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On Totally Irregular Total Labeling of Caterpillars Having Even Number of Internal Vertices with Degree Three

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Abstract. We presume that $G(V,E)$ is a simple, undirected, and connected graph. A function λ from $V \cup E$ to $\{1,2,\ldots,k\}$ is named a totally irregular total k-labeling if the set of vertex-weights and the set of edge-weights of G consist of different values. The minimum integer k in such a way that G has a totally irregular total k-labeling is mentioned as total irregularity strength of G, denoted by $ts(G)$. We investigate the total irregularity strength of the caterpillars that have an even number of internal vertices with degree three. The results are as follows: $ts(S_n,\underbrace{3,3,\ldots,3}_{t},S_n) = \frac{2n+(t-1)}{2}$ and $ts(S_{m-1},\underbrace{3,3,\ldots,3}_{t},S_m) = \frac{2m+(t-2)}{2}$ for even number t.

INTRODUCTION

The concept of graph labeling had been developed rapidly. Many results have been found related to various types of labelings [5]. "A mapping from a set of elements (vertex, edge or both of them) of $G(V,E)$ into a set of integers is named as labeling. When the domain is $V \cup E$, the mapping is mentioned as a total labeling" [13].

Let f be a total labeling. The weights of a vertex or an edge are as follows: $wt(u) = f(u) + \sum_{uv\in E} f(uv)$ and $wt(xy) = f(x) + f(y) + f(xy)$. The concepts of edge (vertex) irregular total labelings were initiated by Bača *et al.* [4]: "a total k-labeling $f : V \cup E \to \{1,2,\ldots,k\}$ is defined to be an edge irregular total k-labeling of G if every two different edges $e_1, e_2 \in E$ have the weights $wt(e_1) \neq wt(e_2)$ and to be a vertex irregular total k-labeling if for every two distinct vertices u and v have weights $wt(u) \neq wt(v)$. The minimum number k for which G has an edge irregular total k-labeling is called the total edge irregularity strength of G, tes(G). Analogously, the total vertex irregularity strength of G is the minimum k for which G has a vertex irregular total k-labeling" [4]. Moreover, "f is a totally irregular total k-labeling of G if the weights of any two distinct vertices are distinct and any two different edges have different weights. The total irregularity strength of G, $ts(G)$, is the minimum number k for which G has a totally irregular total k-labeling" [14].

Bača *et al.* gave the bounds for tvs of any graph which contains p vertices, has minimum degree δ and maximum degree $\triangle$ [4]:

$$\left\lceil \frac{p+\delta}{\triangle+1} \right\rceil \leq tvs(G) \leq p + \triangle - 2\delta + 1.$$

The bounds was updated by Anholcer *et al.* [3] in the following way:

$$tvs(G) \leq 3\left\lceil \frac{p}{\delta} \right\rceil + 1.$$

Nurdin [17] provided the tvs of any tree which has n pendant vertices and there are no $2-$degree vertices:

$$tvs(T) = \left\lceil \frac{n+1}{2} \right\rceil. \quad (1)$$

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Further, Nurdin [17] provided the tvs of a Caterpillar $T_{n,m}$ which contains n pendant vertices and there are m vertices on the main path, i.e.: $tvs(T_{n,m}) = \lceil \frac{n+1}{2} \rceil$. Ivanco and Jendrol et al. [12] proposed the exact value of tes of a tree T as follows:

$$tes(T) = \max \left\{ \left\lceil \frac{|E(T)|+2}{3} \right\rceil, \left\lceil \frac{\triangle(T)+1}{2} \right\rceil \right\}. \tag{2}$$

Meanwhile, Marzuki, *et al.* [14] observed the ts of any graph in the following

$$ts(G) \geq \max \text{ of } \{tes(G), tvs(G)\}. \tag{3}$$

Many results related to exact values of tvs, tes, and ts of any graph classes have been found. Readers may refer to [1, 5, 6, 7, 8, 9, 10, 11, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25]. "A caterpillar is a tree in which all the vertices are within distance 1 of a central path" [16]. It is also a chain graph, that is "a graph consisting of r blocks, i.e., $B_1, B_2, \ldots, B_r$ in which for each index i, B_i and B_{i+1} have a unique cut vertex in such a way that the block cut vertex graph is a path" [2]. The chain of length r is notated by $C[B_1, B_2, \ldots, B_r]$. The caterpillars having t internal vertices with degree three $S_{n,\underbrace{3,3,\ldots,3}_{t},n}$ can also be called as the star chain graphs $C[B_1, B_2, \ldots, B_r]$ where $B_1 = B_r = S_n$ and $B_2 = B_3 = \ldots = B_{r-1} = S_4$.

An open problem of determining the ts of caterpillars $S_{n,3,3,\ldots,3,n}$ for t-times of 3's was stated in [11]. To solve the problem, we investigate the ts of caterpillars $S_{n,\underbrace{3,3,\ldots,3}_{t},n}$ and caterpillars $S_{m,\underbrace{3,3,\ldots,3}_{t},m-1}$ where t is even.

MAIN RESULTS

The results of ts of caterpillars having an even number of internal vertices of degree three are presented in this section.

Ts of caterpillars $S_{n,\underbrace{3,3,\ldots,3}_{t},n}$

A caterpillar $S_{n,\underbrace{3,3,\ldots,3}_{t},n}$ is a graph that is created from a double-star $S_{n,n}$ by subdividing the bridge which connects the centers of two stars such that there are t inserted vertices with degree three. The inserted vertex is also mentioned as an internal vertex [11]. We assume that t is an even number. The vertex set is

$$V = \{u_i | 1 \leq i \leq n\} \cup \{v_i | 1 \leq i \leq n\} \cup \{x_i | i = 1, 2, \ldots, t\} \cup \{x'_i | i = 1, 2, \ldots, t\}.$$

Meanwhile, the edge set is

$$E = \{u_i u_n | 1 \leq i \leq n-1\} \cup \{v_i v_n | 1 \leq i \leq n-1\} \cup \{x_i x_{i+1} | i = 1, 2, \ldots, t-1\} \cup \{x_i x'_i | i = 1, 2, \ldots, t\} \cup \{u_n x_1\} \cup \{v_n x_t\}.$$

An illustration of the caterpillar is given in Figure 1.

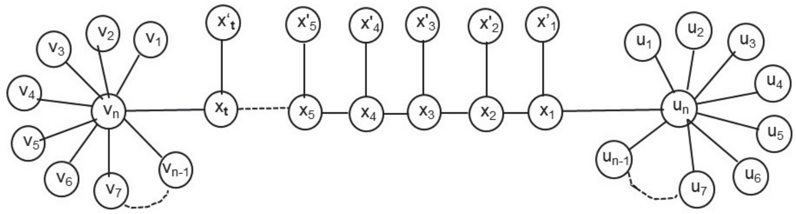

FIGURE 1. The caterpillar $S_{n,\underbrace{3,3,\ldots,3}_{t},n}$.

Theorem 1 *Let t be an even number. Given the caterpillars* $S_{n,\underbrace{3,3,\ldots,3}_{t},n}$ *where*

$$n \geq \begin{cases} 7+\frac{t+2}{3}, & \text{if } t=4 \mod 6, 4\leq t\leq 16,\\ 7+\frac{t-6}{3}, & \text{if } t=0 \mod 6, 6\leq t\leq 18,\\ 7+\frac{t-2}{3}, & \text{if } t=2 \mod 6, 8\leq t\leq 20,\\ \frac{1}{2}t+1, & \text{if } t\geq 22. \end{cases} \tag{4}$$

Then, the ts of the caterpillars is as follows:

$$ts(S_{n,\underbrace{3,3,\ldots,3}_{t},n}) = \left\lceil \frac{2n+(t-1)}{2} \right\rceil.$$

Proof. The graphs $S_{n,\underbrace{3,3,\ldots,3}_{t},n}$ consist of $2n+2t$ vertices and $2(n+t)-1$ edges. According to (1):

$$tvs(S_{n,\underbrace{3,3,\ldots,3}_{t},n}) = \left\lceil \frac{2(n-1)+(t+1)}{2} \right\rceil = \left\lceil \frac{2n+(t-1)}{2} \right\rceil.$$

Meanwhile, the tes can be obtained through (2) as follows:

$$tes(S_{n,\underbrace{3,3,\ldots,3}_{t},n}) = \max\left\{ \left\lceil \frac{2n+2t+1}{3} \right\rceil, \left\lceil \frac{n+1}{2} \right\rceil \right\} = \left\lceil \frac{2n+2t+1}{3} \right\rceil.$$

Based on observation from Marzuki *et al.* [14], we have the lower bound:

$$ts(S_{n,\underbrace{3,3,\ldots,3}_{t},n}) \geq \max\{tvs(S_{n,\underbrace{3,3,\ldots,3}_{t},n}), tes(S_{n,\underbrace{3,3,\ldots,3}_{t},n})\} = \left\lceil \frac{2n+(t-1)}{2} \right\rceil$$

for even number t which satisfies (4).

Let $k = \left\lceil \frac{2n+(t-1)}{2} \right\rceil$. Next, we prove that k is the upper bound. To achieve this goal, we construct a total labeling ϕ from each vertex and edge $v, e \in V \cup E$ to $\{1, 2, \ldots, k\}$ in the following tables.

TABLE I. Label of each vertex in the caterpillars

v	$\phi(v)$	cases for i
u_i	1	$1 \leq i \leq n$
v_i	$\left\lceil \frac{2n+(t-1)}{2} \right\rceil - (\frac{t}{2}+2)$	$1 \leq i \leq n-1$
v_n	1	
x'_i	$\left\lceil \frac{2n+(t-1)}{2} \right\rceil$	$1 \leq i \leq t$
x_i	$\left\lceil \frac{2n+(t-1)}{2} \right\rceil + (i-t)$	$i = 1,2,\ldots,t.$

TABLE II. Label of each edge in the caterpillars

v	$\phi(e)$	cases for i
u_iu_n	i	$1 \leq i \leq n-1$
v_iv_n	$i+2$	$1 \leq i \leq n-1$
$x_ix'_i$	$\left\lceil \frac{2n+(t-1)}{2} \right\rceil + (i-t)$	$1 \leq i \leq t$
x_ix_{i+1}	$\left\lceil \frac{2n+(t-1)}{2} \right\rceil$	$1 \leq i \leq t-1$
u_nx_1	$\left\lceil \frac{2n+(t-1)}{2} \right\rceil - 1$	
v_nx_t	$\left\lceil \frac{2n+(t-1)}{2} \right\rceil - (t-1)$	

According to the above labels, we evaluate the weights of elements $v, e \in V \cup E$ to $\{1,2,\ldots,k\}$ as follows:

TABLE III. Weights of all vertices in $S_{n,\underbrace{3,3,\ldots,3}_{t},n}$

v	$wt(v)$	cases for i
u_i	i+1	$1 \leq i \leq n-1$
u_n	$\left\lceil \frac{n^2+n+(t-1)}{2} \right\rceil$	
v_i	$i+n$	$1 \leq i \leq n-1$
v_n	$\left\lceil \frac{n^2+5n-(t+1)}{2} \right\rceil$	
x_1	$4\left\lceil \frac{2n+(t-1)}{2} \right\rceil + 1 - 2t$	
x_t	$4\left\lceil \frac{2n+(t-1)}{2} \right\rceil - (t-1)$	
x_i	$4\left\lceil \frac{2n+(t-1)}{2} \right\rceil + 2(i-t)$	$2 \leq i \leq t-1$
x'_i	$2\left\lceil \frac{2n+(t-1)}{2} \right\rceil + (i-t)$	$i = 1,2,\ldots,t.$

TABLE IV. Weights of all edges in $S_{n,\underbrace{3,3,\ldots,3}_{t},n}$

e	$wt(e)$	Cases for i
u_iu_n	$i+2$	$1 \le i \le n-1$
v_iv_n	$i+\left\lceil\frac{2n+(t-1)}{2}\right\rceil-\frac{t}{2}+1$	$1 \le i \le n-1$
x_ix_i'	$3\left\lceil\frac{2n+(t-1)}{2}\right\rceil+2(i-t)$	$1 \le i \le t$
x_ix_{i+1}	$3\left\lceil\frac{2n+(t-1)}{2}\right\rceil+(2i-2t+1)$	$i=1,2,\ldots,t-1$
u_nx_1	$2\left\lceil\frac{2n+(t-1)}{2}\right\rceil+(1-t)$	
v_nx_t	$2\left\lceil\frac{2n+(t-1)}{2}\right\rceil-t+2.$	

We observe that vertex-labels and edge-labels are at most $\left\lceil\frac{2n+(t-1)}{2}\right\rceil$. Further, all edges have distinct weights and also no vertices have a same weight under the the labeling ϕ. It shows that

$$ts(S_{n,\underbrace{3,3,\ldots,3}_{t},n}) = \left\lceil\frac{2n+(t-1)}{2}\right\rceil.$$

Example 1 *Figure 2 demonstrates totally irregular 12-total labeling of* $S_{9,\underbrace{3,3,\ldots,3}_{6},9}$. *The red color numbers indicate the weights of vertices or edges. Whereas, the numbers with black colors show the labels of vertices or edges.*

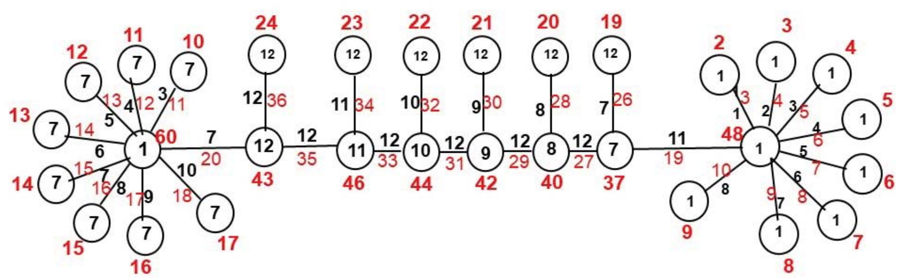

FIGURE 2. The figure that indicates $ts(S_{9,\underbrace{3,3,\ldots,3}_{6},9}) = 12$.

Ts of caterpillars $S_{m-1,\underbrace{3,3,\ldots,3}_{t},m}$

Let t be an even number. The caterpillars $S_{m-1,\underbrace{3,3,\ldots,3}_{t},m}$ are formed from double stars $S_{m-1,m}$ by subdividing the bridge which is connected the stars S_{m-1} and S_m such that there are t inserted internal vertices of degree three. The vertex set is

$$V = \{u_i | 1 \le i \le m\} \cup \{v_i | 1 \le i \le m-1\} \cup \{x_i | 1 \le i \le t\} \cup \{x_i' | 1 \le i \le t\}$$

and the edge set is

$$E = \{u_iu_m | 1 \le i \le m-1\} \cup \{v_iv_{m-1} | 1 \le i \le m-2\} \cup \{x_ix_{i+1} | 1 \le i \le t-1\} \cup \{x_ix_i' | 1 \le i \le t\} \cup \{u_mx_1\} \cup \{v_{m-1}x_t\}.$$

Theorem 2 *Let* $S_{m-1,\underbrace{3,3,\ldots,3}_{t},m}$ *be the caterpillars where*

$$m \geq \begin{cases} 8, & \textit{if } t = 4,6, \\ t+1, & \textit{if } 8 \leq t \leq 14, \\ \frac{t}{2}+4, & \textit{if } t \geq 16. \end{cases} \tag{5}$$

Then, $ts(S_{m-1,\underbrace{3,3,\ldots,3}_{t},m}) = \left\lceil \frac{2m+(t-2)}{2} \right\rceil$.

Proof. The caterpillars $S_{m-1,\underbrace{3,3,\ldots,3}_{t},m}$ consist of $2m+2t-1$ vertices and $2m+2t-2$ edges. According to (1):

$$tvs(S_{m-1,\underbrace{3,3,\ldots,3}_{t},m}) = \left\lceil \frac{2m+(t-2)}{2} \right\rceil.$$

Meanwhile, the tes can be obtained through (2) as follows:

$$tes(S_{m-1,\underbrace{3,3,\ldots,3}_{t},m}) = \max\left\{ \left\lceil \frac{2m+2t}{3} \right\rceil, \left\lceil \frac{m+1}{2} \right\rceil \right\} = \left\lceil \frac{2m+2t+1}{3} \right\rceil.$$

According to evaluation from Marzuki *et al.* [14], we get the lower bound:

$$ts(S_{m-1,\underbrace{3,3,\ldots,3}_{t},m}) \geq \left\lceil \frac{2m+(t-2)}{2} \right\rceil$$

for the value t which satisfies (5). Let $k = \left\lceil \frac{2m+(t-2)}{2} \right\rceil$. Further, we verify that k is the upper bound. To realize, we construct a total labeling ϕ from the union of V and E into $\{1,2,\ldots,k\}$ in tables below.

TABLE V. Label of all vertices in the caterpillars.

v	$\phi(v)$	cases for i
u_i	1	$1 \leq i \leq m$
v_i	$\left\lceil \frac{2m+(t-2)}{2} \right\rceil - (\frac{t}{2}+1)$	$1 \leq i \leq m-2$
v_{m-1}	1	
x'_i	$\left\lceil \frac{2m+(t-2)}{2} \right\rceil$	$1 \leq i \leq t$
x_i	$\left\lceil \frac{2m+(t-2)}{2} \right\rceil + (i-t)$	$i = 1,2,\ldots,t.$

TABLE VI. Label of all edges in the caterpillars.

e	$\phi(e)$	cases for i
u_iu_m	i	$1 \leq i \leq m-1$
v_iv_{m-1}	$i+2$	$1 \leq i \leq m-2$
x_ix_i'	$\left\lceil \frac{2m+(t-2)}{2} \right\rceil + (i-t)$	$1 \leq i \leq t$
x_ix_{i+1}	$\left\lceil \frac{2m+(t-2)}{2} \right\rceil$	$1 \leq i \leq t-1$
u_mx_1	$\left\lceil \frac{2m+(t-2)}{2} \right\rceil$	
$v_{m-1}x_t$	$\left\lceil \frac{2m+(t-2)}{2} \right\rceil - (t-3).$	

In pursuance of the above labeling, we calculate the weights of each vertex and each edge $v, e \in V \cup E$ as in Table VII and Table VIII.

TABLE VII. Vertex-weights.

v	$wt(v)$	cases for i
u_i	i+1	$1 \leq i \leq m-1$
u_m	$\left\lceil \frac{m^2+m+t}{2} \right\rceil$	
v_i	$i+m$	$1 \leq i \leq m-2$
v_{m-1}	$\left\lceil \frac{m^2+3m-t}{2} \right\rceil$	
x_1	$4m-2$	
x_t	$4m+t-1$	
x_i	$4m+2i-4$	$2 \leq i \leq t-1$
x_i'	$2m+i-2$	$1 \leq i \leq t$

TABLE VIII. Weights of edges.

e	$wt(e)$	Cases for i
u_iu_m	$i+2$	$1 \leq i \leq m-1$
v_iv_{m-1}	$m+i+1$	$1 \leq i \leq m-2$
x_ix_i'	$3m+2i-(\frac{t}{2}+3)$	$1 \leq i \leq t$
x_ix_{i+1}	$3m+2i-(\frac{t}{2}+2)$	$1 \leq i \leq t-1$
u_mx_1	$2m$	
$v_{m-1}x_t$	$2m+1$	$t=4$
	$2m+2$	$t \geq 6.$

It is clear that the elements of the caterpillars have labels which are at most $\left\lceil \frac{2m+(t-2)}{2} \right\rceil$ and no elements have the same weight under the the labeling ϕ. It proves

$$ts(S_{m-1,\underbrace{3,3,\ldots,3}_{t},m}) = \left\lceil \frac{2m+(t-2)}{2} \right\rceil.$$

Example 2 *Figure 3 illustrates the value* $ts(S_{9,\underbrace{3,3,\ldots,3}_{8},10}) = 13.$

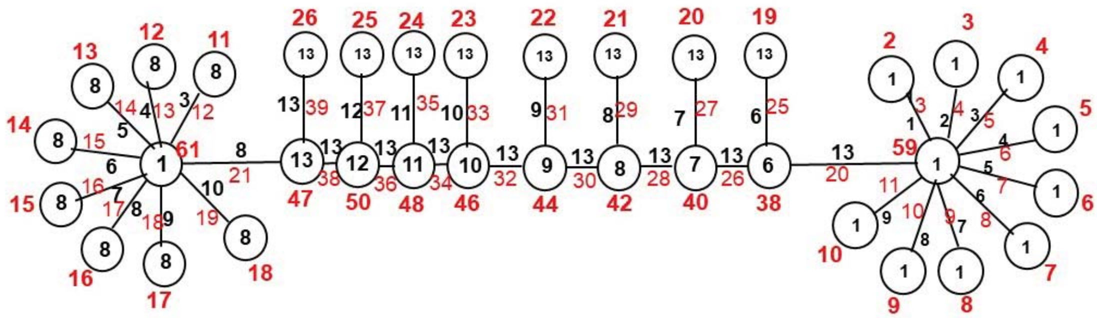

FIGURE 3. Totally irregular total 13-labeling of $S_{9,\underbrace{3,3,\ldots,3}_{8},10}$.

CONCLUSION

In this paper, we have proved the ts of caterpillars that have an even number of internal vertices of degree 3. We got the results: $ts(S_{n,\underbrace{3,3,\ldots,3}_{t},n}) = \left\lceil \frac{2n+(t-1)}{2} \right\rceil$ and $ts(S_{m-1,\underbrace{3,3,\ldots,3}_{t},m}) = \left\lceil \frac{2m+(t-2)}{2} \right\rceil$. The labels of vertices and edges of the caterpillars under the totally irregular total k-labelings were constructed in the theorems.

In upcoming research, we are interested to investigate: ts of caterpillars that have an odd number of internal vertices of degree three and ts of the caterpillars having odd (even) number of internal vertices of degree $q \geq 4$.

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