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Submission date: 12-Apr-2023 09:12AM (UTC+0700)
Submission ID: 2062122235
File name: Exact_meromorphic_solutions_of_cubic_Ostrovsky_equation.pdf (643.85K)
Word count: 2641
Character count: 13121

Journal of Physics: Conference Series

PAPER · OPEN ACCESS

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To cite this article: Herry F. Lalus and N P Aryani 2021 J. Phys.: Conf. Ser. **1918** 022027

Exact meromorphic solutions of cubic Ostrovsky equation: Kudryashov method

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Abstract. We analyze the cubic Ostrovsky equation which is a modification of Ostrovsky equation. This equation can be found in physical phenomena such as wave mechanism due to a rotating media. We use the Kudryashov method to solve this equation. Since this equation is one type of nonlinear partial differential equation, we initially transformed this equation into its nonlinear ordinary differential equation form. Next, we compute the Laurent series of this equation and we get two types of Laurent series with a second-order pole; the coefficients of these series are complex numbers so that the symmetrical form is complex conjugation. Based on these two Laurent series, we construct two types of solutions, each of which consists of elliptic solution (doubly periodic) in the form of Weierstrass-$\wp$ functions, simply periodic solutions, and rational solution.

1. Introduction

One of the most interesting topics in mathematics and nonlinear physics is the problem of solving nonlinear partial differential equations. Nonlinear partial differential equations play a very important role as a language in explaining dynamics in nonlinear physics. Many important and current phenomena require nonlinear partial differential equations to explain them [1–6]. Because this equation generally contains differentiation terms that are spatially dispersed and nonlinear terms that have the opposite properties, the combination of these terms in an equation will have many implications, especially in one of the most important physical applications, namely the soliton phenomenon. Solitons are a type of solution to nonlinear partial differential equations that most people are interested in because of their localized and stable nature over a long time and spatial range. This is what is widely applied/ studied in various fields such as [7,8], [9].

By looking at the brief explanation above, it is important to have a good understanding of how to analyze the solutions to these equations. Nowadays, many methods have been developed for analyzing the solution of nonlinear partial differential equations such as the Exp method [10], Tanh method [11], and Tanh-Coth Method [12]. However, these methods generally produce specific solutions in the form of certain specific functions that correspond to the method's naming. For this reason, a more general method is needed so that more interpretations can be obtained and analyzed. The method in question is the Kudryashov method, which was developed in 2010 ([13],[14],[15]). This method is based on the existence of Laurent series (which has more general properties than Taylor series) of a differential equation which makes it possible to produce negative term expansions ([13,15,16]). Given this negative

term, solutions with doubly periodic (which of course have more general properties) can be formed, such as elliptic functions.

The Ostrovksy equation is a type of nonlinear partial differential equation [17]. This equation already has several modifications ([5,7,17–20,20–23]), one of which is the cubic Ostrovsky equation (referring to the existence of a cubic nonlinear term). This equation can be found in physical problems such as the wave mechanism caused by the rotating [5]. In this paper, we analyze the meromorphic exact solution of the Ostrovsky cubic equation based on the existence of Laurent series. Therefore, this paper generally consists of several parts, namely first is an introduction, second on the Kudryashov method, third on the meromorphic solutions of the cubic Ostrovksy equation, and finally the conclusion.

Methods

In this study, we use the method introduced by Kudryahov to solve nonlinear differential equations ([13,15]). This method is very powerful because it allows us to obtain more general solutions of a nonlinear differential equation in three forms, namely simply periodic solutions, doubly periodic solutions, and rational solutions. These general solutions provide the possibility for us to obtain certain specific solutions based on the selection of certain particular circumstances. The basis on which this method is able to produce general solutions is because before constructing an exact meromorphic solution of a nonlinear differential equation, what must be done first is to find the existence of Laurent series of the equation. It is well known that Laurent series provides a complex expansion from a negative expansion to an infinite positive expansion term. Thus, it is clear that we can construct doubly periodic solutions such as the Weierstrass functions $\wp$ and ζ. This is not found in other methods. Thus, this method can be an excellent reference in finding meromorphic exact solutions of a nonlinear differential equation.

This method can be broadly explained as follows: first, what is being studied is a nonlinear partial differential equation, then the equation must first be transformed into a nonlinear ordinary differential equation, of course by using the appropriate transformation variables. Next, we find the Laurent series of this equation. If the equation has Laurent series, then proceed by applying the theorems in [15]. It can be seen from [15] that there are necessary conditions that must be met by an equation in order to find an exact meromorphic solution, namely for Theorem 1 is $a_{-1} = 0$, while for Theorem 2 it is $\sum_{i\in I} a_{-1}^{(i)} + a_{-1}^{(i_0)} = 0$, where a_j is the series coefficient. What distinguishes the application of Theorems 1 and 2 is how many types of Laurent series a nonlinear differential equation has. If an equation has only one type of Laurent series, then we use Theorem 1, but if the studied equation has more than one type of Laurent series, then we use Theorem 2. The further detailed and explicit steps of this method have been shown very clearly on paper [15].

3. Meromorphic Solutions of the Cubic Ostrovksy Equation

The Kudryashov method used in this paper refers to paper [15]. As previously explained, this paper examines the Ostrovsky Cubic equation, which is one type of modification of the Ostrovsky equation. The equation reviewed in this paper is

$$\frac{\partial^2 u}{\partial x \partial t} + C_0 \frac{\partial^2 u}{\partial x^2} + 2\left(\left(\frac{\partial u}{\partial x}\right)^2 + u\frac{\partial^2 u}{\partial x^2}\right) + \frac{\partial^4 u}{\partial x^4} - u + u^3 = 0 \tag{1}$$

where $u \equiv u(x,t)$. It is clear that this equation is a nonlinear partial differential equation. Then, equation (1) is transformed into a nonlinear ordinary differential equation using the form

$$u(x,t) \equiv w(z) \tag{2}$$

where $z = x - vt$. Using equation (2), equation (1) can be written

$$\frac{d^4 w}{dz^4} + (2w - C_0)\frac{d^2 w}{dz^2} + 2\left(\frac{dw}{dz}\right)^2 - w + w^3 = 0 \tag{3}$$

Equation (3) is further expanded using equation (6) in [15], and two types of Laurent series are obtained, namely

$$w^{(1)}(z) = -\frac{2(5+\sqrt{5}i)}{z^2} - \frac{C_0}{3+\sqrt{5}i} + a_2^{(1)}z^2 + a_4^{(1)}z^4 + a_6^{(1)}z^6 + \cdots \tag{4}$$

and

$$w^{(2)}(z) = -\frac{2(5-\sqrt{5}i)}{z^2} - \frac{C_0}{3-\sqrt{5}i} + a_2^{(2)}z^2 + a_4^{(2)}z^4 + a_6^{(2)}z^6 + \cdots \tag{5}$$

where

$$a_2^{(1)} = \frac{50+26\sqrt{5}i+3(\sqrt{5}i-5)C_0^2}{180(\sqrt{5}-3i)^2}; \tag{6}$$

$$a_4^{(1)} = -\frac{C_0(-19i+4\sqrt{5}+3(2i+\sqrt{5})C_0^2)}{756(\sqrt{5}-3i)^3}; \tag{7}$$

$$a_6^{(1)} = \frac{-860-1388\sqrt{5}i+12(115+\sqrt{5}i)C_0^2+(7\sqrt{5}i-5)C_0^4}{583200(\sqrt{5}-3i)^4}; \tag{8}$$

$$a_2^{(2)} = \frac{50-26\sqrt{5}i-3(\sqrt{5}i+5)C_0^2}{180(\sqrt{5}+3i)^2} \tag{9}$$

$$a_4^{(2)} = -\frac{C_0(19i+4\sqrt{5}+3(-2i+\sqrt{5})C_0^2)}{756(\sqrt{5}+3i)^3}; \tag{10}$$

$$a_6^{(2)} = \frac{-860+1388\sqrt{5}i+12(115-\sqrt{5}i)C_0^2-(7\sqrt{5}i+5)C_0^4}{583200(\sqrt{5}+3i)^4}. \tag{11}$$

From equations (4) to (11) it can be seen that the solution of equation (3) fulfills the relationship

$$w^{(2)}(z) = \left(w^{(1)}(z)\right)^*. \tag{12}$$

For the first part, we calculate the meromorphic solutions of the first type. Based on the Laurent series, it can be seen that the conditions for the existence of an elliptic solution (doubly periodic solutions) are met. Based on Theorem 2, the Weierstrass−$\wp$ elliptic solution form is

$$w(z) = a_{-2}^{(1)}\wp(z;g_2,g_3) + h_0 \tag{13}$$

with

$$a_{-2}^{(1)} = -2(5+\sqrt{5}i). \tag{14}$$

Laurent series for equation (13) around $z = 0$ is

$$w(z) = \frac{a_{-2}^{(1)}}{z^2} + h_0 + \frac{a_{-2}^{(1)}g_2}{20}z^2 + \frac{a_{-2}^{(1)}g_3}{28}z^4 + O(|z|^4), \qquad 0 < |z| < \tilde{\epsilon}_1. \tag{15}$$

By comparing equation (15) with equation (4), we get

$$h_0 = -\frac{C_0}{3+\sqrt{5}i}; \tag{16}$$

$$g_2 = \frac{-4-6i\sqrt{5}+3C_0^2}{18(\sqrt{5}-3i)^2}; \quad g_3 = -\frac{C_0(-19i+4\sqrt{5}+3(2i+\sqrt{5})C_0^2)}{216(50i+23\sqrt{5})} \tag{17}$$

and

$$C_0^{(\pm)} = \pm\frac{\sqrt{\frac{2}{3}(13i+5\sqrt{5})}}{\sqrt{\sqrt{5}-i}}. \tag{18}$$

Hence, the meromorphic elliptic solution of the first type takes the form

$$w(z) = -2\left(5+\sqrt{5}i\right)\wp(z;g_2,g_3) - \frac{C_0}{3+\sqrt{5}i} \tag{19}$$

with the values g_2 and g_3 as shown in equation (17).

Next, we compute the simply periodic solution of the first type, namely

$$w(z) = -\sqrt{L}a_{-2}^{(1)}\frac{d}{dz}\cot\left(\sqrt{L}z\right) + h_0 = La_{-2}^{(1)}\left(1+\cot^2\left(\sqrt{L}z\right)\right) + h_0 \tag{20}$$

where $a_{-2}^{(1)}$ is equal to equation (14), and

$$\sqrt{L} = \frac{\pi}{T}. \tag{21}$$

Laurent series of equation (20) around $z = 0$ is

$$w(z) = \frac{a_{-2}^{(1)}}{z^2} + h_0 + \frac{a_{-2}^{(1)}L}{3} + \frac{a_{-2}^{(1)}L^2}{15}z^2 + \frac{2a_{-2}^{(1)}L^3}{189}z^4 + O(|z|^4), \qquad 0 < |z| < \tilde{\epsilon}_2. \tag{22}$$

Next, we compare this series with the coefficients of the series $w^{(1)}(z)$ (see equation (4)), we determine the parameters of simply periodic solutions (20), namely

$$L_j = \pm\frac{(-1)^{3/4}\sqrt{\left(\sqrt{5}-5i\right)\left(6\sqrt{5}-4i+3C_0^2i\right)}}{2\left(\sqrt{5}-3i\right)\left(\sqrt{6\sqrt{5}-30i}\right)}\ ;\ j = 1,2\ ; \tag{23}$$

$$h_0 = \frac{1}{126}\left(-27C_0 + 9\sqrt{5}C_0i + 420L_j + i84\sqrt{5}L_j\right). \tag{24}$$

Thus, simply periodic solutions of the first type can be written

$$w(z) = \frac{i}{42}\left\{3\left(\sqrt{5}+3i\right)C_0 - 56\left(\sqrt{5}-5i\right)L_j\right\} - 2L_j\left(5+\sqrt{5}i\right)\cot^2\left(\sqrt{L_j}z\right); j = 1,2. \tag{25}$$

By relating the parameters in equation (23) and (24), equation (25) satisfies equation (3). Next, we compute the rational solution of the first type. From equation (4) it can be concluded that the rational solution of the first type has the form

$$w(z) = -\frac{2\left(5+\sqrt{5}i\right)}{z^2} - \frac{C_0}{3+\sqrt{5}i}. \tag{26}$$

.Now we continue the analysis on calculating the meromorphic solution of the second type taking advantage of the symmetry in equation (12). These solutions possess poles of two types at the same type, and without losing the generality, we suppose that the point $z = 0$ is a pole of the first type. The elliptic solutions have the form

$$w(z) = a_{-2}^{(1)}\wp(z;g_2,g_3) + a_{-2}^{(2)}\wp(z-\gamma;g_2,g_3) + \tilde{h}_0 \tag{27}$$

where $a_{-2}^{(1)}$ is equal to equation (14), and

$$a_{-2}^{(2)} = -2\left(5-\sqrt{5}i\right). \tag{28}$$

Next, equation (27) is expanded around the point $z = 0$, we have

$$w(z) = \frac{a_{-2}^{(1)}}{z^2} + \tilde{h}_0 + a_{-2}^{(2)}A - a_{-2}^{(2)}Bz + \left(3A^2a_{-2}^{(2)} - \frac{g_2}{4}a_{-2}^{(2)} + \frac{a_{-2}^{(1)}g_2}{20}\right)z^2 + O(|z|^2), \tag{29}$$

$$0 < |z| < \tilde{\epsilon}_3\ ,$$

and expanded around $z = \gamma$, obtained

$$w(z) = \frac{a_{-2}^{(2)}}{(z-\gamma)^2} + \tilde{h}_0 + a_{-2}^{(1)}A + a_{-2}^{(1)}B(z-\gamma) + \left(3A^2a_{-2}^{(1)} - \frac{g_2}{4}a_{-2}^{(1)} + \frac{a_{-2}^{(2)}g_2}{20}\right)(z-\gamma)^2 + O(|z-\gamma|^2), \qquad 0 < |z-\gamma| < \tilde{\epsilon}_4\ . \tag{30}$$

where $A \equiv \wp(\gamma)$; $B \equiv \wp_z(\gamma)$, and there is a relationship $B^2 = 4A^3 - g_2A - g_3$. Equation (27) can also be expressed in the form

$$w(z) = \left(a_{-2}^{(1)} - a_{-2}^{(2)}\right)\wp(z; g_2, g_3) + \frac{a_{-2}^{(2)}}{4}\left(\frac{\wp_z + B}{\wp - A}\right)^2 + \tilde{h}_0 - a_{-2}^{(2)}A. \quad (31)$$

Next, we compare the series in equations (29) and (30) with equations (4) and (5), we have

$$A = \frac{C_0}{40 + 4\sqrt{5}i}\ ;\tilde{h}_0\ ;\ B = 0 \quad (32)$$

$$g_2 = \frac{38i - 8\sqrt{5} - 3(11i + \sqrt{5})C_0^2}{36(-89i + 11\sqrt{5})} \quad (33)$$

$$g_3 = \frac{C_0}{3600}\frac{5(-115i + \sqrt{5}) + 3(95i + 31\sqrt{5})C_0^2}{(-517i + 16\sqrt{5})}. \quad (34)$$

Therefore, we obtain an elliptic meromorphic solution of the second type in the form

$$w(z) = \frac{C_0}{3 + \sqrt{5}i} - 4i\sqrt{5}\wp - \frac{40(115 + \sqrt{5}i)\wp_z^2}{(C_0 - 40\wp - 4i\sqrt{5}\wp)^2}. \quad (35)$$

Equation (35) can also be expressed in another form by using the relationship $\wp_z^2 = 4\wp^3 - g_2\wp - g_3$, namely

$$w(z) = \frac{100}{9}\frac{\eta_1 - \eta_2\wp + \eta_3\wp^2 - \eta_4\wp^3}{\eta_5(iC_0 + \eta_6\wp)^2} \quad (36)$$

where $\eta_1 = \alpha_1 + 10C_0\alpha_3$; $\eta_2 = \alpha_2 + 40\alpha_5$; $\eta_3 = 10C_0\alpha_4$; $\eta_4 = 40\alpha_6$; $\eta_5 = \alpha_7\alpha_8$; and

$$\alpha_1 = 3(1517i + 493\sqrt{5})C_0^3 \quad (37)$$

$$\alpha_2 = 60(3623i + 1303\sqrt{5})C_0^2 \quad (38)$$

$$\alpha_3 = -1322i + 23\sqrt{5} \quad (39)$$

$$\alpha_4 = 504(-437i + 533\sqrt{5}) \quad (40)$$

$$\alpha_5 = -13105i + 1552\sqrt{5} \quad (41)$$

$$\alpha_6 = 216(-11885i + 7121\sqrt{5}) \quad (42)$$

$$\alpha_7 = (2\sqrt{5}i + 20)^3 \quad (43)$$

$$\alpha_8 = 10i + 3\sqrt{5} \quad (44)$$

$$\eta_6 = 4(-10i + \sqrt{5}) \quad (45)$$

Next, we compute the simply periodic solution of the second type. Based on Theorem 2, we get

$$w(z) = La_{-2}^{(1)}(1 + \cot^2(\sqrt{L}z)) + La_{-2}^{(2)}\left(1 + \cot^2\left(\sqrt{L}(z - \gamma)\right)\right) + h_0, \quad (46)$$

where $a_{-2}^{(1)}$ can be seen in equation (14), $a_{-2}^{(2)}$ in equation (28), and $\sqrt{L}$ in equation (21). Then, equation (46) is expanded around the point $z = 0$ is obtained

$$w(z) = \frac{a_{-2}^{(1)}}{z^2} + \frac{a_{-2}^{(1)}L}{3} + a_{-2}^{(2)}(L + A^2) + h_0 + 2a_{-2}^{(2)}A(L + A^2)z + O(|z|);\ 0 < |z| < \tilde{\epsilon}_5. \quad (47)$$

Equation (46) is expanded around the point $z = \gamma$ is obtained

$$w(z) = \frac{a_{-2}^{(2)}}{(z - \gamma)^2} + \frac{a_{-2}^{(2)}L}{3} + a_{-2}^{(1)}(L + A^2) + h_0 - 2a_{-2}^{(1)}A(L + A^2)(z - \gamma) + O(|z - \gamma|), \quad (48)$$
$$0 < |z - \gamma| < \tilde{\epsilon}_6.$$

As previously done, equations (47) and (48) are compared with equations (4) and (5). From the results of this comparison obtained

$$A = 0\ ; h_0 = 0\ ; L = \frac{3C_0}{28(5 + \sqrt{5}i)}. \quad (49)$$

Thus, a meromorphic solution of the second type of period takes the form

$$w(z) = -\frac{3C_0}{14}\csc\left(\frac{\sqrt{3}}{2}\sqrt{\frac{C_0}{35+7\sqrt{5}i}}z\right) + \frac{3(5i+\sqrt{5})C_0}{14(-5i+\sqrt{5})}\csc\left(\frac{\sqrt{3}}{2}\sqrt{\frac{C_0}{35+7\sqrt{5}i}}(z-\gamma)\right). \quad (50)$$

It can be seen from equation (50) that we can choose specific solutions in the form of other functions such as hyperbolic functions by modifying terms that have an imaginary part, or other trigonometric functions by utilizing the phase difference provided in the terms containing $(z-\gamma)$ or $z-z_0$ which is the complete form of this function. It can also be seen that for this second type there is no rational solution due to the inconsistency of the series terms when compared with Laurent series of the differential equation (3).

4. Conclusion

We have performed an analysis to find a meromorphic solution to the cubic Ostrovksy equation using the Kudryashov method. Since this equation has two types of Laurent series, we derive its meromorphic solution in two types based on Theorem 2 in the method section. For the first type, we obtain a simply periodic solution, one doubly periodic solution in the form of the Weierstrass-$\wp$ function and one rational solution. The constants for each solution have also been shown. For the second type of solution, we obtained one simply periodic solution and one doubly periodic solution. The constants corresponding to these solutions have also been shown.

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PAGE 1

PAGE 2

PAGE 3

PAGE 4

PAGE 5

PAGE 6

PAGE 7