

On Totally Irregular Total Labeling of Caterpillars Having Even Number of Internal Vertices with Degree Three

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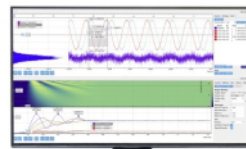
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On Totally Irregular Total Labeling of Caterpillars Having Even Number of Internal Vertices with Degree Three

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Abstract. We presume that $G(V, E)$ is a simple, undirected, and connected graph. A function λ from $V \cup E$ to $\{1, 2, \dots, k\}$ is named a totally irregular total k -labeling if the set of vertex-weights and the set of edge-weights of G consist of different values. The minimum integer k in such a way that G has a totally irregular total k -labeling is mentioned as total irregularity strength of G , denoted by $ts(G)$. We investigate the total irregularity strength of the caterpillars that have an even number of internal vertices with degree three. The results are as follows: $ts(S_n, \underbrace{3, 3, \dots, 3}_t, S_n) = \frac{2n+(t-1)}{2}$ and $ts(S_{m-1}, \underbrace{3, 3, \dots, 3}_t, S_m) = \frac{2m+(t-2)}{2}$ for even number t .

INTRODUCTION

The concept of graph labeling had been developed rapidly. Many results have been found related to various types of labelings [5]. "A mapping from a set of elements (vertex, edge or both of them) of $G(V, E)$ into a set of integers is named as labeling. When the domain is $V \cup E$, the mapping is mentioned as a total labeling" [13].

Let f be a total labeling. The weights of a vertex or an edge are as follows: $wt(u) = f(u) + \sum_{uv \in E} f(uv)$ and $wt(xy) = f(x) + f(y) + f(xy)$. The concepts of edge (vertex) irregular total labelings were initiated by Bača *et al.* [4]: "a total k -labeling $f: V \cup E \rightarrow \{1, 2, \dots, k\}$ is defined to be an edge irregular total k -labeling of G if every two different edges $e_1, e_2 \in E$ have the weights $wt(e_1) \neq wt(e_2)$ and to be a vertex irregular total k -labeling if for every two distinct vertices u and v have weights $wt(u) \neq wt(v)$. The minimum number k for which G has an edge irregular total k -labeling is called the total edge irregularity strength of G , $tes(G)$. Analogously, the total vertex irregularity strength of G is the minimum k for which G has a vertex irregular total k -labeling" [4]. Moreover, " f is a totally irregular total k -labeling of G if the weights of any two distinct vertices are distinct and any two different edges have different weights. The total irregularity strength of G , $ts(G)$, is the minimum number k for which G has a totally irregular total k -labeling" [14].

Bača *et al.* gave the bounds for tvs of any graph which contains p vertices, has minimum degree δ and maximum degree Δ [4]:

$$\left\lceil \frac{p + \delta}{\Delta + 1} \right\rceil \leq tvs(G) \leq p + \Delta - 2\delta + 1.$$

The bounds was updated by Anholcer *et al.* [3] in the following way:

$$tvs(G) \leq 3 \left\lceil \frac{p}{\delta} \right\rceil + 1.$$

Nurdin [17] provided the tvs of any tree which has n pendant vertices and there are no 2-degree vertices:

$$tvs(T) = \left\lceil \frac{n+1}{2} \right\rceil. \quad (1)$$

Further, Nurdin [17] provided the tv_s of a Caterpillar $T_{n,m}$ which contains n pendant vertices and there are m vertices on the main path, i.e.: $tv_s(T_{n,m}) = \lceil \frac{n+1}{2} \rceil$. Ivanco and Jendrol et al. [12] proposed the exact value of tes of a tree T as follows:

$$tes(T) = \max \left\{ \left\lceil \frac{|E(T)|+2}{3} \right\rceil, \left\lceil \frac{\Delta(T)+1}{2} \right\rceil \right\}. \quad (2)$$

Meanwhile, Marzuki, *et al.* [14] observed the ts of any graph in the following

$$ts(G) \geq \max \{tes(G), tv_s(G)\}. \quad (3)$$

Many results related to exact values of tv_s , tes , and ts of any graph classes have been found. Readers may refer to [1, 5, 6, 7, 8, 9, 10, 11, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25]. "A caterpillar is a tree in which all the vertices are within distance 1 of a central path" [16]. It is also a chain graph, that is "a graph consisting of r blocks, i.e., B_1, B_2, \dots, B_r in which for each index i , B_i and B_{i+1} have a unique cut vertex in such a way that the block cut vertex graph is a path" [2]. The chain of length r is notated by $C[B_1, B_2, \dots, B_r]$. The caterpillars having t internal vertices with degree three $S_{n, \underbrace{3, 3, \dots, 3}_t, n}$ can also be called as the star chain graphs $C[B_1, B_2, \dots, B_r]$ where $B_1 = B_r = S_n$ and

$$B_2 = B_3 = \dots = B_{r-1} = S_4.$$

An open problem of determining the ts of caterpillars $S_{n, \underbrace{3, 3, \dots, 3}_t, n}$ for t -times of 3's was stated in [11]. To solve the problem, we investigate the ts of caterpillars $S_{n, \underbrace{3, 3, \dots, 3}_t, n}$ and caterpillars $S_{m, \underbrace{3, 3, \dots, 3}_t, m-1}$ where t is even.

MAIN RESULTS

4

The results of ts of caterpillars having an even number of internal vertices of degree three are presented in this section.

Ts of caterpillars $S_{n, \underbrace{3, 3, \dots, 3}_t, n}$

A caterpillar $S_{n, \underbrace{3, 3, \dots, 3}_t, n}$ is a graph that is created from a double-star $S_{n,n}$ by subdividing the bridge which connects the centers of two stars such that there are t inserted vertices with degree three. The inserted vertex is also mentioned as an internal vertex [11]. We assume that t is an even number. The vertex set is

$$V = \{u_i | 1 \leq i \leq n\} \cup \{v_i | 1 \leq i \leq n\} \cup \{x_i | i = 1, 2, \dots, t\} \cup \{x'_i | i = 1, 2, \dots, t\}.$$

Meanwhile, the edge set is

$$E = \{u_i u_n | 1 \leq i \leq n-1\} \cup \{v_i v_n | 1 \leq i \leq n-1\} \cup \{x_i x_{i+1} | i = 1, 2, \dots, t-1\} \cup \{x_i x'_i | i = 1, 2, \dots, t\} \cup \{u_n x_1\} \cup \{v_n x_t\}.$$

An illustration of the caterpillar is given in Figure 1.

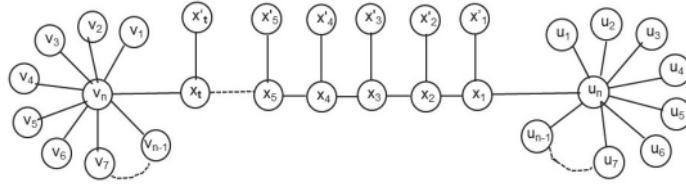


FIGURE 1. The caterpillar $S_{n,3,3,\dots,3,n}$.

Theorem 1 Let t be an even number. Given the caterpillars $S_{n,3,3,\dots,3,n}$ where

$$n \geq \begin{cases} 7 + \frac{t+2}{3}, & \text{if } t = 4 \pmod{6}, 4 \leq t \leq 16, \\ 7 + \frac{t-6}{3}, & \text{if } t = 0 \pmod{6}, 6 \leq t \leq 18, \\ 7 + \frac{t-2}{3}, & \text{if } t = 2 \pmod{6}, 8 \leq t \leq 20, \\ \frac{1}{2}t + 1, & \text{if } t \geq 22. \end{cases} \quad (4)$$

Then, the ts of the caterpillars is as follows:

$$ts(S_{n,3,3,\dots,3,n}) = \left\lceil \frac{2n + (t-1)}{2} \right\rceil.$$

Proof. The graphs $S_{n,3,3,\dots,3,n}$ consist of $2n + 2t$ vertices and $2(n+t) - 1$ edges. According to (1):

$$tvs(S_{n,3,3,\dots,3,n}) = \left\lceil \frac{2(n-1) + (t+1)}{2} \right\rceil = \left\lceil \frac{2n + (t-1)}{2} \right\rceil.$$

Meanwhile, the tes can be obtained through (2) as follows:

$$tes(S_{n,3,3,\dots,3,n}) = \max \left\{ \left\lceil \frac{2n + 2t + 1}{3} \right\rceil, \left\lceil \frac{n+1}{2} \right\rceil \right\} = \left\lceil \frac{2n + 2t + 1}{3} \right\rceil.$$

Based on observation from Marzuki *et al.* [14], we have the lower bound:

$$ts(S_{n,3,3,\dots,3,n}) \geq \max \{tvs(S_{n,3,3,\dots,3,n}), tes(S_{n,3,3,\dots,3,n})\} = \left\lceil \frac{2n + (t-1)}{2} \right\rceil$$

for even number t which satisfies (4).

Let $k = \left\lceil \frac{2n + (t-1)}{2} \right\rceil$. Next, we prove that k is the upper bound. To achieve this goal, we construct a total labeling ϕ from each vertex and edge $v, e \in V \cup E$ to $\{1, 2, \dots, k\}$ in the following tables.

TABLE I. Label of each vertex in the caterpillars

v	$\phi(v)$	cases for i
u_i	$\frac{1}{2} \left\lceil \frac{2n+(t-1)}{2} \right\rceil - \left(\frac{1}{2} + 2\right)$	$1 \leq i \leq n$
v_i	$\frac{1}{2} \left\lceil \frac{2n+(t-1)}{2} \right\rceil - \left(\frac{1}{2} + 2\right)$	$1 \leq i \leq n-1$
v_n	$\frac{1}{2} \left\lceil \frac{2n+(t-1)}{2} \right\rceil$	$1 \leq i \leq t$
x_i'	$\frac{1}{2} \left\lceil \frac{2n+(t-1)}{2} \right\rceil + (i-t)$	$i = 1, 2, \dots, t.$

TABLE II. Label of each edge in the caterpillars

v	$\phi(e)$	cases for i
$u_i u_n$	i	$1 \leq i \leq n-1$
$v_i v_n$	$i+2$	$1 \leq i \leq n-1$
$x_i x_i'$	$\frac{1}{2} \left\lceil \frac{2n+(t-1)}{2} \right\rceil + (i-t)$	$1 \leq i \leq t$
$x_i x_{i+1}$	$\frac{1}{2} \left\lceil \frac{2n+(t-1)}{2} \right\rceil$	$1 \leq i \leq t-1$
$u_n x_1$	$\frac{1}{2} \left\lceil \frac{2n+(t-1)}{2} \right\rceil - 1$	
$v_n x_t$	$\frac{1}{2} \left\lceil \frac{2n+(t-1)}{2} \right\rceil - (t-1)$	

According to the above labels, we evaluate the weights of elements $v, e \in V \cup E$ to $\{1, 2, \dots, k\}$ as follows:

TABLE III. Weights of all vertices in $S_{n, \underbrace{3, 3, \dots, 3}_t, n}$

v	$wt(v)$	cases for i
u_i	$i+1$	$1 \leq i \leq n-1$
u_n	$\frac{1}{2} \left\lceil \frac{n^2+n+(t-1)}{2} \right\rceil$	
v_i	$i+n$	$1 \leq i \leq n-1$
v_n	$\frac{1}{2} \left\lceil \frac{n^2+5n-(t+1)}{2} \right\rceil$	
x_1	$4 \left\lceil \frac{2n+(t-1)}{2} \right\rceil + 1 - 2t$	
x_t	$4 \left\lceil \frac{2n+(t-1)}{2} \right\rceil - (t-1)$	
x_i	$4 \left\lceil \frac{2n+(t-1)}{2} \right\rceil + 2(i-t)$	$2 \leq i \leq t-1$
x_i'	$2 \left\lceil \frac{2n+(t-1)}{2} \right\rceil + (i-t)$	$i = 1, 2, \dots, t.$

TABLE IV. Weights of all edges in $S_{n,3,3,\dots,3,n}$

e	$wt(e)$	Cases for i
$u_i u_n$	$i + 2$	$1 \leq i \leq n - 1$
$v_i v_n$	$i + \left\lceil \frac{2n+(t-1)}{2} \right\rceil - \frac{t}{2} + 1$	$1 \leq i \leq n - 1$
$x_i x'_i$	$3 \left\lceil \frac{2n+(t-1)}{2} \right\rceil + 2(i - t)$	$1 \leq i \leq t$
$x_i x_{i+1}$	$3 \left\lceil \frac{2n+(t-1)}{2} \right\rceil + (2i - 2t + 1)$	$i = 1, 2, \dots, t - 1$
$u_n x_1$	$2 \left\lceil \frac{2n+(t-1)}{2} \right\rceil + (1 - t)$	
$v_n x_t$	$2 \left\lceil \frac{2n+(t-1)}{2} \right\rceil - t + 2.$	

We observe that vertex-labels and edge-labels are at most $\left\lceil \frac{2n+(t-1)}{2} \right\rceil$. Further, all edges have distinct weights and also no vertices have a same weight under the labeling ϕ . It shows that

$$ts(S_{n,3,3,\dots,3,n}) = \left\lceil \frac{2n+(t-1)}{2} \right\rceil.$$

Example 1 Figure 2 demonstrates totally irregular 12-total labeling of $S_{9,3,3,\dots,3,9}$. The red color numbers indicate the weights of vertices or edges. Whereas, the numbers with black colors show the labels of vertices or edges.

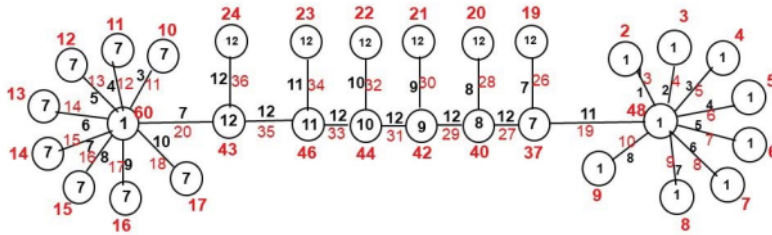


FIGURE 2. The figure that indicates $ts(S_{9,3,3,\dots,3,9}) = 12$.

Ts of caterpillars $S_{m-1,3,3,\dots,3,m}$

Let t be an even number. The caterpillars $S_{m-1,3,3,\dots,3,m}$ are formed from double stars $S_{m-1,m}$ by subdividing the bridge which is connected the stars S_{m-1} and S_m such that there are t inserted internal vertices of degree three. The vertex set is

$$V = \{u_i | 1 \leq i \leq m\} \cup \{v_i | 1 \leq i \leq m - 1\} \cup \{x_i | 1 \leq i \leq t\} \cup \{x'_i | 1 \leq i \leq t\}$$

and the edge set is

$$E = \{u_i u_m | 1 \leq i \leq m - 1\} \cup \{v_i v_{m-1} | 1 \leq i \leq m - 2\} \cup \{x_i x_{i+1} | 1 \leq i \leq t - 1\} \cup \{x_i x'_i | 1 \leq i \leq t\} \cup \{u_m x_1\} \cup \{v_{m-1} x_t\}.$$

Theorem 2 Let $S_{m-1, \underbrace{3, 3, \dots, 3}_t, m}$ be the caterpillars where

$$m \geq \begin{cases} 8, & \text{if } t = 4, 6, \\ t + 1, & \text{if } 8 \leq t \leq 14, \\ \frac{t}{2} + 4, & \text{if } t \geq 16. \end{cases} \quad (5)$$

$$\text{Then, } ts(S_{m-1, \underbrace{3, 3, \dots, 3}_t, m}) = \left\lceil \frac{2m + (t-2)}{2} \right\rceil.$$

Proof. The caterpillars $S_{m-1, \underbrace{3, 3, \dots, 3}_t, m}$ consist of $2m + 2t - 1$ vertices and $2m + 2t - 2$ edges. According to (1):

$$tvs(S_{m-1, \underbrace{3, 3, \dots, 3}_t, m}) = \left\lceil \frac{2m + (t-2)}{2} \right\rceil.$$

Meanwhile, the tes can be obtained through (2) as follows:

$$tes(S_{m-1, \underbrace{3, 3, \dots, 3}_t, m}) = \max \left\{ \left\lceil \frac{2m + 2t}{3} \right\rceil, \left\lceil \frac{m + 1}{2} \right\rceil \right\} = \left\lceil \frac{2m + 2t + 1}{3} \right\rceil.$$

According to evaluation from Marzuki *et al.* [14], we get the lower bound:

$$ts(S_{m-1, \underbrace{3, 3, \dots, 3}_t, m}) \geq \left\lceil \frac{2m + (t-2)}{2} \right\rceil$$

for the value t which satisfies (5). Let $k = \left\lceil \frac{2m + (t-2)}{2} \right\rceil$. Further, we verify that k is the upper bound. To realize, we construct a total labeling ϕ from the union of V and E into $\{1, 2, \dots, k\}$ in tables below.

TABLE V. Label of all vertices in the caterpillars.

v	$\phi(v)$	cases for i
u_i	1	$1 \leq i \leq m$
v_i	$\left\lceil \frac{2m + (t-2)}{2} \right\rceil - (\frac{t}{2} + 1)$	$1 \leq i \leq m - 2$
v_{m-1}	1	
x'_i	$\left\lceil \frac{2m + (t-2)}{2} \right\rceil$	$1 \leq i \leq t$
x_i	$\left\lceil \frac{2m + (t-2)}{2} \right\rceil + (i - t)$	$i = 1, 2, \dots, t.$

TABLE VI. Label of all edges in the caterpillars.

e	$\phi(e)$	cases for i
$u_i u_m$	i	$1 \leq i \leq m-1$
$v_i v_{m-1}$	$i+2$	$1 \leq i \leq m-2$
$x_i x'_i$	$\left\lceil \frac{2m+(t-2)}{2} \right\rceil + (i-t)$	$1 \leq i \leq t$
$x_i x_{i+1}$	$\left\lceil \frac{2m+(t-2)}{2} \right\rceil$	$1 \leq i \leq t-1$
$u_m x_1$	$\left\lceil \frac{2m+(t-2)}{2} \right\rceil$	
$v_{m-1} x_t$	$\left\lceil \frac{2m+(t-2)}{2} \right\rceil - (t-3)$.	

In pursuance of the above labeling, we calculate the weights of each vertex and each edge $v, e \in V \cup E$ as in Table VII and Table VIII.

TABLE VII. Vertex-weights.

v	$wt(v)$	cases for i
u_i	$i+1$	$1 \leq i \leq m-1$
u_m	$\left\lceil \frac{m^2+m+t}{2} \right\rceil$	7
v_i	$i+m$	$1 \leq i \leq m-2$
v_{m-1}	$\left\lceil \frac{m^2+3m-t}{2} \right\rceil$	
x_1	$4m-2$	
x_i	$4m+t-1$	
x_i	$4m+2i-4$	$2 \leq i \leq t-1$
x'_i	$2m+i-2$	$1 \leq i \leq t$

TABLE VIII. Weights of edges.

e	$wt(e)$	Cases for i
$u_i u_m$	$i+2$	$1 \leq i \leq m-1$
$v_i v_{m-1}$	$m+i+1$	$1 \leq i \leq m-2$
$x_i x'_i$	$3m+2i - (\frac{i}{2} + 3)$	$1 \leq i \leq t$
$x_i x_{i+1}$	$3m+2i - (\frac{i}{2} + 2)$	$1 \leq i \leq t-1$
$u_m x_1$	$2m$	
$v_{m-1} x_t$	$2m+1$	$t=4$
	$2m+2$	$t \geq 6$.

It is clear that the elements of the caterpillars have labels which are at most $\left\lceil \frac{2m+(t-2)}{2} \right\rceil$ and no elements have the same weight under the the labeling ϕ . It proves

$$ts(S_{m-1, \underbrace{3, 3, \dots, 3}_t, m}) = \left\lceil \frac{2m+(t-2)}{2} \right\rceil.$$

Example 2 Figure 3 illustrates the value $ts(S_{9, \underbrace{3, 3, \dots, 3}_8, 10}) = 13$.

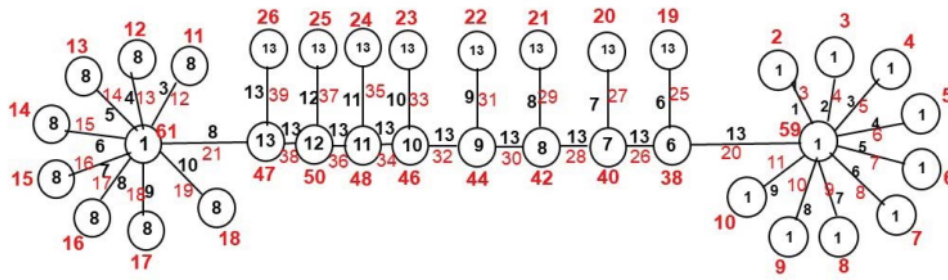


FIGURE 3. Totally irregular total 13-labeling of $S_{9,3,3,\dots,3,10}$.

CONCLUSION

In this paper, we have proved the ts of caterpillars that have an even number of internal vertices of degree 3. We got the results: $ts(S_{n,3,3,\dots,3,n}) = \left\lceil \frac{2n+(t-1)}{2} \right\rceil$ and $ts(S_{m-1,3,3,\dots,3,m}) = \left\lceil \frac{2m+(t-2)}{2} \right\rceil$. The labels of vertices and edges of the caterpillars under the totally irregular total k -labelings were constructed in the theorems.

In upcoming research, we are interested to investigate: ts of caterpillars that have an odd number of internal vertices of degree three and ts of the caterpillars having odd (even) number of internal vertices of degree $q \geq 4$.

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PAGE 1

PAGE 2

PAGE 3

PAGE 4

PAGE 5

PAGE 6

PAGE 7

PAGE 8

PAGE 9

PAGE 10
