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A typical oscillating perturbation on protonium

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Abstract. Protonium is a type of onium consisting of a proton (p) and an anti-proton ($\bar{p}$) which is called antiprotonic hydrogen. Since the proton and the anti-particle have the same mass, and each of them is a fermion, the Hamiltonian form of this system consists of the kinetic energy terms of each particle, the potential energy, and the spin interactions of both (hyperfine structure). To simplify the analysis, the Hamiltonian is then converted into a Hamiltonian which contains the relative motion terms of the two particles. This new Hamiltonian contains their reduced mass because the two particles orbit each other towards a certain center of mass. In the initial state, $t < 0$, the system is in the ground state and then at $0 < t < \tau$, the system is given a disturbance of the form $V(t) = (V_0 + V_1\, r \sin\theta\, e^{i\phi})\, cos(\omega t)\, \hat{\sigma}_p\, .\hat{x}$, where V_0 is a constant, V_1 is a constant and very small, $\hat{\sigma}_p$ is the Pauli matrix for proton, and $\hat{x}$ is the three-dimensional position operator in spherical coordinates. We analyze this system using a time-dependent perturbation theory that begins by defining the initial state, then calculating the transition amplitude for the new state when the perturbation is applied to the system. From the results of this calculation, we compute the system transition probability. We find that the transition probability of this system can only occur for the following quantum number terms, namely n even positive integer, $l = 2$, $S = 1$, and with the certain combinations of m and M.

1. Introduction

Research on subatomic particles has always been an interesting topic to do. This is not only because of its mathematical beauty and physical interpretation, but also because there are so many counter-intuitive physical realities, so that it makes scientists even more curious to reveal them; such as quantum tunneling effects [1–4], particle decay and transmutation problems [5–8], spin quantity [9–11], interactions between subatomic particles [12],[13] , and many other physical properties.

The increasing and better understanding of the quantum reality of subatomic particles and their interactions will certainly have a very good impact on the integrated understanding of how our universe works, of course by involving other theories such as the General Theory of Relativity [14], Quantum Field Theory [15], Quantum Chromodynamic Theory [16], Quantum Gravity Theory [17], String Theory [18], even the Theory of Everything [19]. The properties of particles and their interactions at the subatomic scale that are studied theoretically will also be the basis for how experimental scientists carry out a proof for the sake of proving each theory and theoretical framework developed.

In this paper, we examine the behavior of a subatomic system of onium, specifically for protonium, which is a system of particle interactions consisting of a proton (p) and its anti-particle ($\bar{p}$). More specifically, what we do in this research is to analyze the probability of the transition of this system when disturbed by a perturbation of form $V(t) = (V_0 + V_1 r \sin\theta\, e^{i\phi})\, cos(\omega t)\, \hat{\sigma}_p\, .\hat{x}$ which is done on the ground-state system, where the definition of each variable at this potential has been described in the

abstract section. Because this perturbation is time-dependent we use the theory of time-dependent perturbation to analyze this system. Even from the potential form containing the cosine function, we can see that this perturbation is an oscillating perturbation. Our detailed analysis can be found in the fourth section of this paper. This paper generally consists of four parts, namely, first, an introduction, second on protonium, third on the theory of time-dependent perturbation, fourth on the probability of transition of a quantum system subject to perturbation, and the last is a conclusion.

2. Protonium

Protonium is one type of onium that is very interesting to study because it is a system consisting of particles and anti-particles [20], meaning that this particle interaction occurs in particles that are only distinguished by their charge, while their spin and mass are the same. The system has a peculiarity in that there is an excess of the number of symmetries compared to other types of onium, namely the similarity of the mass of the two particles. Protonium as a system is a boson, but its constituents are fermions. Protonium has an average life span of 1 μs, with a binding energy of -0.75 KeV [21]. Pure protonium has a Hamiltonian form [22]

$$\hat{H} = -\frac{\hbar^2}{2\mu_p}\nabla_p^2 - \frac{\hbar^2}{2\mu_{\bar{p}}}\nabla_{\bar{p}}^2 + \frac{1}{2}\frac{\mu_p\mu_{\bar{p}}}{\mu_p+\mu_{\bar{p}}}\omega^2\left(\vec{r}_p\cdot\vec{r}_{\bar{p}}\right)^2 + g\,\vec{\sigma}_p\cdot\vec{\sigma}_{\bar{p}} \quad (1)$$

where $g \ll \hbar\omega$; μ_p and $\mu_{\bar{p}}$, respectively, are the proton and anti-proton masses; g is the Lande factor; $\vec{\sigma}_p$ and $\vec{\sigma}_{\bar{p}}$ are the Pauli spin matrix for protons and anti-protons, respectively.

To facilitate analysis, the form $\hat{H}$ equation (1) can be converted into Hamiltonian form with the coordinates of the center of mass, namely

$$\hat{H} = -\frac{\hbar^2}{2\mathrm{M}}\nabla_R^2 - \frac{\hbar^2}{2\mu}\nabla_r^2 + \frac{\mu}{2}\omega^2 r^2 + 2g\left(S(S+1) - \frac{3}{2}\right) \quad (2)$$

where s stands for spin, next

$$\mathrm{M} = \mu_p + \mu_{\bar{p}} = 2\mu_p;\ \ \mu = \frac{\mu_p\mu_{\bar{p}}}{\mu_p+\mu_{\bar{p}}} = \frac{1}{2}\mu_p;\ \ \vec{r} = \vec{r}_p - \vec{r}_{\bar{p}}\ ;\ \ \vec{R} = \frac{\mu_p\vec{r}_p + \mu_{\bar{p}}\,\vec{r}_{\bar{p}}}{\mu_p+\mu_{\bar{p}}} = \frac{1}{2}(\vec{r}_p + \vec{r}_{\bar{p}}) \quad (3)$$

and total spin

$$\hat{S} = \hat{s}_p + \hat{s}_{\bar{p}} = \frac{1}{2}(\vec{\sigma}_p + \vec{\sigma}_{\bar{p}}). \quad (4)$$

It can be seen that, from equation (2), the first term of the right side is the motion of the system as a whole; the second and third terms are Hamiltonian terms for an isotropic harmonic oscillator without spin; and the last term of the right-hand side arises from the product of the Pauli spin matrix, which uses the basis $|s_p s_{\bar{p}}\,; SM\rangle$, where the indices correspond to the Pauli spin matrix, and which are associated with protons and anti-protons. Meanwhile, the bases of each particle are $|s_p m_p\rangle$ and $|s_{\bar{p}} m_{\bar{p}}\rangle$, respectively. Based on equation (2), the energy state of the system without perturbation is easily obtained

$$E_{nS} = \frac{P^2}{2\mathrm{M}} + \left(n + \frac{3}{2}\right)\hbar\omega + 2g\left(s(s+1) - \frac{3}{2}\right) \quad (5)$$

where $n = 0,1,2,3,\ldots$

For the energy level of internal motion, we must ignore the first term on the right side of equation (2). For the ground state of internal motion, $n = 0$; $S = 0$, then

$$E_{00} = \frac{3}{2}\hbar\omega - g \quad (6)$$

3. Time-Dependent Perturbation Theory

3.1. Dyson series

Suppose a quantum system has a state $|\alpha\rangle$ at time $t = t_0$, then at time t, the system is in the state $|\alpha, t_0; t\rangle_I$, where the form $|\ \ \rangle_I$ represents the state in the interaction picture. For a quantum system that

is subjected to time-dependent perturbation, in the interaction picture, the state equation using the time evolution operator $U_I(t,t_0)$ can be expressed in the form ([23,24])

$$|\alpha,t_0;t\rangle_I = U_I(t,t_0)|\alpha,t_0;t_0\rangle_I \tag{7}$$

where $U_I(t,t_0) = e^{\frac{iH_0t}{\hbar}}U(t,t_0)e^{\frac{-iH_0t_0}{\hbar}}$; $U(t,t_0)$ is the operator of the evolution of time in the Shrodinger picture; and H_0 is the time-independent Hamiltonian. The time evolution operator in equation (7) satisfies the equation

$$i\hbar\frac{dU_I(t,t_0)}{dt} = V_I(t)U_I(t,t_0) \tag{8}$$

where $V_I(t)$ is a potential interaction picture that has a form

$$V_I(t) = e^{\frac{iH_0t}{\hbar}}V(t)e^{\frac{-iH_0t}{\hbar}}. \tag{9}$$

Furthermore, equation (8) becomes

$$\int_{U_I(t,t_0)|t_0}^{U_I(t,t_0)} dU_I(t',t_0) = \frac{1}{i\hbar}\int_{t_0}^{t} V_IU_I(t',t_0)(t')dt'. \tag{10}$$

By using the initial condition $U_I(t,t_0)|_{t=t_0} = 1$, the integral result of equation (10) is obtained, namely

$$U_I(t,t_0) = 1 + \frac{1}{i\hbar}\int_{t_0}^{t} V_I(t')U_I(t',t_0)dt'. \tag{11}$$

Furthermore, assuming that $V_I(t)$ is small, then the solution of equation (11) can be approximated successively. For the first-order approximation, entering $U_I(t',t_0) = 1$, we get

$$U_I^{(1)}(t,t_0) = 1 + \frac{1}{i\hbar}\int_{t_0}^{t} V_I(t')dt'. \tag{12}$$

For the second order approximation, we use $U_I(t',t_0) = U_I^{(1)}(t',t_0)$ and then inserting this form and equation (12) into equation (11), we get

$$\begin{aligned} U_I^{(2)}(t,t_0) &= 1 + \frac{1}{i\hbar}\int_{t_0}^{t} V_I(t')\left(1 + \frac{1}{i\hbar}\int_{t_0}^{t'} V_I(t'')dt''\right)dt' \\ &= 1 + \frac{1}{i\hbar}\int_{t_0}^{t} V_I(t')dt' + \left(\frac{1}{i\hbar}\right)^2\int_{t_0}^{t} V_I(t')dt'\int_{t_0}^{t'} V_I(t'')dt''. \end{aligned} \tag{13}$$

The higher orders can be obtained using the same method. Therefore, the general form of this approximation can be expressed in the form

$$\begin{aligned} U_I(t,t_0) = 1 &+ \frac{1}{i\hbar}\int_{t_0}^{t} V_I(t')dt' + \left(\frac{1}{i\hbar}\right)^2\int_{t_0}^{t} V_I(t')dt'\int_{t_0}^{t'} V_I(t'')dt'' + \cdots \\ &+ \left(\frac{1}{i\hbar}\right)^n \int_{t_0}^{t'} dt' \int_{t_0}^{t'} dt'' \ldots \times \int_{t_0}^{t^{(n-1)}} dt^{(n)} V_I(t')V_I(t'')\ldots V_I\left(t^{(n)}\right) + \cdots \end{aligned} \tag{14}$$

This form is known as the Dyson Series which is used to calculate the state vector up to the required perturbation order.

3.2. Transition Probability

The probability of transitioning a quantum system with the initial unperturbed $|i\rangle$ to $|n\rangle$ state is ([23,24])

$$P(i \to n) = |\langle n|U_I(t,t_0)|i\rangle|^2 = |C_n|^2 \tag{15}$$

where $n \neq i$, and

$$C_n = C_n^{(0)} + C_n^{(1)} + C_n^{(2)} + \cdots = \langle n|U_I(t,t_0)|i\rangle. \tag{16}$$

Using equations (9), (16) and (14), we get

$$C_n^{(0)} = \delta_{ni} \quad ; \text{(independent of } t\text{)} \tag{17}$$

$$C_n^{(1)} = \frac{1}{i\hbar}\int_{t_0}^{t} \langle n|V_I(t')|i\rangle dt' = \frac{1}{i\hbar}\int_{t_0}^{t} e^{i\omega_{ni}t'}\langle n|V(t')|i\rangle dt' \ ; \tag{18}$$

$$C_n^{(2)} = \left(\frac{1}{i\hbar}\right)^2 \sum_m \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \, e^{i\omega_{nm}t'}\langle n|V(t')|m\rangle e^{i\omega_{mi}t''}\langle m|V(t'')|i\rangle. \tag{19}$$

where

$$e^{i\omega_{ni}t} \equiv e^{i(E_n - E_i)t/\hbar}. \tag{20}$$

Equation (15) is a statement of the probability of a quantum state transition to a certain order in $V_I(t)$. However, for values in high orders generally have a very small contribution to the value of the transition probability in low orders, especially for first-order. Therefore, most of the transition probability analyzes are only carried out in first-order because this order alone is sufficient to represent the physical state of a quantum system such as the problem of atoms and nuclear physics.

4. Transition Probability of Perturbed System

The quantum system studied in this paper is protonium as previously described, but in a state when protonium is subjected to oscillating perturbation. In its initial state, protonium is in the ground state, and then at a certain time, the protonium experiences a perturbation potential in the form of

$$V(t) = \left(V_0 + V_1\, r \sin\theta\, e^{i\phi}\right)\cos(\omega t)\ \vec{\sigma}_p \cdot \hat{x} \ \ ; \ 0 < t < \tau\,, \tag{21}$$

where V_0 is a constant, $\hat{\sigma}_p$ is the Pauli matrix for protons, and $\hat{x}$ is the three-dimensional position operator in spherical coordinates; V_1 is very small; $\frac{g\tau}{\hbar} \ll 1$; $V(t)$ is zero when $t < 0$ and $t > \tau$.

Before being perturbed, the system is in a ground state. If we suppose that the Hamiltonian in the basic state is

$$\hat{H}_0 = -\frac{\hbar^2}{2\mu}\nabla_r^2 + \frac{\mu}{2}\omega^2 r^2 + 2g\left(S(S+1) - \frac{3}{2}\right) \tag{22}$$

then

$$\hat{H}_0|n\rangle = E_n|n\rangle, \tag{23}$$

and furthermore, the equation for the state after being subjected to perturbation $V(t)$ is

$$\left(\hat{H}_0 + V(t)\right)|\psi(t)\rangle = i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle\,. \tag{24}$$

By expanding the state to base $|n\rangle$ with the form

$$|\psi(t)\rangle = \sum_n C_n(t)e^{-iE_nt/\hbar}|n\rangle \tag{25}$$

and put into equation (25), then multiplied by the form $\langle m|e^{iE_mt/\hbar}$ from the left side and add up for all m obtained

$$\frac{dC_n(t)}{dt} = \frac{1}{i\hbar}\sum_n \langle n|V(t)|m\rangle e^{-\frac{i(E_m - E_n)t}{\hbar}} C_m(t). \tag{26}$$

As previously explained, at first the system is in a ground state, and we define this state in the form $|000;00\rangle$ and the final state is in the form $|nlm;SM\rangle$, which arises from the direct product

$$|nlm;SM\rangle = |nlm\rangle \otimes |SM\rangle. \tag{27}$$

Since proton and anti-proton are fermions with spin-$\frac{1}{2}$ each, the total spin state $|SM\rangle$ of the protonium system contains the base state of each particle, namely []

$$|11\rangle = |s_p(\uparrow)s_{\bar{p}}(\uparrow)\rangle \tag{28}$$

$$|10\rangle = \frac{1}{\sqrt{2}}(|s_p(\uparrow)s_{\bar{p}}(\downarrow)\rangle + |s_p(\downarrow)s_{\bar{p}}(\uparrow)\rangle) \tag{29}$$

$$|1-1\rangle = |s_p(\downarrow)s_{\bar{p}}(\downarrow)\rangle \tag{30}$$

and

$$|00\rangle = \frac{1}{\sqrt{2}}(|s_p(\uparrow)s_{\bar{p}}(\downarrow)\rangle - |s_p(\downarrow)s_{\bar{p}}(\uparrow)\rangle)\,. \tag{31}$$

Since this type of perturbation involves a three-dimensional vector $\hat{x}$, the first analysis is to calculate the shape of $\vec{\sigma}_p \cdot \hat{x}$ in spherical coordinates, namely

$$\vec{\sigma}_p \cdot \hat{x} = \sin\theta\cos\phi\,\sigma_{px} + \sin\theta\sin\phi\,\sigma_{py} + \cos\theta\sigma_{pz} \tag{32}$$

where $\hat{x} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$; with $x = r\sin\theta\cos\phi$; $y = r\sin\theta\sin\phi$; $z = r\cos\theta$; and $\vec{\sigma}_p = \sigma_{px}\hat{\imath} + \sigma_{py}\hat{\jmath} + \sigma_{pz}\hat{k}$. Then, the operator in equation (32) is worked on state $|00\rangle$, then we get the form

$$\begin{aligned}(\vec{\sigma}_p \cdot \hat{x})|00\rangle = \frac{1}{\sqrt{2}}\{&\sin\theta\cos\phi\,\sigma_{px}|s_p(\uparrow)s_{\bar{p}}(\downarrow)\rangle - \sin\theta\cos\phi\,\sigma_{px}\,|s_p(\downarrow)s_{\bar{p}}(\uparrow)\rangle \\ &+ \sin\theta\sin\phi\sigma_{py}\,|s_p(\uparrow)s_{\bar{p}}(\downarrow)\rangle \\ &- \sin\theta\sin\phi\sigma_{py}\,|s_p(\downarrow)s_{\bar{p}}(\uparrow)\rangle + \cos\theta\sigma_{pz}\,|s_p(\uparrow)s_{\bar{p}}(\downarrow)\rangle \\ &- \cos\theta\sigma_{pz}\,|s_p(\downarrow)s_{\bar{p}}(\uparrow)\rangle\}.\end{aligned} \tag{33}$$

By using relationships

$$\sigma_{px}|s_p(\uparrow)\rangle = |s_p(\downarrow)\rangle;\;\; \sigma_{px}|s_p(\downarrow)\rangle = |s_p(\uparrow)\rangle; \tag{34}$$

$$\sigma_{py}|s_p(\uparrow)\rangle = i|s_p(\downarrow)\rangle;\;\; \sigma_{py}|s_p(\downarrow)\rangle = -i|s_p(\uparrow)\rangle; \tag{35}$$

$$\sigma_{pz}|s_p(\uparrow)\rangle = |s_p(\uparrow)\rangle;\;\; \sigma_{pz}|s_p(\downarrow)\rangle = -|s_p(\downarrow)\rangle; \tag{36}$$

and by utilizing the form of equations (28), (29), (30); then equation (33) becomes

$$(\vec{\sigma}_p \cdot \hat{x})|00\rangle = \frac{1}{\sqrt{2}}\sin\,\theta e^{i\phi}|1-1\rangle + \cos\,\theta|10\rangle - \frac{1}{\sqrt{2}}\sin\,\theta e^{-i\phi}|11\rangle. \tag{37}$$

Next, we calculate the form of the average value containing $r\sin\theta\, e^{i\phi}\vec{\sigma}_p \cdot \hat{x}$, namely

$$\begin{aligned}&\langle nlm; SM|r\sin\theta\, e^{i\phi}\vec{\sigma}_p \cdot \hat{x}(|00\rangle \otimes |000\rangle) \\ &= \langle nlm; SM|(r\sin\theta\, e^{i\phi})(\frac{1}{\sqrt{2}}\sin\,\theta e^{i\phi}|1-1\rangle + \cos\,\theta|10\rangle \\ &- \frac{1}{\sqrt{2}}\sin\,\theta e^{-i\phi})\,|11\rangle\,)Y_0^0 \otimes |0\rangle\end{aligned} \tag{38}$$

where Y_l^m is a spherical harmonic function. By performing several calculation steps, it is obtained

$$\begin{aligned}&\langle nlm; SM|r\sin\theta\, e^{i\phi}\vec{\sigma}_p \cdot \hat{x}|\;00;000\rangle \\ &= \langle nlm; SM|r(\sqrt{\frac{16\pi}{15}}Y_2^2|1-1\rangle - \sqrt{\frac{8\pi}{15}}Y_2^1|10\rangle + \frac{1}{3}\sqrt{\frac{8\pi}{5}}Y_2^0|11\rangle \\ &- \frac{2}{3\sqrt{2}}|11\rangle)\,Y_0^0 \otimes |0\rangle\end{aligned} \tag{39}$$

The next form of equation (39) can be written

$$\begin{aligned}&\langle nlm; SM|r\sin\theta\, e^{i\phi}\vec{\sigma}_p \cdot \hat{x}|\;00;000\rangle \\ &= \langle nl|r|00\rangle\delta_{l,2}\delta_{S,1}\left(\sqrt{\frac{4}{15}}\,\delta_{m,2}\delta_{M,-1} + \frac{1}{3}\sqrt{\frac{2}{5}}\delta_{m,0}\delta_{M,1} - \sqrt{\frac{2}{15}}\,\delta_{m,1}\delta_{M,0}\right) \\ &- \frac{2}{3\sqrt{2}}\langle nl|r|00\rangle\delta_{l,0}\delta_{S,1}\delta_{m,0}\delta_{M,1}\end{aligned} \tag{40}$$

where

$$\langle nl|r|00\rangle = \int_0^\infty R_{nl}R_{00}r^3\,dr\,. \tag{41}$$

For $l = 2$,

$$\langle n2|r|00\rangle = \int_0^\infty R_{n2}R_{00}r^3 dr. \tag{42}$$

Next, we calculate the form of the average value which contains $\vec{\sigma}_p \cdot \hat{x}$, namely

$$\begin{aligned}&\langle nlm; SM|\vec{\sigma}_p \cdot \hat{x}(|00\rangle \otimes |000\rangle)\\ &= \langle nlm; SM|(\frac{1}{\sqrt{2}}\sin\,\theta e^{i\phi}|1-1\rangle + \cos\,\theta|10\rangle - \frac{1}{\sqrt{2}}\sin\,\theta e^{-i\phi})\,|11\rangle\,)Y_0^0\\ &\otimes |0\rangle\,.\end{aligned} \tag{43}$$

By performing several calculation steps, it is obtained

$$\begin{aligned}&\langle nlm; SM|\vec{\sigma}_p \cdot \hat{x}|\ 00; 000\rangle\\ &= \langle nlm; SM|r(\sqrt{\frac{4\pi}{3}}Y_1^1|1-1\rangle - \sqrt{\frac{4\pi}{3}}Y_1^0|10\rangle - \sqrt{\frac{4\pi}{3}}Y_1^{-1}|11\rangle)\ Y_0^0 \otimes |0\rangle\,.\end{aligned} \tag{44}$$

The next form of equation (44) can be written

$$\langle nlm; SM|\vec{\sigma}_p \cdot \hat{x}|\ 00; 000\rangle = \sqrt{\frac{1}{3}}\delta_{n0}\delta_{l1}\delta_{S1}\left(\delta_{m0}\delta_{M0} - \delta_{m1}\delta_{M,-1} - \delta_{m1}\delta_{M1}\right) \equiv 0, \tag{45}$$

since $l = 0$ for $n = 0$.

Next, we calculate $C_n(\tau)$, that is

$$C_n(\tau) = \frac{1}{i\hbar}\int_0^\infty e^{i\omega_{n_0}t}\langle nlm; SM|V(t)|\ 00; 000\rangle\,dt \tag{46}$$

$$C_n(\tau) = \frac{1}{i\hbar}\int_0^\tau e^{in\omega t}\,\langle nlm; SM|(V_0 + V_1 r\sin\theta\, e^{i\phi})\cos(\omega t)\ \vec{\sigma}_p \cdot \hat{x}|\ 00; 000\rangle\,dt \tag{47}$$

$$C_n(\tau) = \frac{1}{i\hbar}\int_0^\tau e^{in\omega t}\,\langle nlm; SM|(V_0 + V_1 r\sin\theta\, e^{i\phi})\,\vec{\sigma}_p \cdot \hat{x}|\ 00; 000\rangle\ \cos(\omega t)\,dt \tag{48}$$

$$C_n(\tau) = \frac{1}{i\hbar}\langle nlm; SM|(V_0 + V_1 r\sin\theta\, e^{i\phi})\,\vec{\sigma}_p \cdot \hat{x}|\ 00; 000\rangle\int_0^\tau e^{in\omega t}\ \cos(\omega t)\,dt \tag{49}$$

$$C_n(\tau) = \frac{1}{i\hbar}\langle nlm; SM|V_1 r\sin\theta\, e^{i\phi}\ \vec{\sigma}_p \cdot \hat{x}|\ 00; 000\rangle\int_0^\tau e^{in\omega t}\ \cos(\omega t)\,dt, \tag{50}$$

because $\langle nlm; SM|\vec{\sigma}_p \cdot \hat{x}|\ 00; 000\rangle$ so it doesn't contribute to $C_n(\tau)$. By using Euler's formula, which is the inversion of the form $e^{\pm i\theta} = \cos\theta + \pm i\sin\theta$, the integral result is

$$\int_0^{\tau} e^{in\omega t}\cos(\omega t)\,dt = \frac{(n-1)\cos\big((n+1)\omega\tau\big)+(n+1)\cos\big((n-1)\omega\tau\big)}{2i(n+1)(n-1)\omega} + \frac{(n-1)\sin\big((n+1)\omega\tau\big)+(n+1)\sin\big((n-1)\omega\tau\big)}{2(n+1)(n-1)\omega} + \frac{n}{(n+1)(n-1)\omega i}. \quad (51)$$

From equations (40) and (51), equation (50) can be written

$$C_n(\tau) = \frac{V_1}{\hbar}\left(\langle nl|r|00\rangle\delta_{l,2}\delta_{S,1}\left(\frac{2}{\sqrt{15}}\delta_{m,2}\delta_{M,-1}+\frac{1}{3}\sqrt{\frac{2}{5}}\delta_{m,0}\delta_{M,1}-\sqrt{\frac{2}{15}}\delta_{m,1}\delta_{M,0}\right) - \frac{2}{3\sqrt{2}}\langle nl|r|00\rangle\delta_{l,0}\delta_{S,1}\delta_{m,0}\delta_{M,1}\right)\left(-\frac{(n-1)\cos\big((n+1)\omega\tau\big)+(n+1)\cos\big((n-1)\omega\tau\big)}{2(n+1)(n-1)\omega} + \frac{(n-1)\sin\big((n+1)\omega\tau\big)+(n+1)\sin\big((n-1)\omega\tau\big)}{2i(n+1)(n-1)\omega} - \frac{n}{(n+1)(n-1)\omega}\right). \quad (52)$$

Or, equation (52) can also be written in the form

$$C_n(\tau) = \frac{V_1}{2\hbar}\left(\frac{e^{-i(n-1)\omega\tau}}{(n-1)\omega} - \frac{e^{i(n+1)\omega\tau}}{(n+1)\omega} - \frac{2n}{(n+1)(n-1)\omega}\right)\left(\langle nl|r|00\rangle\delta_{l,2}\delta_{S,1}\left(\frac{2}{\sqrt{15}}\delta_{m,2}\delta_{M,-1}+\frac{1}{3}\sqrt{\frac{2}{5}}\delta_{m,0}\delta_{M,1} - \sqrt{\frac{2}{15}}\delta_{m,1}\delta_{M,0}\right) - \frac{2}{3\sqrt{2}}\langle nl|r|00\rangle\delta_{l,0}\delta_{S,1}\delta_{m,0}\delta_{M,1}\right). \quad (53)$$

For a three-dimensional harmonic oscillator, $n = l + 2n_r$, and based on equation (53), $C_n(\tau)$ is non-zero if $l = 2$, then $n = 2 + 2n_r = 2(1 + n_r)$ is an even number. Therefore, we get

$$C_{2\eta-1}(\tau) = 0, \quad (54)$$

and

$$C_{2\eta}(\tau) = \frac{V_1}{2\hbar}\langle 2\eta; 2|r|00\rangle\delta_{S,1}\left(\frac{e^{-i(2\eta-1)\omega\tau}}{(2\eta-1)\omega} - \frac{e^{i(2\eta+1)\omega\tau}}{(2\eta+1)\omega} - \frac{4\eta}{(2\eta+1)(2\eta-1)\omega}\right)\left(\frac{2}{\sqrt{15}}\delta_{m,2}\delta_{M,-1}+\frac{1}{3}\sqrt{\frac{2}{5}}\delta_{m,0}\delta_{M,1} - \sqrt{\frac{2}{15}}\delta_{m,1}\delta_{M,0}\right), \quad (55)$$

where $l = 2;\ \eta = 1, 2, 3, \ldots$; and $\langle 2\eta; 2|r|00\rangle = \int_0^{\infty} R_{(2\eta),2}R_{00}r^3dr$.

The expression for equation (55) is a relatively general form because it allows several possible transitions depending on η. Of course, we can easily select a particular state to obtain results similar to harmonic perturbations such as [24], [23], and [25]. Based on this equation (55), we can state the probability of a system transition, namely

$$P = \left|C_{2\eta}(\tau)\right|^2 = \frac{V_1^2}{4\hbar^2}|\langle 2\eta;2|r|00\rangle|^2\delta_{S,1}\left(\frac{2+24\eta^2}{(\omega-4\eta^2\omega)^2} - \frac{2}{(2\eta-1)(2\eta+1)\omega^2}\cos(4\eta\omega\tau)\right.$$
$$- \frac{8\eta}{(2\eta+1)(2\eta-1)^2\omega^2}\cos\left((2\eta-1)\omega\tau\right)$$
$$\left. + \frac{8\eta}{(2\eta+1)^2(2\eta-1)\omega^2}\cos\left((2\eta+1)\omega\tau\right)\right)\left(\frac{2}{\sqrt{15}}\,\delta_{m,2}\delta_{M,-1}\right. \quad (56)$$
$$\left. + \frac{1}{3}\sqrt{\frac{2}{5}}\delta_{m,0}\delta_{M,1} - \sqrt{\frac{2}{15}}\,\delta_{m,1}\delta_{M,0}\right)^2.$$

Equation (56) is a general form of transition probability when protonium is perturbed by the perturbation form as shown in equation (21). Perturbation of this type contains sinusoidal terms in the polar and azimuthal parts simultaneously, the time-dependent oscillation term with oscillation frequency ω; and finally, there is a scalar product factor between the protonium Pauli spin matrix and its three-dimensional position operator. The contribution of several of these factors simultaneously makes the total perturbation seem quite complex. However, by performing various calculation steps, the system transition probability form is finally obtained as shown by equation (56). This form of transition probability can easily be reduced to well-known forms as in the case of constant perturbation or harmonic perturbation, of course, by the choice of special circumstances. However, if one looks closely, there is something that is somewhat surprising, namely the absence of a constant potential V_0 in the system transition probability formulation. It can be seen that V_0 does not contribute at all to the transition probability of this system. However, we can understand that the presence of the factor $\vec{\sigma}_p \cdot \hat{x}$ as a multiplier of the constant V_0 makes the average value zero. This can be seen in the form of perturbation if we choose to omit (for comparison only) the scalar product factor of the Pauli spin matrix and this three-dimensional position operator; if this form does not exist, then the system transition probability is guaranteed to appear constant V_0 (either the form $\cos\omega t$ is omitted or not). This means, it is true that the presence of the scalar product completely eliminates the effect of V_0. But keep in mind that its presence only removes the influence of constant perturbation factors (as far as currently known), not for all multipliers. In fact, the perturbation term containing the factor $V_1\, r \sin\theta\, e^{i\phi}$ does not become zero when multiplied by the scalar product reviewed earlier, thus indicating that V_1 has a strong influence on the system transition probability, unlike V_0. Finally, when viewed again from equation (56) above, the general conditions for quantum numbers that must be met in order for a quantum transition to occur for this system are n even positive integer, $l = 2, S = 1$, and with the combination of m and M as shown in the last terms of the right-hand side of the equation.

5. Conclusion

Based on our analysis, it has been found that a protonium system when disturbed by perturbation as shown in equation (21) will produce a transition probability form as shown in equation (56). This system transition is possible only with the terms of the quantum number n positive integers (even numbers), $l = 2, S = 1$, and a certain combination of m and M according to the Delta-Kronecker function, with the pairs $\delta_{m,2}\delta_{M,-1}$; $\delta_{m,0}\delta_{M,1}$; and $\delta_{m,1}\delta_{M,0}$. It has also been found that the presence of the scalar product factor of the protonium Pauli matrix $\vec{\sigma}_p$ with the three-dimensional position operator $\hat{x}$ as a multiplier of a constant potential V_0 eliminates the effect of this potential totally on the system.

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