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Students' Spatial Ability in Learning Non-Euclid Geometry Through Ethnomathematics Approach

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Abstract

Background and Objective: Non-Euclid geometry is an abstract and difficult subject to learn, but it is mandatory for students. Ethnomathematics approach as a learning approach to improve students' spatial abilities. The purpose of this study was to discover new elements of the spatial capabilities of Non-Euclid Geometry through ethnomathematics learning.

Method: This study applies a micro-genetic method with a factorial experimental research design. The sample of this study was 100 students of Mathematics education. There are three research instruments that have been valid and reliable through expert tests and field trials. Data collection is carried out in two ways: tests and in-depth interviews. Quantitative data were analyzed through structural equation modeling, and qualitative data were analyzed through fixed comparisons. Experimental data were analyzed using ANCOVA.

Results: The results are that the learning of Non-Euclid Geometry through an ethnomathematics approach has a positive impact on students' spatial abilities. The spatial ability of students who were given the Ethnomathematical Learning Approach was higher than those who were given the conventional learning approach for Non- Euclid Geometry Materials.

Keywords: Spatial ability; geometry; non-Euclid; ethnomathematics

INTRODUCTION

Geometry is a system built from abstract concepts, namely the set of all points. It was a difficult course for mathematics education students (Widada, Herawaty, Ma'rifah, & Yunita, 2019; Nugroho, Widada, & Herawaty, 2019). Geometric objects are abstract, so students often have difficulty understanding them (Wu & Ma, 2006)(Widada, Herawaty, et al., 2019). Based on the mathematics curriculum, geometry taught in schools and colleges is Euclid Geometry, even though real problems are more related to Non-Euclid Geometry. There are two disadvantages of Euclid Geometry, namely that Euclid attempted to define all elements in geometry. Points, lines and planes are the three primitive concepts that he co-defines. Euclid defines a point as something that has no parts. He defines long lines without width. The words "section", "length" and "width" are not found in Euclid's Geometry. Also, almost every part of the proof of the theorem uses geometric drawings, but in practice these drawings are misleading (Nugroho et al., 2021).

In learning geometry systems, it presents its own challenges for students. Students still have a fairly good understanding when learning Euclid Geometry, but they have difficulty in learning Non- Euclid Geometry (Widada, Nugroho, Sari, & Pambudi, 2019). Therefore, a geometry learning approach is needed that can be a bridge from Euclid Geometry to Non-Euclid Geometry. Such an approach should be able to improve the abstraction ability of geometric concepts. It is an approach to learning based on real life and student culture.

Basically, geometry is the introduction of the properties of geometric shapes and objects. Spatial ability is the most important factor in understanding geometry and solving geometry problems (Eskisehir & Ozlem, 2015). Spatial ability is an intelligence to understand geometry. It was the core skill of human life. These abilities can be improved through formal education, namely by utilizing tools and technology developed through good planning (Yurt & Tünkler, 2016). Students get their own challenges during Geometry learning (Widada, Herawaty, Widiarti, Aisyah, & Tuzzahra, 2020). That is especially during students studying Non- Euclid Geometry. Spatial ability will play a very important role.

Space and form are the main studies in Geometry, so spatial ability is a basic competency to learn it (Güven & Kosa, 2008). The ability really helps students to understand solve to the problems of geometry (Kayhan, 2005). Spatial ability is a cognitive process that is interconnected with daily activities. In learning geometry and mathematics in general, spatial thinking skills are needed (Eskisehir & Ozlem, 2015). Spatial ability is a major factor in intelligence that can be improved through digital media (Dilling & Vogler, 2021). There are five components of spatial intelligence when viewed from the theory of multiple intelligences (Maier, 1998b). These components are first, spatial perception, namely fixation of perpendicular and horizontal directions; second, visualization is the ability to describe situations when components move in comparison to each other; third, mental rotation. It is the rotation of a three-dimensional solid; fourth, spatial relationship, as the ability to recognize the relationship between the parts of a solid object; Finally, spatial orientation, which is an ability to enter certain spatial situations.

Spatial geometry is one of the mathematics learning materials that must be developed from an early age (Bosnyak & Kondor, 2008). From the outset, students must effectively perform spatial representation conventions. To improve it, learning geometry must start from objects in real life (Widada, Nugroho, et al., 2019). Geometry learning is a student activity as a reflection and mathematization process. It's like a mathematician's work. The learning of geometry is the process of searching, and discovering new techniques, to reinvent statements, concepts and other mathematical objects(Karadag, 2009). It was a mathematical process aimed at investigating problem situations. Students can connect real objects with the concept of geometry. Students are able to perform abstractions, idealizations and generalizations through the surrounding experiences and cultures. A contextual problem-based learning approach is more appropriate for teaching geometry. Through this approach, students can represent mathematical activities horizontally and vertically to achieve formal mathematics (Freudenthal, 1991; Fauzan, Slettenhaar, Plomp, 2002; Gravemeijer, 1994; Treffers, 1991).

Integrated learning based on daily life has a positive effect on the ability to understand mathematics (Andriani et al., 2020). It can be seen that the ability to understand mathematical concepts of students who study with a real-world approach is higher than that of students who study conventionally. It increases even more for local culture-based learning. The research shows that learning geometry through an ethnomathematical approach can improve students' mathematical abilities. Also, other studies have consistently shown similar results. The ethnomathematical approach can improve students' mathematical understanding abilities which are higher than conventional methods (Widada, Herawaty, & Lubis, 2018). For example, the implementation of an ethnomathematical approach can simplify mathematical concepts to become more meaningful (Herawaty, Widada, Adhitya, Sari, & Novianita, 2020). Also, students are able to achieve the geometric principle of the product of two vectors (Widada, Herawaty, Sari, & Riyani, 2020). Other research, that there is an increase in the ability of mathematical representation of students who learn through an ethnomathematical approach (Widada, Nugroho, Sari, & Pambudi, 2019). These results consistently show that the ethnomathematical approach makes a positive contribution to improving the ability to understand geometry.

In learning geometry, mathematical connection with reality becomes a significant process. It arises from horizontal mathematization. Such is Freudenthal's idea that contextual problems as frameworks are inherent in mathematics itself (Gravemeijer, 2008; Plomp & Nieveen, 2013). The concepts of geometric data can be connected to the real world (Frassia & Serpe, 2017). It is more real-world connections in non- Euclid Geometry concepts. However, based on the indicators of Maier's spatial ability (Maier, 1998a), it can be concluded that this ability is for Euclid's Geometry. It is shown from the indicators of spatial perception and spatial visualization that students can make the perception of an object vertically and horizontally, and they can visualize the motion of solid objects. It is for platonic solid objects. On these objects, the axiom of Euclid alignment applies. The axiom is through one point outside a line there is exactly one that parallels that line. Another statement that suggests that the elements of such spatial abilities are for the frame of Euclid Geometry is that students can determine the position of an object if it rotates in a certain direction. This shows that the angle of one round is full 36 0. and there is a right angle in the frame. Right angles exist only in Euclid Geometry. Therefore, new indicators are needed that can accommodate real objects outside of platonic objects are very necessary to compile. It was a technique for bridging the spatial ability elements of Euclid Geometry towards the spatial ability elements in Non-Euclid Geometry. The geometric systems in question are Lobachevsky Geometry and Riemann Geometry.

The fundamental difference between the three geometric systems is the axiom of their alignment. The axiom of parallelism in Lobachevsky geometry is that there are two or more lines passing through the point P that do not lie on the line g and are parallel to the line. In contrast to the axiom of alignment in Riemann Geometry, that is, there are no lines parallel to the other lines. Two revelations are not accommodated in the indicators of the elements of spatial ability (Maier, 1998b). Therefore, new elements were needed to accommodate the indicators of spatial ability in Lobachevsky Geometry and Riemann Geometry. It was a way to connect non-platonic geometric objects in accommodating the indicators of other elements of spatial ability.

Non-platonic real objects are found a lot in everyday life. Those are geometric objects that are already familiar to the students. Like real objects that are used daily. Those are some objects in the local culture or objects commonly used by the community. Such as an indicator of visualizing the alignment of lines on objects based on local culture. Since the axioms of alignment are the fundamental differentiator of the three geometric systems, the starting-point of the bridge from Euclid to Non-Euclid Geometry is about the axioms of alignment. Through the local culture (=fishing gear "bubu") students can express the properties of the axioms of Lobachevsky's alignment (Herawaty, Khrisnawati, Widada, & Mundana, 2020). Another study found that through the medium of large oranges students were able to connect objects in Riemann geometry (Widada, Herawaty, Widiarti, Aisyah, et al., 2020). The two studies were conducted through learning with a local cultural approach (ethnomathematics of the term from (D'Ambrosio, 1998; 2001; Orey & Rosa, 2004; and François, 2010b). To find a description of the cognitive processes of the subjects in the two studies was analyzed through the genetic decomposition of students (Borji et al., 2018; Hankeln, 2021). It was the theory of APOS (action-process-object-scheme) (Dubinsky & McDonald, 2001; Dubinsky et al., 2000; Widada et al., 2019); Widada, et al., 2020); Dubinsky & Wilson, 2013).

During geometry learning, students can interpret the process of solving problems and the achievement of geometric concepts. It's a mathematical process that can be analyzed through the student's genetic decomposition of students (Cooley et al., 2007; Widada et al., 2019; 2020). Genetic decomposition is a structured collection of students' cognitive processes in the achievement of mathematical concepts (Cooley et al., 2007; Widada, 2001). The analysis of genetic decomposition is carried out through in-depth exploration of the activities of actions, processes, objects, and schemas in students' mental processes. It is an implementation of APOS Theory (ie. Action-Process-Object-Schema)

(Dubinsky & McDonald, 2001; Dubinsky et al., 2000; Widada, 2017).

The theory can be used to analyze spatial abilities. One of them is the ability that demands the position of the object being observed horizontally or vertically. Also, the ability to display the rules of change or displacement of the elements that make up a building, either three-dimensional to two-dimensional or vice versa. In cognitive activity, one of the mental activities is the spatial ability to rotate two-dimensional and three-dimensional objects precisely and accurately. It is also an ability to understand the arrangement of an object and its parts and their relationship to one another. The last is observing an object from various circumstances (Maier, 1998b). Based on the description, the problem of this research is how students' spatial abilities during learning Non-Euclid Geometry through an ethnomathematics approach. It is an attempt to produce an increase in new theory about the spatial ability characteristics of Non-Euclid Geometry. Characteristics were analyzed using The Frame Analysis Method based on student genetic decomposition (through Action-Process-Object-Schema Theory). Thus, there are three objectives of this study, namely determining ethnomathematics visualization, mental visualization, iconic relationships, symbolic relationships, formal geometry (non-Euclid) are valid and reliable indicators of the spatial ability of Non-Euclid geometry; Producing a model of structural equations of the relationship between Euclid geometric spatial ability and Non-Euclid geometry; and describing the characteristics of new spatial ability elements about the concepts and principles of Non-Euclid Geometry through an ethnomathematics approach reviewed from APOS Theory.

This research also answers quantitative research problems. The research problem is how to increase the students' spatial ability in Non-Euclid Geometry after learning through an ethnomathematical approach? To answer this, there are nine specific problems, namely: Are there differences in spatial ability between students who study Lobachevsky geometry and Riemann geometry after controlling for the effect of Euclid geometry's spatial ability?; Is there a difference in spatial ability between students who are given ethnomathematical and conventional approaches after controlling for the influence of Euclid Geometry spatial ability?; Is there an interaction effect of Geometry Material and Learning Approach on Non- Euclid Geometry spatial ability after controlling for Euclid Geometry spatial ability effect?; Is there a linear effect of covariate Euclid Geometry spatial ability on Non- Euclid Geometry spatial ability?; Does Euclid's Geometry spatial ability, Geometry Material and Learning Approach together affect spatial ability?; Is the student's spatial ability for Lobachevsky Geometry higher than Riemann Geometry taught by the Ethnomathematical Learning Approach after controlling for the effect of Euclid's Geometry spatial ability?; Is the students' spatial ability for Lobachevsky Geometry higher than Riemann Geometry taught with Conventional Learning Approach after controlling for the effect of Euclid Geometry spatial ability?; Is the spatial ability of students who are given an Ethnomathematical Learning Approach higher than students who are given a conventional Learning Approach for Lobachevsky Geometry Materials after controlling for the effect of Euclid Geometry spatial ability?; Is the spatial ability of students who are given an Ethnomathematical Learning Approach higher than students who are given a conventional Learning Approach for Riemann Geometry Materials after controlling for the effect of Euclid Geometry spatial ability?

METHODS

Participants

The participant is students of mathematics and mathematics education at universities in the Bengkulu Province, Indonesia. We selected 100 students through a simple random technique. Based on the students' work, 4 subjects were selected to be interviewed in depth. The selection is based on the initial characteristics for each group of geometric spatial abilities.

This research implements a mixed method between qualitative and quantitative. The focus of this research is to explore students' spatial abilities during non- Euclid Geometry learning. The learning is carried out through an ethnomathematical approach. Students' understanding of non- Euclid geometry concepts was analyzed based on APOS mental activity. To comply with the COVID-19 health protocol, the implementation of learning is carried out hybrid (online and offline according to the provisions of the head of the research site). This study also has exploratory characteristics.

Instrument

The researcher used a spatial ability test instrument to explore quantitative and qualitative data. Quantitative assessment rubric with ratio scale data. Quantitative data is also used as the basis for determining the research subjects who will be interviewed in depth. Interviews were conducted during and after learning geometry through an ethnomathematical approach. This research instrument has been reviewed and validated by 5-experts, it has also been tested on 30 students. Based on the Expert Test (panelist) of the Lobachevsky Geometric Spatial Ability Test Instrument it was obtained that the average validity index of Aiken's is 0.85 and each grain > 0.80 with a Cronbach Alpha coefficient of 0.744. The test showed that all items of the Lobachevsky Geometry Spatial Ability Test were valid and reliable. The results of the Riemann Geometry Spatial Ability Test Expert Test that each item showed Aiken's validity index of more than 0.80 and the average was 0.84 with a Cronbach Alpha coefficient of 0.770. This test gives the meaning that experts agree that the instrument of the Riemann Geometry Spatial Ability Test is valid and reliable. The third instrument is the Euclid Geometry Spatial Ability Test. The panelists also agreed that each item was valid and reliable, with the average Aiken's validity index of 0.85 with a Cronbach Alpha coefficient of 0.770. Thus, these three research instruments are valid and reliable based on expert tests. Furthermore, all these instruments were tested on 100 mathematics education students at a university in Bengkulu, Indonesia. The data of the trial results were analyzed using the help of the Lisrel 8.8 program. Analysis of the test data of the Euclid Geometry spatial ability test instrument obtained the result that each test item was valid, this was shown from the T-value of each item more than 1.96, with an average of 6.824. The instrument is also reliable with a Cronbach alpha of 0.79. For each item of the instrument the spatial ability of Lobachevsky Geometry is valid with a T-value of more than 1.96 and the average T-value is 5.695. Its Alpha Cronbach is 0.88 which means reliable. Also, the T-value of each item of the instrument's spatial ability of Riemann Geometry > 1.96 which means it is valid with the average T-value being 6.767 and the Cronbach alpha of 0.89. Thus, these three research instruments are standard and feasible to be used for data collection.

Data Collection Techniques

Learning is carried out during using the zoom meeting media. Also, supported by social media Whatsapp Group, Youtube and E-mail. Learning is carried out for twelve meetings in a period of eight weeks. After the lesson was finished, we conducted a spatial ability test via google form for all students out of 100 students. The test results were analyzed using the existing assessment rubric. However, to determine the four research subjects, a pretest was carried out. So that we can conduct in-depth interviews during the learning process. Interviews were conducted using Whatsapp media via video call. This in-depth interview was conducted to explore students' spatial abilities on Non- Euclid Geometry. This interview was recorded to obtain complete and accurate data. In the process of collecting data, the researcher sent a google form link from the research instrument to students. The arguments in the reason column are made openly for students to explore. An open field that has certain answers according to the students' cognitive processes.

Experimental Research Design

To answer a specific research problem regarding the effect of the ethnomathematical approach on geometric spatial abilities, experimental research was conducted. The design used is a 2x2 factorial design, as shown in Table 1.1.

Table 1.1 Experimental Design (Factorial 2x2)

Content of Geometry	Learning Approach	
	Ethnomathematics (B1)	Conventional (B2)
Lobachevsky (A1)	A1B1	A1B2
Riemann (A2)	A2B1	A2B2

Based on the research design (Table 1.1), A1B1 is Lobachevsky Geometry and Ethnomathematics Approach, A2B1 is Riemann Geometry and Ethnomathematics Approach, A1B2 is Lobachevsky Geometry and Conventional Approach, and A2B2 is Riemann Geometry and Conventional Approach. In this study, the covariate is Euclid Geometric Spatial Ability (X), and the dependent variable is Non-Euclid Geometric Spatial Ability (Lobachevsky Geometry and Riemann Geometry) (Y). Based on the research sample of 100 students, divided into four groups according to the experimental design. The four groups are Group 1: Students Study Lobachevsky Geometry with Ethnomathematical Approach; Group 2: Students Learn Riemann Geometry with Ethnomathematical Approach; Group 3: Students Study Lobachevsky Geometry with Conventional Approach; Group 4: Students Learn Riemann Geometry with Conventional Approach.

Data Analysis

The research data were analyzed through a micro-genetic approach (Siegler & Crowley, 1991). The micro-genetic approach is an approach to data analysis of learner activity that can generate a more precise description of cognitive changes than is possible. Data analysis used the frame analysis method (FAM) (Karadag, 2009). It was a spiral and cyclic structure with many cycles interacting with each other. Each cycle after the first one interacts with the previous cycle. The collection of mental activities obtained through the frame method is collected in pieces of statement (tree) through genetic decomposition analysis to find the characteristics of spatial ability elements of Non-Euclid Geometry (forest) (Karadag, 2009)(Widada, Herawaty, Jumri, & Wulandari, 2020). The cognitive processes in each of the above stages are always analyzed using the genetic decomposition of students (Widada, Herawaty, Nugroho, & Anggoro, 2019). Researchers analyzed the data using genetic decomposition analysis techniques. Researchers follow pre-analysis, microanalysis, and tree-to-forest sub-stages for each test item and its arguments. The description of each subject of study is categorized in action-process-object-scheme (Dubinsky & McDonald, 2000). Each stage of mental activity (APOS) can be represented in real activity (Widada, 2011).

The analysis process through a micro-genetic approach with the frame analysis method is carried out based on the following stages. The stages are Pre-analysis Stage-1, Pre-analysis Stage-2, Pre-analysis Stage-3, Microanalysis Stage-1, Microanalysis Stage-2, Microanalysis Stage-3, Tree-to-forest Stage-1, Tree-to-forest Stage-2, and Tree-to-forest Stage-3. To obtain a new theory, a process of theorizing is carried out through a fixed comparison method (Glaser & Strauss, 2006). Experimental data were analyzed using covariate analysis. To facilitate the analysis of the data, the SPSS application program was used.

RESULTS AND DISCUSSION

Based on data from the Euclid and Non-Euclid geometry spatial ability test results on 100 mathematics education students, it was analyzed using the Application Program of Lisrel 8.8. The result is poured in the following exposure. Based on data from the Euclid and Non-Euclid geometry spatial ability test results on 100 mathematics education students, it was analyzed using the Application Program of of Lisrel 8.8. The result is poured in the following exposure.

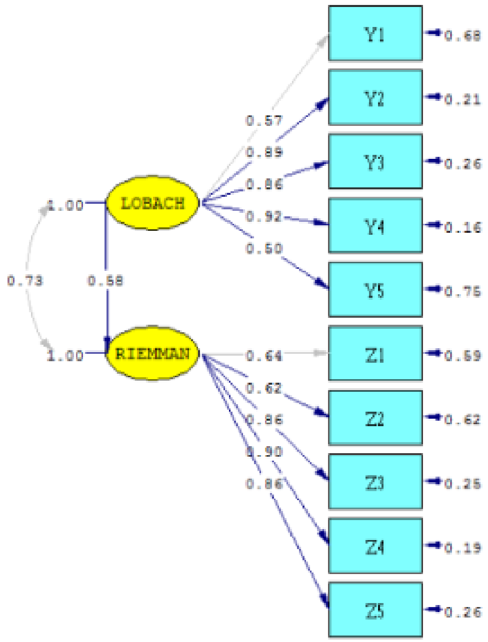

Figure 2.1a Standardized solutions of Lobachevsky and Riemann

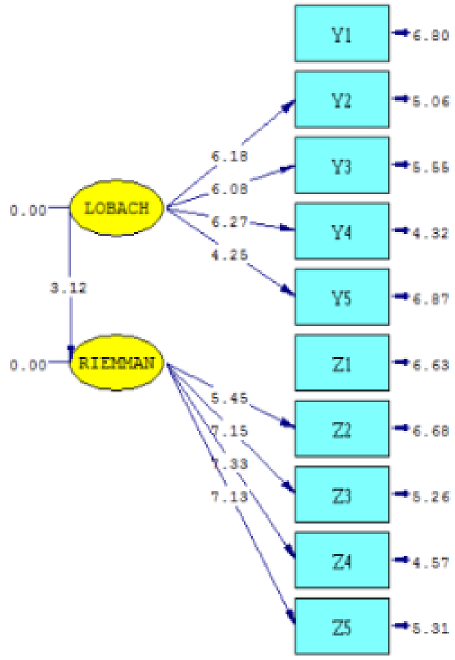

Figure 2.1b T-value of Lobachevsky and Riemann

Figures 2.1a and 2.1b are outputs of lisrel with elements of Lobachevsky's Geometric Spatial Ability being Y1: visualization of ethnomathematics; Y2: mental visualization; Y3: iconic relationships; Y4: symbolic relationships, and Y5: formal geometry (Lobachevsky). Whereas the elements of Riemann's Geometric Spatial Ability are Z1: ethnomathematics visualization; Z2: mental visualization; Z3: iconic relationships; Z4: symbolic relationships; Z5: formal geometry (Riemann). Based on Figures 2.1a and 2.1b, the calculation results are obtained as stated in Tables 2.1a and 2.1b.

Table 2.1a Validity and reliability of elements of spatial ability of Lobachevsky Geometry

Indicator	Standardized Loading Factors (SLF)≥ 0.50	Standard Errors	t-value > 1.96	Declaration of Validity	Reliability	
					CR≥0.70	VE≥0.50
Y1	0.57	0.68	**	Good	0.88	0.70
Y2	0.89	0.21	6.18	Good		
Y3	0.86	0.26	6.08	Good		
Y4	0.92	0.16	6.27	Good		
Y5	0.60	0.76	4.25	Good		

The captions for Tables 2.1a and 2.1b are CR: Construct Reliability, VE: Variance Extracted. Based on Table 2.1a, the t-value of each element > 1.96; also, CR > 0.70 and the VE > = to 0.50; that means that the five elements of the Lobachevsky Spatial Ability of Geometry (Y1-Y5) are valid and reliable. That means that the elements Y1, Y2, Y3, Y4 and Y5 are valid and reliable elements of the spatial capabilities of Lobachevsky's Geometry.

Table 2.1b Validity and reliability of elements of spatial ability of Riemann Geometry

Indicator	Standardized Loading Factors (SLF)≥ 0.50	Standard Errors	t-value > 1.96	Declaration of Validity	Reliability	
					CR≥0.70	VE≥0.50
Z1	0.64	0.59	**	Good	0.89	0.65
Z2	0.62	0.62	5.45	Good		
Z3	0.86	0.26	7.16	Good		
Z4	0.90	0.19	7.33	Good		
Z5	0.86	0.26	7.13	Good		

Based on Table 2.1b. the t-value of each element > 1.96; also, CR > 0.70 and VE > = to 0.50; it means that all five elements of the Riemann Geometry Spatial Ability (Z1-Z5) are valid and reliable. That means that the elements Z1, Z2, Z3, Z4 and Z5 are valid and reliable elements of the spatial capabilities of Riemann Geometry. It is thus concluded that ethnomathematics visualization, mental visualization, iconic relationships, symbolic relationships, constructing formal (non-Euclid) geometries are valid and reliable elements of the spatial abilities of Non-Euclid Geometry.

Based on these conclusions, it can be stated as a theory that the spatial appearance of Geometry Non-Euclid is the ability to think geometricly through a process of abstraction, idealization and generalization based on the five elements of mental and physical activity, namely ethnomathematics visualization, mental visualization, iconic relationships, symbolic relationships, and building a formal geometry of Non-Euclid. This is to complement the previous theory (Maier, 1998).

To conduct empiric tests of the elements of the spatial ability of Euclid Geometry was also carried out test the spatial ability of Euclid Geometry. The results of the analysis assisted by the Application Program of Lisrel 8.8 are as follows.

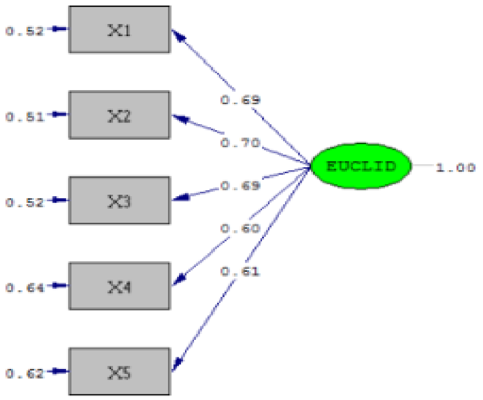

Figure 2.2a Standardized solutions of Euclid

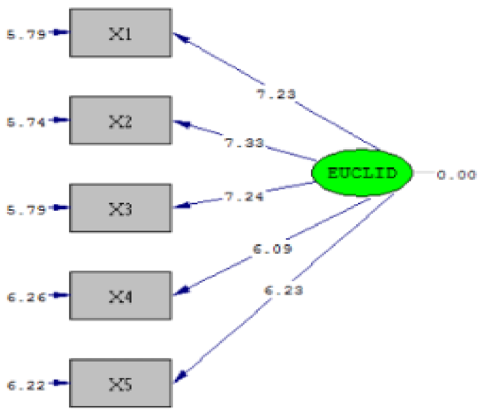

Figure 2.2b T-value of Euclid

Figures 2.2a and 2.2b with elements of Euclid's Geometric Spatial Ability are X1: Spatial Perception; X2: Visualization; X3: Mental Rotation; X4: Spatial Relationships; and X5:

Spatial Orientation. Based on Figures 2.2a and 2.2b obtained Table 2.2.

Table 2.1a Validity and reliability of elements of spatial ability of Euclid Geometry

Indicator	Standardized Loading Factors (SLF)≥ 0.50	Standard Errors	t-value > 1.96	Declaration of Validity	Reliability	
					CR≥0.70	VE≥0.50
X1	0.69	0.52	7.23	Good	0.79	0.50
X2	0.70	0.51	7.33	Good		
X3	0.69	0.52	7.24	Good		
X4	0.60	0.64	6.09	Good		
X5	0.61	0.62	6.23	Good		

Based on Table 2.2, the t-value of each element > 1.96; also, the CR > 0.70 and the VE > = to 0.50. The means that the five elements of Euclid's Geometric Spatial Ability are X1: Spatial Perception; X2: Visualization; X3: Mental Rotation; X4: Spatial Relationships; X5: Spatial Orientation is valid and reliable. It can be concluded that Spatial Perception, Visualization, Mental Rotation, Spatial Relationships, Spatial Orientation are valid and reliable elements of Euclid's geometric spatial ability empirically. Thus, Euclid's geometric spatial appearance is the ability to think geometry through a process of abstraction, idealization and generalization based on five mental activities in the form of Spatial Perception, Visualization, Mental Rotation, Spatial Relationships, and Spatial Orientation.

Based on the two conclusions above, a new statement is obtained as a theory. It is that the spatial appearance of geometric systems is the ability to think geometry through a process of abstraction, idealization and generalization based on ten mental and physical activities namely spatial perception, ethnomathematics visualization, immaterial visualization, mental visualization, mental rotation, iconic relationships, spatial relationships, symbolic relationships, spatial orientation and building formal geometry (Geometry Systems).

The aforementioned test data are also used to answer of the research problems: How does the structural equation model the relationship between the spatial ability of Euclid geometry and Non-Euclid geometry? A Theoretical Model of Structural Equations the relationship between the spatial ability of Euclid geometry and Non-Euclid geometry can be seen in Figure 2.3.

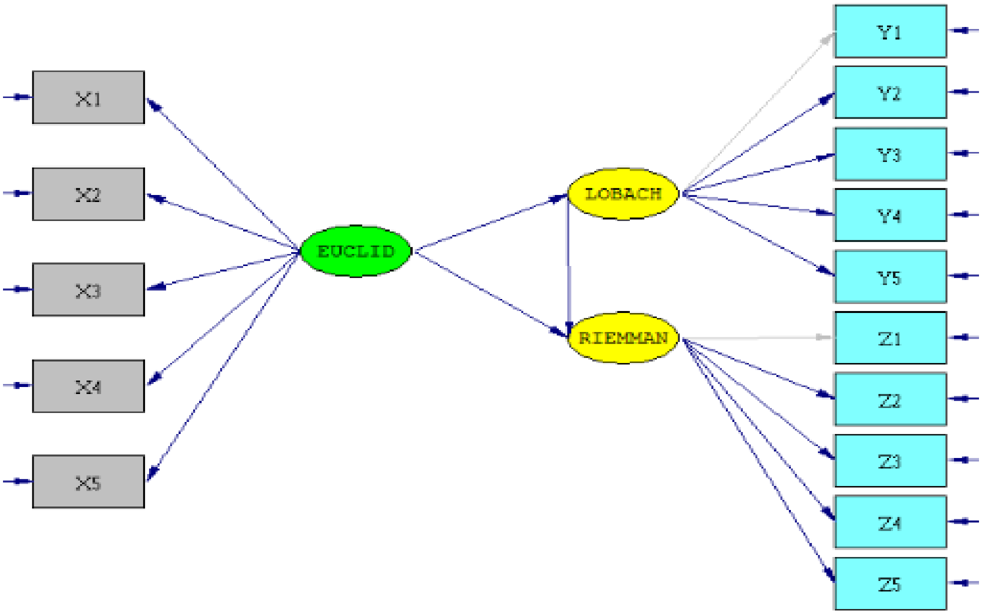

Figure 2.3 Theoretical Model of Structural Equations of the relationship between Euclid's geometric spatial abilities, Lobachevsky's geometry, and Riemann geometry

Figure 2.3 shows a Theoretical Model of Structural Equations the relationship between Euclid's geometric spatial abilities, Lobachevsky's geometry, and Riemann geometry. The elements are X1-X5, Y1-Y5 and Z1-Z5 are the same as the previous beam. Theoretical models are tested for compatibility with structural empiric equations. Based on the output of data analysis using Lisrel 8.8, obtained as stated in Table 2.3.

Table 2.3 Overall Model Fit Test Results

Statistics	Benchmark Values for Model Fit	Fit the Model from Empirical Data
X2 = 30.34; P-value = 0.073	P-value ≥ 0.05	Good Fit
RMSEA = 0.015	≤ 0.08	Good Fit
NFI = 0.92	≥ 0.90	Good Fit
NNFI = 0.91	≥ 0.90	Good Fit
CFI = 0.91	≥ 0.90	Good Fit
IFI = 0.91	≥ 0.90	Good Fit
RFI = 0.93	≥ 0.90	Good Fit
RMR = 0.01	≤ 0.05	Good Fit
SRMR = 0.02	≤ 0.05	Good Fit
GFI = 0.91	≥ 0.90	Good Fit
AGFI = 0.90	≥ 0.90	Good Fit

Based on Table 2.3, it appears that X2 = 30.34 P-value = 0.073≥ 0.05 means good fit. RMSEA = 0.015 ≤ 0.08; means good fit. NFI = 0.92 ≥ 0.90 means good Fit. NNFI = 0.91 ≥ 0.90 good fit. CFI = 0.91 ≥ 0.90 good fit. IFI = 0.91 ≥ 0.90 good fit. RFI = 0.93 ≥ 0.90 good fit. RMR = 0.01 ≤ 0.05 good fit. SRMR = 0.02 ≤ 0.05 good fit. GFI = 0.91 ≥ 0.90 good fit, lastly AGFI = 0.90 ≥ 0.90 which means marginal fit. This shows that the match test of the complete structural equation model is suitable. It also means that the theoretical structural equation model is compatible with the empirical structural equation model. Thus, the overall match test of such models shows the fit model.

An Empirical Model of Structural Equations the relationship between the spatial abilities of

Euclid geometry and Non-Euclid geometry can be presented in Figure 2.3.

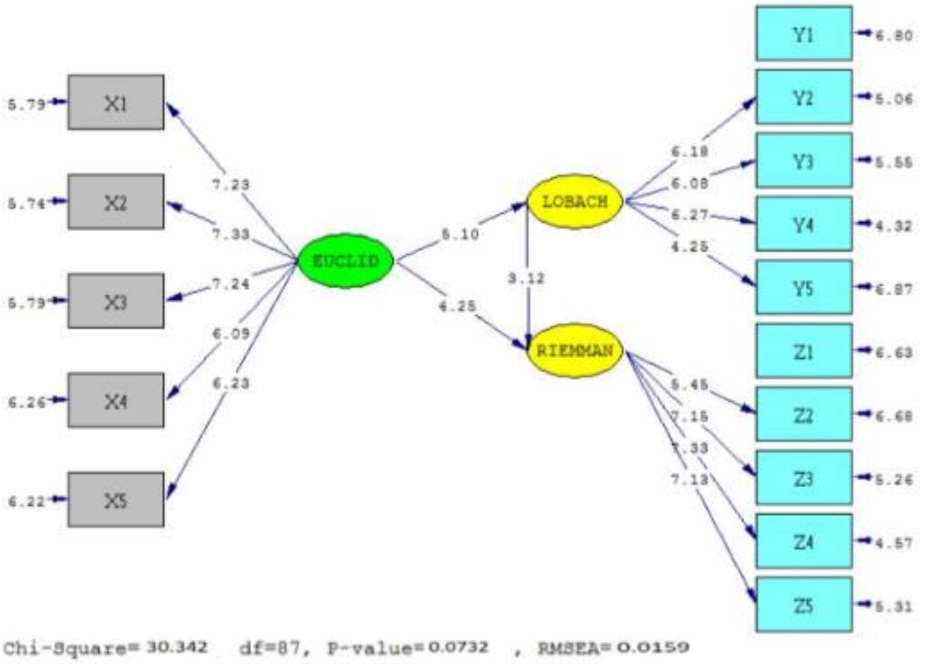

Figure 2.3. Empiric model of structural equations for T-test

Based on Figure 2.3 is an empiric model of structural equations that shows it fits into its theoretical model. The T-test on the empiric model showed that the spatial ability of Euclid Geometry had a direct effect on the spatial ability of Lobachevsky Geometry; Euclid Geometry spatial ability has a direct effect on the spatial ability of Riemann Geometry; the spatial ability of Lobachevsky Geometry has a direct effect on the spatial ability of Riemann Geometry; and the spatial ability of Euclid Geometry had an indirect effect on the spatial ability of Riemann Geometry through the spatial ability of Lobachevsky Geometry. This is in accordance with the results of previous studies that there is a direct relationship of Euclid geometry to Non- Euclid geometry, in particular hyperbolic geometry, in secondary education through dynamic geometry(Favian, López, Jesús, & Velázquez, 2021). Spatial ability can be measured by understanding the ability to perceive observed objects both horizontally and vertically (Rahmatulwahidah & Zubainur, 2017). It is to show the rules of change from three-dimensional to two-dimensional or vice versa, rotating two-dimensional and three-dimensional.

Furthermore, to describe the characteristics of new spatial ability elements about the concepts and principles of Non-Euclid Geometry through an ethnomathematics approach in terms of APOS Theory, in-depth interview data were used. Based on descriptive data from in-depth interviews of research subjects, snippets of interviews of four research subjects (Ar, Dk, Br, and Fr) with the interviewer (P) can be presented. This interview is to explore the spatial abilities of Lobachevsky Geometry. This exposure is divided into five elements of the spatial ability of Lobachevsky Geometry, namely ethnomathematics visualization, iconic visualization, mental relationships, symbolic relationships, and building formal Lobachevsky Geometry.

Visualization of Ethnomathematics

Excerpts of interviews of four research subjects that have the character of ethnomathematics visualization elements are as follows.

P: Given the line g and point P beyond line g with P and g located on the surface of the fishing gear "bubu", determine the line that goes through point P and parallel line g, then determine the other lines that go through point P and parallel line g.

Ar: Well, please wait for me to work on your orders using the actual bubu, as available.

Ar: [after 8 minutes]... I can tell you the picture is as follows.

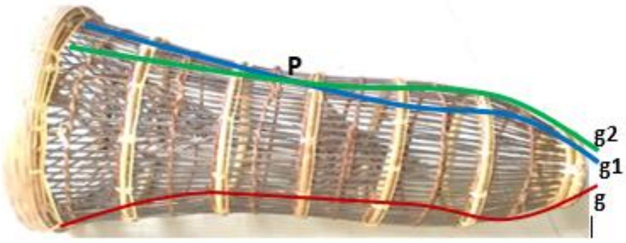

Figure 2.4 Ethno visualization of two lines parallel to g created by Ar

Dk: [after 7 minutes] ... Here's a picture from me.

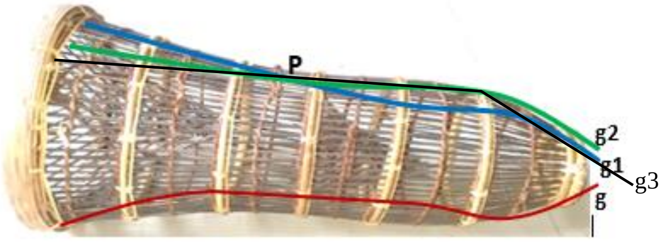

Figure 2.5 Ethno visualization of three lines parallel to g created by Dk

Br: [after 5 minutes] ... I can describe it as follows.

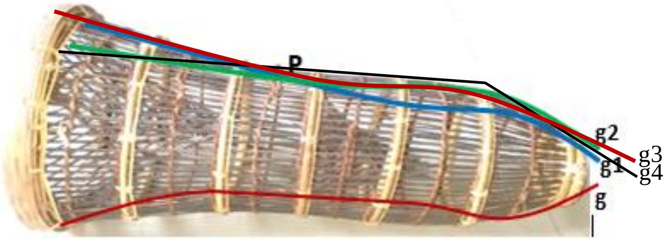

Figure 2.6 Ethno visualization of four lines parallel to g created by Br

Fr: [after 7 minutes] ... The picture of the line you requested is as follows.

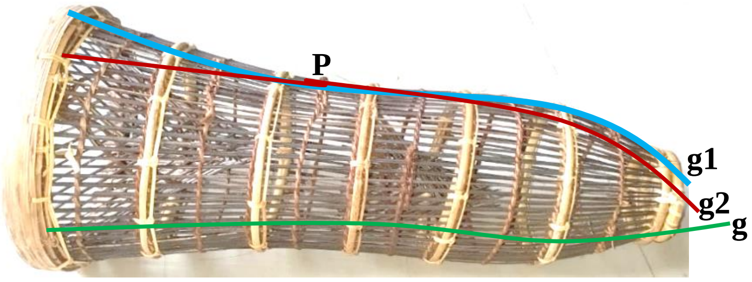

Figure 2.7 Ethno visualization of two lines parallel to g created by Fr

Based on in-depth interviews and Figure 2.4-2.7, the characteristic of ethnomathematics visualization in Lobachevsky Geometry is that students can visualize the alignment of lines in Lobachevsky Geometry based on local cultural objects. The characteristic based on the APOS Theory is that students can carry out action and process activities so as to produce visual objects about the alignment of lines in Lobachevsky Geometry based on local cultural objects. It supports the results of the study that the improvement of the geometry ability of triggered through modeling that helps students understand the world around them; can obtain a flexible way of thinking, and is able to reflect and analyze the real world (Frassia & Serpe, 2017).

Iconic Visualizations

Snippets of interviews of four research subjects that have the character of iconic visualization elements are as follows.

P: Determine the drawing of the lines that go through the P point and parallel the g line on the local culture-based object "bubu" placed on a flat plane?

Ar: Well, please wait for me to work on your orders using the actual bubu, as available.

Ar: Ok, based on the image from me then I made the following.

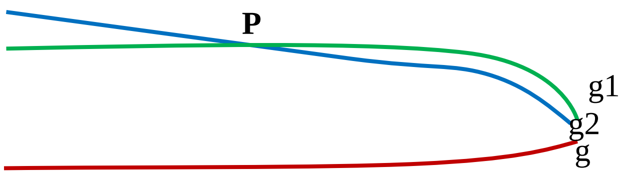

Figure 2.8 Iconic visualization of two lines parallel to g created by Ar

Dk: [after 7 minutes] ... Here's a picture from me.

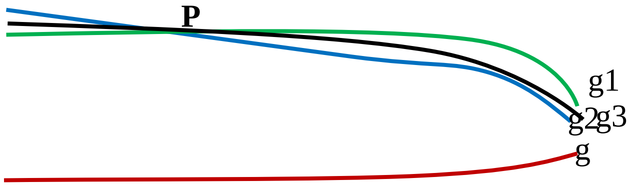

Figure 2.9 Iconic visualization of three lines parallel to g created by Dk

Br: [after 5 minutes] ... I can describe it as follows.

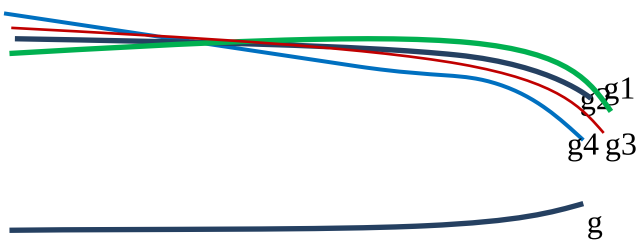

Figure 2.10 Iconic visualization of four lines parallel to g created by Br

Fr: [after 7 minutes] ... The picture of the line you requested is as follows.

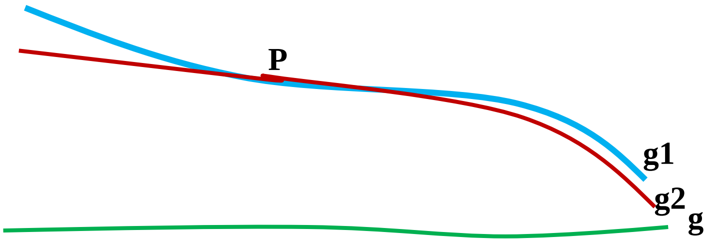

Figure 2.11 Iconic visualization of two lines parallel to g created by Fr

Based on in-depth interviews and Figure 2.4-2.7, the characteristic of ethnomathematics visualization in Lobachevsky Geometry is that students can visualize the alignment of lines in Lobachevsky Geometry based on local cultural objects. The characteristic based on the APOS Theory is that students can carry out action and process activities so as to produce visual objects about the alignment of lines in Lobachevsky Geometry based on local cultural objects. It supports the results of the study that the improvement of the geometry ability of triggered through modeling that helps students understand the world around them; can obtain a flexible way of thinking, and is able to reflect and analyze the real world (Frassia & Serpe, 2017).

Iconic Visualizations

Snippets of interviews of four research subjects that have the character of iconic visualization elements are as follows.

P: Determine the drawing of the lines that go through the P point and parallel the g line on the local culture-based object "bubu" placed on a flat plane?

Ar: Well, please wait for me to work on your orders using the actual bubu, as available.

Ar: Ok, based on the image from me then I made the following.

Based on in-depth interviews and Figure 2.8-2.11, then the characteristic of Iconic Visualization is that student can visualize the alignment of lines on Lobachevsky Geometry in iconic representations (images) based on local cultural objects. The characteristic based on the APOS Theory is that students can carry out action and process activities so as to produce objects in the form of iconic representations of the alignment of lines in Lobachevsky Geometry based on local cultural objects. It supports the results of research that improving geometry skills through modeling helps students understand the world around them; can acquire a flexible way of thinking, and be able to reflect and analyze the real world (Frassia & Serpe, 2017).

Mental Relationships

Excerpts of interviews of four research subjects that have the character of mental relationship elements are as follows.

P: If given a picture of the lines going through the point P and paralleling the g line on a local culture-based object "bubu", make a statement of the alignment of the lines?

Ar: Based on the visual representation I made, I stated that: "there are two lines parallel to the g line and those two lines through the P point."

Dk: In my opinion, "through the point P beyond the line g there are three lines parallel to the line g."

Br: Based on the image I made earlier, I can create in the form of a sentence that is "Four lines can be made that go through a P point parallel to the g line.".

Fr: It can be stated that "there are two lines that go through point P that are not members of line g and those two lines are parallel to g.

Based on the above snippet of the interview, the student who is in the Mental Relationships group has the character that he can make an understanding/statement about the alignment of lines on Lobachevsky Geometry based on iconic representations. The characteristic based on APOS Theory is that students can carry out action, process and encapsulation activities so as to produce mental objects in the form of understanding/ statements about the alignment of lines in Lobachevsky Geometry based on iconic representations. This gives the meaning that through an ethnomathematics approach, students are able to build geometric mental objects, according to the mental constructions of APOS (Dubinsky & McDonald, 2000a).

Symbolic Relationships

Excerpts of interviews of four research subjects that have the character of symbolic relationship elements are as follows.

P: Make a statement about the lines that go through the point P and parallel the g line on the local culture-based object "bubu", and make a symbolic representation of it?

Ar: According to the statement I have made, it can be created symbolically as follows.

$$(P \notin g)\ (\exists\ g1\ \&\ g2).\ g1//g\ \&\ g2//g\ (g1 \cap g2 = P).$$

Dk: For the symbolic I adjusted to the statement I gave that is:

$$(P \notin g)\ (\exists\ g1 \wedge g2 \wedge g3).\ (P \epsilon g1,\ P \epsilon g2,\ p\ \epsilon g3)\ g1 \wedge g2 \wedge g3//g.$$

Br: Based on the statement I made, it can be symbolically written as follows.

$$(\exists\ g1,\ g2,\ g3,\ g4).\ g1 \cap g2 \cap g3 \cap g4 = \{P\}.\ (P \epsilon g1,\ P \epsilon g2,\ p\ \epsilon g3)\ g1//g,\ g2//g,\ g3//g,\ g4//g.$$

Fr: I think the symbolic can be seen in the answers on “this paper...”

$$(\exists\ g1 \wedge g2)\ \&\ g1 \cap g2 = \{P\}.\ (P \epsilon g1,\ P \epsilon g2,\ p\ \epsilon g3)\ g1 \wedge g2//g\ \&\ P \notin g.$$

Based on the above snippet of the interview, students who are in the Symbolic Relationships group have characters can make symbolic connections between points and lines in Lobachevsky Geometry based on mental visualization. Students can carry out activities of actions, processes and objects so as to produce schemes in the form of symbolic relationships between points and lines in Lobachevsky Geometry based on mental visualization. This suggests that geometry learning close to the student's mind improves students' geometry skills (Oladosu, 2014). The ethnomathematics approach is to utilize media that is close to the student's mind, so this approach can affect geometry skills.

Building Formal Geometry

Excerpts of interviews of four research subjects that have the character of the building elements of the Formal Lobachevsky Geometry are as follows.

P: Determine the reasonable effect of the statement about the lines passing through point P and parallel to line g on the local culture-based object "bubu" and prove the veracity of the statement made? (Hint: at least one statement).

Ar: Well, if I observe from the lines that through the points P and g is a line with P not being in g, then I declare that "There are a great many lines that go through point P and are parallel to g."

P: Can you determine how many lines are parallel to the g?

Ar:.... There seems to be an infinite number of them.

Dk: [Separately Dk gave an answer]... I found one statement which is "If P is a point beyond line g, then there is an infinity line parallel to line g."

This snippet of the interview gives the meaning that the student can make a new statement and prove it as a corollary of the statement about the alignment of the lines on the Lobachevsky Geometry. The character is that students can perform activities of actions, processes, objects and relate schemes about the alignment of the lines on Lobachevsky Geometry in order to make new statements and prove them as corollaries of statements in such geometric systems.

Furthermore, the core of the data analysis of this research are: Data on the implementation of the ethnomathematical learning approach of Non-Euclid geometry learning materials. analyzed using the SPSS program. The data is the spatial ability of Lobachevsky geometry, and the spatial ability of Riemann geometry as the dependent variable, with the covariate being the spatial ability of Euclid's geometry. The results of the analysis are presented as follows.

The homogeneity test of variance of the four study groups (A1B1, A1B2, A2B1, and A2B2) is presented in Table 2.4.

Table 2.4 Levene's Test of Equality of Error Variancesa

F	df1	df2	Sig.
3.789	3	96	0.331

Tests the null hypothesis that the error variance of the dependent variable is equal across groups. Dependent Variable is Spatial Ability. aDesign: Intercept + A * B + X + A * B * X.

Based on Table 2.4, Levene's test value of variance error is F = 3.789 with db(3, 96) and p-value = 0.331> 0.05, which means Ho is accepted. Thus, the conclusion is that the average parameter of the four sample data groups has the same variance. It is also said that the four variances are homogeneous. Furthermore, the regression alignment is tested as follows.

Table 2.5 Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Intercept	3328.835	1	3328.835	194.453	.000
A * B	161.449	3	53.816	3.144	.029
X	704.614	1	704.614	41.160	.000
A * B * X	249.414	3	83.138	4.856	.064
Error	1574.944	92	17.119		
Total	545484.000	100			

Dependent Variable: Spatial Ability; R Squared = .914 (Adjusted R Squared = .908)

Based on Table 2.5 obtained F = 4.856 with db(3, 92) and p-value = 0.064> 0.05. It means that Ho is accepted. The conclusion is the regression coefficient of the four groups is homogeneous. The meaning is that the four regression equations are parallel. The four regression equations are Y11 = 6 3.350 + 0.503X; Y21 = 61.261+ 0.503X; Y12 = 40.363+ 0.503X; and Y22 = 37.271 + 0.503X.

Since the four variances are homogeneous and the four regression equations are parallel, it can be continued by testing the research hypothesis. Successively the hypothesis test is as follows.

Table 2.6 Tests of Between-Subjects Effects

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	16,583.643a	4	4,145.911	215.891	.000
Intercept	3,534.831	1	3,534.831	184.070	.000
A	124.794	1	124.794	6.498	.029
B	15,718.409	1	15,718.409	818.507	.000

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
A * B	126.865	1	126.865	6.606	.024
X	755.003	1	755.003	39.315	.000
Error	1,824.357	95	19.204		
Total	545,484.000	100			

Dependent Variable: Spatial Ability; [a.]R Squared = .901 (Adjusted R Squared = .897)

Based on Table 2.6, it is obtained that Fo (A) = 184.070. db = (1, 99) and p-value = 0.00<0.05, Ho is rejected. Thus, there is a difference in spatial ability between students with Lobachevsky Geometry and Riemann Geometry after controlling for the effect of Euclid Geometry spatial ability. Fo(B) = 6.498, db (1, 99) and p-value=0.029<0.05, Ho is rejected. Thus, there is a difference in spatial ability between students who were given ethnomathematical and conventional approaches after controlling for the influence of Euclid Geometry spatial ability. Fo(AB) = 818,507, db (1, 99) and p-value=0.00<0.05, Ho is rejected. Thus, there is an interaction effect of Geometry Material and Learning Approach on Non- Euclid Geometry spatial ability after controlling for the effect of Euclid Geometry spatial ability. Fo(X) = 6.606, db (1, 99) and p-value=0.024<0.05, Ho is rejected. Thus, there is a linear effect of covariate Euclid Geometry spatial ability on Non-Euclid Geometry spatial ability. In the corrected model line, it is obtained Fo = 215,891 with db(4, 99) and p-value = 0.00<0.05 Ho is rejected. Thus, the spatial ability of Euclid's Geometry, Geometry Materials and Learning Approach together affect the spatial ability. Next, consider Table 2.7.

Table 2.7 Parameter Estimates I

Parameter	B	Std. Error	t	Sig.
Intercept	49.995	10.721	4.663	.000
[A=1.00]	1.010	.698	.374	.709
X	.487	.234	2.078	.040

Dependent Variable: Spatial Ability

Based on Table 2.7, the t test with t count = 0.374 and p-value = 0.709 > 0.05 means Ho is accepted. Thus, the average spatial ability of students who studied Lobachevsky geometry was not higher than that of students who studied Riemann geometry after controlling for the effect of Euclid geometry's spatial ability.

Table 2.8 Parameter Estimates II

Parameter	B	Std. Error	t	Sig.
Intercept	35.646	3.532	10.093	.000
[B=1.00]	25.089	.880	28.526	.000
X	.537	.076	7.040	.000

Dependent Variable: Spatial Ability

Based on Table 2.8, the t-test with t-count = 28.52 and p-value = 0.000 <0.05 means Ho is rejected. Thus, the average non- Euclid Geometry spatial ability of students who were given the Ethnomathematical Learning Approach was higher than the students who were given the conventional Learning Approach after controlling for the effect of Euclid Geometry spatial ability.

Table 2.9 Parameter Estimates III

Parameter	B	Std. Error	t	Sig.
Intercept	-5.992	.913	-6.562	.000
[A=1.00]*[B=1.00]	1.521	.259	5.870	.000
X	.113	.007	15.128	.000

Dependent Variable: Spatial Ability

Based on Table 2.9, the t-test with t-count = 5.870 and p-value = 0.000 <0.05 means Ho is rejected. It means that there is an interaction effect between Geometry Material and Learning Approach factors on Non-Euclid Geometry spatial ability after controlling for the influence of Euclid Geometry spatial ability.

Table 2.10 Parameter Estimates IV

Parameter	B	Std. Error	t	Sig.
Intercept	37.271	3.866	9.641	.000
X	0.503	.080	6.270	.000
[A=1.00] * [B=1.00]	2.088	1.286	1.624	.108
[A=1.00] * [B=2.00]	-.092	1.260	-.073	.942

Dependent Variable: Spatial Ability

Based on Table 2.1 0. in the third row it is found that the t test with t count = 1.624 and p-value = 0.108/2 = 0.054 > 0.05 means Ho is accepted. It means that students' spatial ability for Lobachevsky Geometry is not higher than Riemann Geometry taught with Ethnomathematical Learning Approach after controlling for the influence of Euclid's Geometry spatial ability. Also, the t-test in the 4th row obtained t count = -0.073 and p-value = 0.942/2 = 0.471> 0.05 means Ho is accepted. Thus, students' spatial ability for Lobachevsky Geometry is not higher than Riemann Geometry taught with Conventional Learning Approach after controlling for the effect of Euclid Geometry spatial ability. The final test, see Table 2.11.

Table 2.11 Parameter Estimates V

Parameter	B	Std. Error	t	Sig.
Intercept	37.271	3.866	9.641	.000
X	0.503	.080	6.270	.000
[A=1.00] * [B=1.00]	26.171	1.268	20.644	.000
[A=2.00] * [B=1.00]	23.990	1.276	18.796	.000

Dependent Variable: Spatial Ability

Based on Table 2.11, in the third row obtained t count = 2 0.644 and p-value = 0.000 < 0.05 means Ho is rejected. It means that the spatial ability of students who are given the Ethnomathematical Learning Approach is higher than the students who are given the conventional Learning Approach for Lobachevsky Geometry Materials after controlling for the influence of Euclid's Geometry spatial ability. For the 4th row, t count = 18,796 and p-value 0.000 < 0.05 means Ho is rejected. It means that the spatial ability of students who are given the Ethnomathematical Learning Approach is higher than the students who are given the conventional Learning Approach for Riemann Geometry Materials after controlling for the effect of Euclid's Geometry spatial ability.

Based on the hypothesis testing from Table 2.6 to Table 2.11, it can be concluded as follows.

1) There is a difference in spatial ability between students with Lobachevsky Geometry and

Riemann Geometry after controlling for the effect of Euclid's Geometry spatial ability.

2) There are differences in spatial ability between students who were given ethnomathematical and conventional approaches after controlling for the influence of Euclid Geometry spatial ability.

3) There is an interaction effect of Geometry Material and Learning Approach on the spatial ability of Non- Euclid Geometry after controlling for the effect of Euclid Geometry spatial ability.

4) There is a linear effect of covariate Euclid Geometry spatial ability on Non- Euclid Geometry spatial ability.

5) The spatial ability of Euclid's Geometry, Geometry Material and Learning Approach together affect the spatial ability.

6) Students' spatial ability for Lobachevsky Geometry material is not higher than Riemann Geometry material taught with Ethnomathematical Learning Approach after controlling for the influence of Euclid's Geometry spatial ability.

7) The students' spatial ability for Lobachevsky Geometry is not higher than the Riemann Geometry material taught with the Conventional Learning Approach after controlling for the influence of Euclid's Geometry spatial ability.

8) The spatial ability of students who were given the Ethnomathematics Learning Approach was higher than the students who were given the conventional Learning Approach for Lobachevsky Geometry after controlling for the effect of Euclid's Geometry spatial ability.

9) The spatial ability of students who were given the Ethnomathematics Learning Approach was higher than the students who were given the conventional Learning Approach for Riemann Geometry after controlling for the effect of Euclid's Geometry spatial ability.

Discussion

Based on the application of learning through an ethnomathematics approach, a micro-analysis of student activities during learning is carried out. It was an analysis of genetic decomposition by applying the APOS Theory (Dubinsky, Dautermann, Leron, & Zazkis, n.d.). The results of the analysis are as follows.

(1) Action:

To facilitate students to take "action" that is to understand the real problems related to the Lobachevsky Alignment Axiom, namely "Bubu"(Traditional fishing gear of the Bengkulu people). Students choose one place of bonding each stick bubu, One place is named after the point P, choose one stick at the base of the bubu that is not bound at point P, name the one stick with the line g, Choose two sticks tied to the bond at point P, merela name the two sticks with the lines g1 and g2. It has also been produced by Widada, et. al. (W Widada, Herawaty, Hudiria, Prakoso, et al., 2020). See Figure 2.1.

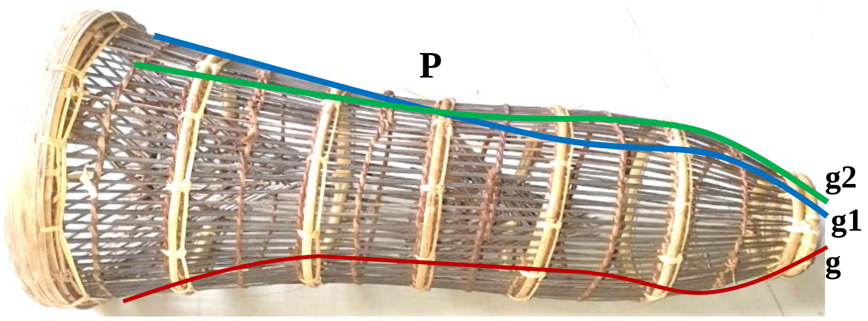

Figure 2.12 Student Action through Ethnomathematics Media

The action is a cognitive process in the form of Ethnomathematics-Visualization. Students who successfully make mathematical connections, that is, connect how the ideas of geometry related to bubu; connect new problems to old ones by asking, "Where did I see problems like this before?"; like to see how the ideas or concepts of Lobachevsky's Geometry are connected to the real world; can easily relate ideas familiar to new concepts or skills; and like to know when others think of solution strategies in a different way. It has also been researched in the study of Riemann Geometry (W Widada, Herawaty, Widiarti, Aisyah, et al., 2020).

(2) Process:

Process is a contemplated action. A process can be obtained by performing the action repeatedly. This stage is characterized when a person who can think of doing a process without actually doing it and can think about how to reverse or structure a process. To facilitate students to carry out the "process" which is to represent one place of bonding with one point P, representing one stick at the base of the bubu that is not bound at point P there is a line g, and representing two sticks tied to the bond at point P with two lines, namely the lines g1 and g2 (See Figure 2.13).

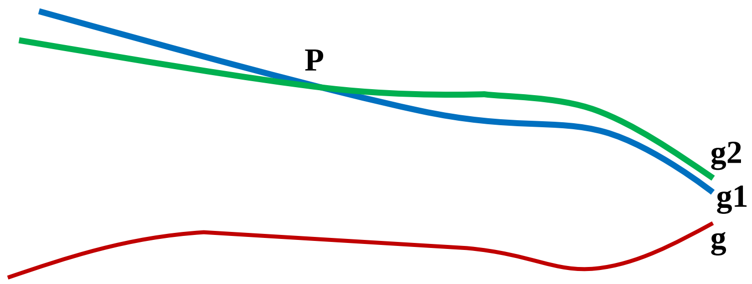

Figure 2.13 Student Process Activity through Ethnomathematics Media

Process activity is cognitive activity in the form of mathematical representations. A student who succeeds in representation (Widada, Nugroho, et al., 2019) i.e. has a list of ways to represent the problem and its solution; uses a series of representations in expressing his thoughts, (words, images or images, graphs or other graphs); uses representations to reveal what he thinks, how he knows them, and how the problem is solved; can easily move from one type of representation to another and know the right or proper representation to use and when to use it.

(3) Object:

An object is a totality that is carried out on a process. This stage is characterized by the ability of a person to perform actions on the object as well as provide reasons or explanations about its properties, as well as be able to re-decompose. To facilitate students to do "objects, this module contains: questions consisting of several concepts, questions that encourage students to provide explanations for what they write, questions that encourage students to re-decipher the properties of a concept, questions that train students to be able to flip through concepts in the material being taught.

The object produced by students in this activity is a statement about the alignment of lines in the ethnomathematics media "bubu". The statement is: "Through the point P beyond the g line, then there are g1 and g2 lines parallel to the g line." The object is a cognitive process in the form of mental relationships. It is that a student who manages to communicate mathematically about Lobachevsky's Parallels that is able to explain his thoughts clearly and succinctly, that is, seeks clarification about two lines parallel to a certain line; seeks to make new statements about the axioms of alignment in special cases; gives explanations or tries to figure out why the statement makes sense (Herawaty, Widada, Handayani, Febrianti, & Abdurrobbil, 2020)(Widada, Nugroho, et al., 2019).

(4) Schema:

The scheme for a particular concept of Lobachevsky Geometry is a collection of actions, processes and objects connected by several principles in general so as to form an interrelated framework in one's mind. To facilitate students to carry out a "scheme" that is to contain: achieving a statement in the form of the Axiom of Lobachevsky's Alignment as a result of a task that connects a general situation involving several concepts studied. Students can make a statement equivalent to the previous statement. The statement is: "If A point is beyond line g, then there are at least two lines parallel to g." The scheme is a mental activity carried out by students in the form of Symbolic Relationships. The activity gives the meaning that the student can make symbolic connections between points and lines based on mental visualization

(5) Thematization:

Thematization is a mental activity it is a thematization of the action-process-object-schematics that resulted in a Formal Lobachevsky Geometry (W Widada, Herawaty, Hudiria, Prakoso, et al., 2020). The result is that student can make a new statement and prove it as a corollary of the statement on the alignment of the lines. It was a response to a stimulus: determine the reasonable consequences (at least one statement) of the statement about the lines that go through the point P and parallel the line g on the object based on the local culture "bubu" and prove the truth of the statement made?

Based on the thematic activities, students are able to compose a statement as follows: "Through one point P outside the g line, there is an infinite line parallel to the g line." Students are trying to prove it, even though no student has been able to complete it completely. This is the basis for further exploration of the results of the analysis of the application of non-Euclid geometry learning through an ethnomathematics approach. This simplification becomes important to bring students from mathematical thinking horizontally through the local culture towards vertical mathematization, namely Lobachevsky Geometry. This means that the ethnomathematics approach became a bridge from Euclid Geometry to Lobachevsky Geometry (Widada, Herawaty, et al., 2019). Based on the process of theorizing (Tree-to-forest) obtained a description of the characteristics of each element of the spatial abilities of the Lobachevsky Geometry and Riemann Geometry.

Based on the results and several relevant studies, the spatial ability characteristics for Non-Euclid Geometry contain five elements, namely Ethnomathematics-Visualization, Iconic Visualization, Mental Relationships, Symbolic Relationships, and Building Non-Euclid Geometry. Visualization of Ethnomathematics has the character that students can carry out action and process activities so as to produce visual objects about the alignment of lines in Non-Euclid Geometry based on local cultural objects; Iconic Visualizations with character students can carry out action and process activities so as to produce objects in the form of iconic representations of the alignment of lines in Non-Euclid Geometry based on local cultural objects; Mental Relationships with character students can carry out action, process and encapsulation activities so as to produce mental objects in the form of understanding/statements about the alignment of lines in Non-Euclid Geometry based on iconic representations; Symbolic Relationships with character students can carry out activities of actions, processes and objects so as to produce schemes in the form of symbolic relationships between points and lines in Non-Euclid Geometry based on mental visualization; and Building Non-Euclid Geometry in character students can perform action activities, processes, objects and connect schemes about the alignment of lines in Non-Euclid Geometry to make new statements and prove them as corollary of statements in the geometry system. The results of this study contributed to the development of theories about the elements of spatial ability, especially in the realm of Geometric Systems (Euclid and Non-Euclid). It was to complement the elements of the initial spatial ability (Maier, 1998b; Kayhan, 2005; Eskisehir & Ozlem, 2016).

CONCLUSION

Learning Euclid and Non-Euclid geometry through ethnomathematics approaches has a positive impact on students' spatial abilities. This research resulted in the statement that spatial representation of geometric systems is the ability to think geometry through a process of abstraction, idealization and generalization based on ten mental and physical activities, namely spatial perception, ethnomathematics visualization, delicious visualization, mental visualization, mental rotation, iconic relationships, spatial relationships, symbolic relationships, spatial orientation and building formal geometry (Euclid and Non-Euclid Geometry). Euclid's Geometric spatial ability had a direct effect on Lobachevsky's Geometric spatial ability; Euclid Geometry spatial ability has a direct effect on the spatial ability of Riemann Geometry; the spatial ability of Lobachevsky Geometry has a direct effect on the spatial ability of Riemann Geometry; and the spatial ability of Euclid Geometry had an indirect effect on the spatial ability of Riemann Geometry through the spatial ability of Lobachevsky Geometry.

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REFERENCES

1. Andriani, D., Widada, W., Herawaty, D., Ardy, H., Nugroho, Ma'rifah, N., … Anggoro, A. F. D. (2020). Understanding the number concepts through learning Connected Mathematics (CM): A local cultural approach. Universal Journal of Educational Research, 8(3), 1055–1061. https://doi.org/10.13189/ujer.2020.080340
2. Borji, V., Font, V., Alamolhodaei, H., & Sánchez, A. (2018). Application of the complementarities of two theories, APOS and OSA, for the analysis of the university students' understanding on the graph of the function and its derivative. Eurasia Journal of Mathematics, Science and Technology Education, 14(6), 2301–2315. https://doi.org/10.29333/ejmste/89514
3. Bosnyak, A., & Kondor, R. N. (2008). The spatial ability and spatial geometrical knowledge of university students majored in mathematics. Acta Didactica Universitatis Comenianae. Mathematics, (8), 1–25.
4. Cooley, L., Trigueros, M., & Baker, B. (2007). Schema Thematization: A Framework and an Example. Journal for Research in Mathematics Education, 38(4), 370–392. https://doi.org/10.2307/30034879
5. D'Ambrosio, U. (1989). Ethnomathematics, Link between Traditions and Modernity. Philosophia Mathematica. ROTTERDAM: SENSE PUBLISHERS. https://doi.org/10.1093/philmat/s2-4.1.3
6. Dilling, F., & Vogler, A. (2021). Fostering Spatial Ability Through Computer-Aided Design: a Case Study. Digital Experiences in Mathematics Education. https://doi.org/10.1007/s40751-021-00084-w
7. Dubinsky, E. (2000). Using a Theory

of Learning in College Mathematics Courses. MSOR Connections, 1(2), 10–15. https://doi.org/10.11120/msor.2001.01020010

8. Dubinsky, E., Dautermann, J., Leron, U., & Zazkis, R. (n.d.). On Learning Fundamental Concepts of Group Theory, 0(376 2).
9. Dubinsky, E., Dautermann, J., Leron, U., & Zazkis, R. (1994). On learning fundamental concepts of group theory. Educational Studies in Mathematics, 2–38.
10. Dubinsky, E., & Mcdonald, M. A. (2001). APOS : a Constructivist Theory of Learning in Undergraduate Mathematics Education. Netherlands: Kluwer Academic Publishers.
11. Dubinsky, E., & McDonald, M. A. (2000a). APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research. Http:/Www.Telri.Ac.Uk/CM/Paper.Pdf.
12. Dubinsky, E., & McDonald, M. A. (2000b). APOS: A Constructivist Theory of Learning in Undergraduate Mathematics Education Research. USA: Georgia State University.
13. Dubinsky, E., & Wilson, R. T. (2013). High school students' understanding of the function concept. The Journal of Mathematical Behavior, 32(1), 83–101. https://doi.org/10.1016/s0732-3123(01)00065-7
14. Eskisehir, O., & Ozlem, K. (2015). Investigation of the relationship between the spatial visualization success and visual/spatial intelligence capabilities of Sixth Grade students. International Journal of Instruction, 8(1), 189–204.
15. Fauzan, A., Slettenhaar, D., & Plomp, T. (2002). Traditional Mathematics Education vs . Realistic Mathematics Education : Hoping for Changes. Proceedings of the 3rd International Mathematics Education and Society Conference. Copenhagen: Centre for Research in Learning Mathematics, 1–4.
16. Favian, O., López, L., Jesús, O., & Velázquez, R. (2021). From Euclid Geometry To Non- Euclid Geometry , In Particular Hyperbolic Geometry , In Secondary Education Through Dynamic Geometry. Turkish Journal of Computer and Mathematics Education, 12(12), 1195–1221.
17. François, K. (2010a). The Role of Ethnomathematics within Mathematics Education. Centre for Logic and Philosophy of Science (CLWF) Free University Brussels (VUB), (December), 1517–1526.
18. François, K. (2010b). The Role of Ethnomathematics Within Mathematics Education. Proceedings of Cerme, 1517–1526. https://doi.org/10.1007/978-3-319-12688-3
19. Frassia, M. G., & Serpe, A. (2017). Learning Geometry Through Mathematical Modelling: An Example With Geogebra. Turkish Online Journal of Educational Technology, (Special Issue for INTE 2017).
20. Freudenthal, H. (1991). Revisiting Mathematics Educational. Dordrecht: Reidel Publising.
21. Glaser, B. G., & Strauss, A. L. (2006). The Discovery of Grounded Theory, Strategies for Qualitative Research. New Brunswick: Aldine transaction.
22. Gravemeijer, K. (2008). RME Theory and Mathematics. Tools and Processes in Mathematics Teacher Education, 283–302.
23. Güven, B., & Kosa, T. (2008). The effect of dynamic geometry software on student mathematics teachers' spatial visualization skills. Turkish

Online Journal of Educational Technology, 7(4), 100–107.

24. Hankeln, C. (2021). Combining and Contrasting Formal Concept Analysis and APOS Theory. Educational Studies in Mathematics, 105(1), 1–14. https://doi.org/10.1007/s10649-018-9859-7

25. Herawaty, D., Khrisnawati, D., Widada, W., & Mundana, P. (2020). The cognitive process of students in understanding the parallels axiom through ethnomathematics learning. IOP Conf. Series: Journal of Physics: Conf. Series 1470 (2020) 012077 Doi:10.1088/1742-6596/1470/1/012077, 147 0. 1–8. https://doi.org/10.1088/1742-6596/1470/1/012077

26. Herawaty, D., Widada, W., Adhitya, A., Sari, R. D. W., & Novianita, L. (2020). Students ' ability to simplify the concept of function through realistic mathematics learning with the ethnomathematics approach. IOP Conf. Series: Journal of Physics: Conf. Series 1470 (2020) 012031 Doi:10.1088/1742-6596/1470/1/012031, 147 0. 1–8. https://doi.org/10.1088/1742-6596/1470/1/012031

27. Herawaty, D., Widada, W., Handayani, S., Febrianti, R., & Abdurrobbil, F. D. (2020). Students ' obstacles in understanding the properties of the closed sets in terms of the APOS theory. IOP Conf. Series: Journal of Physics: Conf. Series 1470 (2020) 012068 Doi:10.1088/1742-6596/1470/1/012068, 147 0. 1–7. https://doi.org/10.1088/1742-6596/1470/1/012068

28. Karadag, Z. (2009). Analyzing Students Mathematical Thinking in Technologie in Technologie-Supported Environments. University of Toronto. Retrieved from www.statisticshowto.com/wp.../06/Dyscalculia-createspace-v3.pdf

29. Kayhan, E. B. (2005). Investigation Of High School Students' Spatial Ability.

30. Maier, P. H. (1998a). Spatial geometry and spatial ability: How to make solid geometry solid? In Gesellschaft Für Didaktik Der Mathematik (Pp. 63–75). Saarbrücken, Germany.

31. Maier, P. H. (1998b). Spatial Geometry and Spatial Ability - Hoe to Make Solid Geometry Solid? Selected Papers from the Annual Conference of Didactics of Mathematics 1996. Elmar Cohors-Fresenborg et All (Ed).. Osnabrueck, 1998, 69–81.

32. Nugroho, Sukestiyarno, Y. L., & Nurcahyo, A. (2021). Weaknesses of Euclid Geometry : A Step of Learning Needs Analysis of Non- Euclid Geometry through an Ethnomathematics Approach. EDUMATIKA: Jurnal Riset Pendidikan Matematika, 4(November).

33. Oladosu, L. O. (2014). Secondary School Students' Meaning and Learning of Circle Geometry.

34. Orey, D. C., & Rosa, M. (2004). Ethnomathematics: Teaching and Learning Mathematics from a Multicultural Perspective. The Journal of Mathematics and Culture, 1(May 2006), 139–148. Retrieved from http://nasgem.rpi.edu/files/1438

35. Plomp, T., & Nieveen, N. (2013). Educational Design Research Educational Design Research. Educational Design Research, (July), 1–206. https://doi.org/10.1007/978-1-4614-3185-5_11

36. Rahmatulwahidah, N., & Zubainur, C. M. (2017). The Analysis of Students' Spatial Ability at Senior High School in Banda Aceh. Proceedings of AICS-Social Sciences, 7, 745–752.

37. Rosa, M., & Orey, D. C. (2011). Ethnomathematics: the cultural aspects of mathematics. Revista Latinoamericana de Etnomatemática, 4(2), 32–54.

38. Siegler, R. S., & Crowley, K. (1991). The microgenetic method: A direct means for studying cognitive development. American Psychologist. https://doi.org/10.1037//0003-066x.46.6.606

39. Widada, W., Herawaty, D., & Lubis, A. N. M. T. (2018). Realistic mathematics learning based on the ethnomathematics in Bengkulu to improve students' cognitive level. Journal of Physics: Conference Series, 1088. https://doi.org/10.1088/1742-6596/1088/1/012028

40. Widada, W., Herawaty, D., Nugroho, & Anggoro, A. F. D. (2019). The ability to Understanding of the Concept of Derivative Functions for Inter-Level Students During Ethnomathematics Learning. Journal of Physics: Conference Series, 1179(012056), 1–6. https://doi.org/10.1088/1742-6596/1179/1/012056

41. Widada, W, Agustina, A., Serlis, S., Dinata, B. M., & Hasari, S. T. (2019). The abstraction ability of students in understanding the concept of geometry. Journal of Physics: Conference Series, 1318(012082), 1–7. https://doi.org/10.1088/1742-6596/1318/1/012082

42. Widada, W, Herawaty, D., Hudiria, I., Prakoso, Y. A., Anggraeni, Y. R., & Zaid, K. U. (2020). The understanding of the triangle in Lobachevsky Geometry through local culture. International Seminar on Applied Mathematics and Mathematics Education 2020 (2nd ISAMME 2020). Journal of Physics: Conference Series, 1657(012038), 1–7. https://doi.org/10.1088/1742-6596/1657/1/012038

43. Widada, W, Herawaty, D., Widiarti, Y., Aisyah, S., & Tuzzahra, R. (2020). The cognitive process of students in understanding the triangles in Geometry of Riemann through local content. International Seminar on Applied Mathematics and Mathematics Education 2020 (2nd ISAMME 2020). Journal of Physics: Conference Series, 1657(012033), 1–8. https://doi.org/10.1088/1742-6596/1657/1/012033

44. Widada, W, Nugroho, Sari, W. P., & Pambudi, G. A. (2019). The ability of mathematical representation through realistic mathematics learning based on ethnomathematics. Journal of Physics: Conference Series, 1318(012073), 1–8. https://doi.org/10.1088/1742-6596/1318/1/012073

45. Widada, W.. (2011). Mathematics Learning towards Extended Trans Level. Bengkulu: Program Pascacarjana Pendidikan Matematika Universitas Bengkulu.

46. Widada, W.. (2017). Beberapa Dekomposisi Genetik Siswa dalam Memahami Matematika. Jurnal Pendidikan Matematika Raflesia, 1(1), 44–54.

47. Widada, W., Efendi, S., Herawaty, D., & Nugroho, K. U. Z. (2020). The genetic decomposition of students about infinite series through the ethnomathematics of Bengkulu , Indonesia. IOP Conf. Series: Journal of Physics: Conf. Series 1470 (2020) 012078 Doi:10.1088/1742-6596/1470/1/012078, 147 0. 1–9. https://doi.org/10.1088/1742-6596/1470/1/012078

48. Widada, W., Herawaty, D., Beka, Y., Sari, R. M., & Riyani, R. (2020). The mathematization process of students to understand the concept of vectors

through learning realistic mathematics and ethnomathematics. IOP Conf. Series: Journal of Physics: Conf. Series 1470 (2020) 012071 Doi:10.1088/1742-6596/1470/1/012071, 147 0. 1–10. https://doi.org/10.1088/1742-6596/1470/1/012071

49. Widada, W., Herawaty, D., Jumri, R., & Wulandari, H. (2020). Students of the extended abstract in proving Lobachevsky's parallel lines theorem. IOP Conf. Series: Journal of Physics: Conf. Series 1470 (2020) 012098 Doi:10.1088/1742-6596/1470/1/012098, 147 0. 1–10. https://doi.org/10.1088/1742-6596/1470/1/012098

50. Widada, W., Herawaty, D., Ma'rifah, N., & Yunita, D. (2019). Characteristics of Students Thinking in Understanding Geometry in Learning Ethnomathematics. International Journal of Scientific & Technology Research, 8(11), 3496–3503.

51. Widada, W., Nugroho, Sari, W. P., & Pambudi, G. A. (2019). Characteristics of Students Thinking in Understanding Geometry in Learning Ethnomathematics. Journal of Physics: Conference Series, 1318(3), 1–6. https://doi.org/10.1088/1742-6596/1179/1/012056

52. Wu, D., & Ma, H. (2006). The Distributions Of Van Hiele Levels Of Geometric Thinking Among 1st Through 6th Graders. Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education, 5, 409–416.

53. Yurt, E., & Tünkler, V. (2016). A study on the spatial abilities of prospective social studies teachers: A mixed method research. Kuram ve Uygulamada Egitim Bilimleri, 16(3), 965–986. https://doi.org/10.12738/estp.2016.3.0324