thinking skills are also experienced in learning calculus in university. Calculus is a basic subject that is a prerequisite for other courses and is also useful in many areas of life. The big challenge of learning calculus in developing critical thinking skills is the many difficulties that students experience.

The following are some types of student difficulties in learning calculus; (1) drawing graphs [2], [3], (2) solving inequalities, algebraic functions, and limit functions [4-6] ; (3) the concept of limit and continuity of a function [7], (4) understanding the derivative concept of a function [8], (5) understanding the integral concept of a function [3, 9-13] , and (6) determining the area, the boundary, and using the integral formula [3]. Various types of difficulties experienced by these students will have an impact on learning outcomes and on students' mathematical abilities which are expected to develop such as critical thinking skills. To gain knowledge about the difficulties experienced by students and their relationship with critical thinking skills, it is necessary to investigate the types of difficulties and their relation to the level of critical thinking skills and indicators. There are four indicators of critical thinking skills that can



be used for investigation, including the abilities in (1) analyzing problems, (2) concluding and giving an explanation, (3) evaluating, and (4) choosing problem-solving strategies [1].

Based on the background that has been described, the problem in this study is focused on how to describe students' learning difficulties in integral calculus based on critical thinking skills.

2. Research Methods

The method used in this research is descriptive exploratory. There are four types of instruments used, including tests, interviews, documentation, and observation. The research data was taken in the academic year 2018/2019, conducted at Universitas Negeri Semarang, with research subjects of 20 undergraduate students from the mathematics education study program taken from two classes of integral calculus with different learning models, namely Problem-based Learning (P) and Mathematical Problem Posing (Q).

The test results are used to classify students according to their level of critical thinking skills. In this research, the test instrument was used to find out the achievement of students' critical thinking skills. The reliability index of this test is 0.702 and consists of four items problems to measure the achievement of students' critical thinking skills indicators. The four indicators of critical thinking skills used in this study include the abilities in (1) analyzing problems, (2) concluding and giving an explanation, (3) evaluating, and (4) choosing problem-solving strategies [1]. Students' critical thinking skills achievement are grouped by levels as in Table 1. From the two classes of integral calculus, two students were selected from each level of critical thinking skills as research subjects, so that there was a total of 20 students. The selection and code of research subjects are shown in Table 2. Interviews are used to investigate mathematical critical thinking skills and difficulties experienced by students. The results of observation dan documentation during lectures are used to obtain a description of student learning activities. Research data analyzed and described exploratively.

Table 1. The achievement levels

Scores	Levels
$32 < x \leq 40$	Excellent
$26 < x \leq 32$	Good
$22 < x \leq 26$	Adequate
$16 < x \leq 22$	Marginally Adequate
$0 \leq x \leq 16$	Poor

Table 2. Selection of research subjects based on the level of critical thinking skills

Classes	Critical thinking skills levels				
	Excellent (A)	Good (B)	Adequate (C)	Marginally Adequate (D)	Poor (E)
P	A ₁ P	B ₁ P	C ₁ P	D ₁ P	E ₁ P
	A ₂ P	B ₂ P	C ₂ P	D ₂ P	E ₂ P
Q	A ₁ Q	B ₁ Q	C ₁ Q	D ₁ Q	E ₁ Q
	A ₂ Q	B ₂ Q	C ₂ Q	D ₂ Q	E ₂ Q

3. Results and Discussion

3.1. Students distribution based on critical thinking skills levels in integral calculus

The results of student achievement in the test are classified and analysed based on their level of critical thinking skills. Table 3 shows the distribution of students based on the level of critical thinking skills.

Based on Table 3, it is known that there are 17 students (31.48%) who are classified as excellent level in critical thinking skills, 12 students (22.23%) who are classified as good level in critical thinking

skills, 11 students (20.37%) who are classified as adequate level in critical thinking skills, 7 students (12.96%) who are classified as marginally adequate level in critical thinking skills, and 7 students (12.96%) who are classified as poor level in critical thinking skills. From the two classes of integral calculus in the next step, two students were selected from each level of critical thinking skills as research subjects, so that there was a total of 20 students. Based on Table 3, it is known that the two classes with different learning models (Problem-based Learning (P) and Mathematical Problem Posing (Q)) are quite successful in developing critical thinking skills with a large percentage of students at the good and excellent level. This finding is relevant to previous research which states that Problem-based Learning and Mathematical Problem Posing can be used in the development of critical thinking skills [1, 14-17]. On the other hand, data is obtained that students still experience difficulties in learning Integral Calculus as presented in Table 4.

Table 3. Students distribution based on critical thinking skills levels

Students' critical thinking skills levels	Students distribution			
	Classes		Total	Percentage (%)
	P	Q		
Excellent	8	9	17	31.48
Good	6	6	12	22.23
Adequate	6	5	11	20.37
Marginally Adequate	5	2	7	12.96
Poor	3	4	7	12.96
Total	28	26	54	100.00

3.2. Students' learning difficulties in integral calculus based on critical thinking skills

Based on the analysis of results of test and interviews, a recapitulation of students' learning difficulties in integral calculus based on critical thinking skills was obtained as presented in Table 4. Based on Table 4, information is obtained that students at excellent level of critical thinking skills experience difficulties of 50% of the problems given, with six types of difficulties, namely difficulty in (1) finding trigonometric antiderivatives, (2) applying the integral formula for the volume of a rotating object, (3) proving or analysing statements about antiderivatives, (4) proving or explaining definite integral theorems, (5) finding integral bounds, and (6) difficulty caused by forgot the material. In general, students at excellent level of critical thinking skills have no difficulty in drawing graphics and based on the documentation they were able to overcome difficulties up to 87.50%.

Students at good level of critical thinking skills experience a difficulty of 68.75% of the problems given, with eight types of difficulties, namely difficulty in (1) linking definitions and statements, (2) making definite integral examples, (3) drawing the area graph (correct graph result), (4) drawing the volume graph of a rotating object (unable to draw), (5) proving or explaining definite integral theorems, (6) difficulty caused by did not careful in solving problems, (7) difficulty caused by anxiety, and (8) difficulty caused by forgot the material. Based on the documentation they were able to overcome the difficulties up to 18.18%, 45.45% were incomplete, and 36.37% failed to be overcome. As in the 4th indicator or the ability to develop problem-solving strategies, subject B₁P did not able to formulate, compile or choose a problem-solving strategy and implement it, the subject has difficulty since drawing objects on the graph so that unable to draw it. Students at adequate level of critical thinking skills experience a difficulty of 75% of the problems given, with ten types of difficulties, namely difficulty in (1) applying formulas in the integration process, (2) making examples of antiderivatives, (3) drawing the area graph (correct graph result), (4) drawing the volume graph of a rotating object (incorrect graph results), (5) proving or explaining definite integral theorems, (6) difficulty caused by did not understand definite integral theorems, (7) difficulty caused by did not understanding the application of definite integral for finding volume, (8) difficulty caused by did not careful in solving problems, (9) difficulty caused by did not confident, and (10) difficulty caused by forgot the material. Based on the

documentation they have not been able to overcome the difficulties, it was found that 8.33% were incomplete, and 91.67% failed to be overcome.

Students at marginally adequate level of critical thinking skills experience a difficulty of 81,25% of the problems given, with 13 types of difficulties, namely difficulty in (1) finding trigonometric antiderivatives, (2) applying the integral formula for the volume of a rotating object, (3) linking definitions and statements, (4) drawing the area graph (unable to draw), (5) drawing the area graph (correct graph result), (6) drawing the volume graph of a rotating object (unable to draw), (7) drawing the volume graph of a rotating object (correct graph result), (8) proving or explaining definite integral theorems, (9) finding integral bounds, (10) difficulty caused by did not understand definite integral theorems, (11) difficulty caused by did not understanding the application of definite integral for finding areas, (12) difficulty caused by did not careful in solving problems, and (13) difficulty caused by forgot the material. Students at poor level of critical thinking skills experience a difficulty of 100% of the problems given, with ten types of difficulties, namely difficulty in (1) finding trigonometric antiderivatives, (2) determining integration techniques, (3) drawing the area graph (unable to draw), (4) drawing the area graphs (incorrect graph results), (5) drawing the volume graph of a rotating object (unable to draw), (6) proving or analyzing statements about antiderivatives, (7) proving or explaining definite integral theorems, (8) difficulty caused by did not understand the concept of antiderivatives, (9) difficulty caused by did not understand definite integral theorems, and (10) difficulty caused by forgot the material. Students at marginally adequate and poor level of critical thinking skills fail to overcome their difficulties.

Table 4. Students difficulties based on levels and indicators of critical thinking skills

Students' critical thinking skills levels		Students difficulties based on indicators of critical thinking skills							
		1		2		3		4	
		P	Q	P	Q	P	Q	P	Q
Excellent	A ₁	At	Pr1	Frg	Pr2	-	-	-	Bnd
	A ₂	-	-	-	Pr2	-	-	Frm2	Frg
Good	B ₁	-	-	Pr2	Ex2	Dr1(+)	-	Dr2(-)	-
	B ₂	Frg	Lnk	Pr2	Pr2	Crf	-	Anx	Crf
Adequate	C ₁	-	-	Cnf, Und2	Frg	Dr1(+)	Frg	Frm1	Dr2(+,-), Apl2
	C ₂	-	Ex1	Und2	Pr2	-	Dr1(+)	Crf	Dr2(+,-)
Marginally Adequate	D ₁	At	-	Frg	-	Apl1, Bnd	Apl1, Dr1(-)	Frg, Dr2(-)	Dr2(-), Frm2
	D ₂	-	Frg, Lnk	Und2	Frg	Crf	Dr1(+), Bnd	Crf	Dr2(+)
Poor	E ₁	Und1	Pr1, Und1	Und2	Pr2	Tch	Dr1(-)	Dr2(-)	Dr2(-)
	E ₂	Und1	At	Frg	Pr2	Dr1(-)	Dr1(+,-)	Dr2(-)	Dr2(-)

Description of the types of difficulty:

1. Dr1 (-) : Difficulty in drawing the area graph and unable to draw the graph
2. Dr1 (+,-) : Difficulty in drawing the area graphs and the graph result incorrect
3. Dr1 (+) : Difficulty in drawing the area graph but the graph results correct
4. Dr2 (-) : Difficulty in drawing the volume graph of a rotating object and unable to draw the graph
5. Dr2 (+,-) : Difficulty in drawing the volume graph of a rotating object and the graph result incorrect

- 6. Dr2 (+) : Difficulty in drawing the volume graph of a rotating object but the graph results correct
- 7. At : Difficulty in finding trigonometric antiderivatives
- 8. Pr1 : Difficulty in proving or analyzing statements about antiderivatives
- 9. Pr2 : Difficulty in proving or explaining definite integral theorems
- 10. Frm1 : Difficulty in applying formulas in the integration process
- 11. Frm2 : Difficulty in applying the integral formula for the volume of a rotating object
- 12. Tch : Difficulty in determining integration techniques
- 13. Bnd : Difficulty in finding integral bounds
- 14. Und1 : Difficulty caused by did not understand the concept of antiderivatives
- 15. Und2 : Difficulty caused by did not understand definite integral theorems
- 16. Apl1 : Difficulty caused by did not understanding the application of definite integral for finding areas
- 17. Apl2 : Difficulty caused by did not understanding the application of definite integral for finding volume
- 18. Lnk : Difficulty in linking definitions and statements
- 19. Ex1 : Difficulty in making examples of antiderivatives
- 20. Ex2 : Difficulty in making definite integral examples
- 21. Frg : Difficulty caused by forgot the material
- 22. Crf : Difficulty caused by did not careful in solving problems
- 23. Cnf : Difficulty caused by did not confident
- 24. Anx : Difficulty caused by anxiety

Based on Table 4, it is known that: (1) there are 24 types of difficulties identified in integral calculus courses, these difficulties consist of 20 types of difficulties related to calculus material (types of difficulties number 1 to 20) and four types of difficulties that are not related to material (types of difficulties number 21 to 24). (2) The types of difficulties experienced by students based on the student's critical thinking skills level, the higher the critical thinking skills level, the fewer types of difficulties experienced, conversely the lower the critical thinking skills level, the more types of difficulties they experienced. (3) Types of difficulties based on the student's critical thinking skills level, the higher the critical thinking skills level, the types of difficulties experienced can be overcome so that students are able to solve problems correctly. Conversely, the lower the level of critical thinking skills, the types of difficulties they experience fail to overcome so that students are unable to solve problems correctly. And (3) the types of difficulties experienced by students based on the student's critical thinking skills level, the lower the critical thinking skills level, the types of difficulties experienced by students tend to be more fundamental. The fundamental types of difficulties based on the critical thinking skills aspect include understanding integral concepts and drawing graphics.

This finding is relevant to previous research which states that in learning calculus, students can experience difficulties in (1) drawing graphs [2,3] (2) understanding the concept of integral functions [3, 9-13], and (3) determining the area, the boundary, and using the integral formula [3]. When students face difficulties in learning calculus, some students can overcome difficulties, some incomplete, and some have failed. This can be influenced by the benefits of their critical thinking skills [1] and their positive mathematical disposition [18].

4. Conclusion

Some conclusions are obtained based on the results and discussion that have been described, these conclusions include: (1) there were 24 types of student difficulties identified in calculus lectures; these difficulties consist of 20 types of difficulties related to calculus material and four types of difficulties that were not related to material, and (2) these types of difficulties were graded according to the level of student critical thinking skills. A higher level of critical thinking skills had fewer types of difficulties;

on the other hand, a lower level of critical thinking skills had more types of difficulties and more fundamental. The fundamental types of difficulties based on the critical thinking skills aspect include understanding integral concepts and drawing graphics.

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