Development of higher mathematical thinking of senior high school students through training of open-ended problems assisted by the teacher's scaffolding

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Abstract. This article is based on applied research that has started with preliminary research in 2019. The objectives: Obtain the results of the analysis of Open-ended Problems assisted by the teacher's scaffolding, which can be used to develop Higher Mathematical Thinking for Senior High School students. The methods, qualitative approaches. The research subjects took 6 students of SMAN 1 Ungaran, two students from the clever, medium, and lack groups. Data analysis includes data reduction, data presentation, data interpretation, and concluding. Data obtained from test results, intensive interviews, and triangulation. The results: (1) Senior High school teachers found identification of scaffolding level to develop of Higher mathematical thinking, (2) through the practice of open-ended Problems with the help of scaffolding by the teacher, it was successfully used as a means of developing Higher Mathematical Thinking for Senior High School students. The conclusions: (1) The practice of Open-ended Problems assisted by the teacher's scaffolding succeeded in developing Higher Mathematical Thinking for senior high school students. (2) Teachers need to be at a scaffolding level to develop Higher Mathematical Thinking, depending on intelligence level. The more intelligent, the less its guidance.

1. Introduction

One of the factors that can support students' success in learning mathematics is having a mathematical thinking competence, as explained by [1] and [2]. Students should have this mathematical thinking ability since taking elementary education. Thus, when students continue their education at the Senior High school level, they only need to deepen their mathematical thinking competence, known as Higher Mathematical Thinking. Teachers at the basic education level have not provided provisions so that students have competence in mathematical thinking. Therefore, when students are at the high school level, and the teacher intends to provide high school students' provisions to grow Higher Mathematical Thinking, mathematics teachers need to provide guided, planned, and structured assistance. Teacher assistance so that students grow advanced mathematical thinking is called scaffolding by teachers to students. According to [3] and [4], in helping students to develop advanced mathematical thinking, teachers need to: (1) provide divergent questions, which means that the questions the teacher gives to students have more than one way to solve them. (2) Teachers need to assist students who experience obstacles in solving problems.

The thought process is defined as a way of responding or thinking mentally to information or an event. Another opinion about the thought process is that it can be classified into three steps, namely: (1) the formation of the meaning of the incoming information, (2) the formation of opinions by comparing

existing knowledge to form opinions, and (3) take a conclusion. Therefore, it is necessary to find the best way out so that mathematics teachers in Senior High School can guide and train their students to improve their mathematical thinking. Teachers also need to know how to guide and train their students in a planned and structured manner so that their students' mathematical thinking skills can increase and be helpful.

According to [5], [6], and [7], these open-ended problems have the following types. (1) If a problem has not only one correct answer, it is called an open-ended problem with a fluency characteristic. (2) If a problem has not only one solution procedure, it is called an open-ended problem with flexibility. Furthermore, the teacher presented the open-ended problems material through the discovery-based learning model, following the demands of the current 2013 Curriculum.

According to [8], a task or question that is open-ended can be interpreted as a problem whose resolution is not standard. Therefore, [9] wrote that the teacher has to provide the exercises or an open-ended problem whose solution needs high-level mathematical thinking.

For an example of an Open-Ended Problems that have multiple solution procedure.

Linear Equations System in Three Variables:

There are 3 goods, namely goods I, II, and III. The weight of goods I and II is 6 kg. The weight of goods I and III is 5 kg. The weight of goods II and III is 7 kg. Determine at the same time the weight of goods I, II, and III, with several algorithms.

Solutions:

For example, A = weight of goods I, B = weight of goods II, and C = weight of goods III.

You will look for the values of A + B + C.

From the above problems, the following mathematical model can be made.

A + B = 6 …… (1)
A + C = 5 …… (2)
B + C = 7 …… (3)

So thet we get:

The first algorithm:

A + B = 6
A + C = 5
B + C = 7
---------- +
2A + 2B + 2C = 18
A + B + C = 9

The second algorithm:

A + B = 6 A + C = 5
A = 6 – B A = 5 – C

And than 6 – B = 5 – C
C = –1 + B

B + C = 7
C = 7 – B

We get: –1 + B = 7 – B
B = 4

We now, A = 6 – B, if B = 4 then A = 2

If A = 5 – C, B = 4, and A = 2. We get C = 3.

So, A + B + C = 2 + 4 + 3 = 9.

The third algorithm:

A + B = 6 B – C = 1
A + C = 5 B + C = 7
---------- (–) ---------- (+)
B – C = 1 2B = 8 then B = 4

And than A + B = 6 A + C = 5

$$A + 4 = 6 \qquad 2 + C = 5$$
$$A = 2 \qquad C = 3$$
So, $A + B + C = 2 + 4 + 3 = 9$.

Furthermore, the problems mentioned above can be solved with 3 algorithms, namely: (1) Substitution Method, (2) Elimination Method, or (3) Non-routine Method.

If the teacher wants SeniorHigh School students to find some solution procedure to get answers, then teacher guidance is needed. As a result, students can develop their abilities in Higher Mathematical Thinking. Step in the developing of Higher Mathematical Thinking of Senior High School Students.

For high school students, Higher Mathematical Thinking is an ability that must be mastered so that students have good mathematics learning abilities. With Higher Mathematical Thinking, students are expected to be able to think systematically and rationally when facing mathematical problems and problems in everyday life. Furthermore, [10] wrote that through Higher Mathematical Thinking, students are expected to draw the correct conclusions based on facts and correct proof steps. Also, [11] also stated that Higher Mathematical Thinking could help students develop skills in solving problems based on the stages of reasoning and skills that follow the problems at hand. To foster the ability of Higher Mathematical Thinking, teachers need to equip their students with various skills to solve various problems. One of them is to train students' abilities to be able to solve open-ended problems. An open-ended problem is a problem that has more than one correct answer or a problem that has more than one solution way. If students are accustomed to solving open-ended problems, it is hoped that Higher Mathematical Thinking can grow naturally and well in students.

There are students of Senior High School who haven't high math abilities. Thus, if the teachers want to train and develop Higher Mathematical Thinking skills, a gradual, programmed, and measurable teacher scaffold is needed. Referring to thinking [12], [13], and [14], the level scaffolding of teachers to develop Higher Mathematical Thinking skills, the stages are as follows.

Level 1: Students need to be given examples of common problems, and the teacher provides ways to solve these problems. Students are asked to practice problems according to the example given by the teacher. Level 2: Students begin to be given Open-Ended Problems, and the teacher provides examples on how to solve them. Level 3: Students begin to practice independently on solving Open-Ended Problems. Then the teacher only provides proportional assistance. Level 4: The teacher and students give students some Open Ended Problems to do without teacher assistance.

1.1 *Problems*

The problems of this applied research are as follows. How to get the results of the analysis of Open-Ended Problems with the help of scaffolding by the teacher that can be used as a means of developing Higher Mathematical Thinking for Senior High School students?

1.2 *Objective*

This study's objective is to obtain the results of the analysis of Open Ended Problems assisted by scaffolding by the teacher, which can be used to develop Higher Mathematical Thinking for Senior High School students.

2. Methods

2.1. Approach and Research Subjects

The research method, with a qualitative approach. This study's subjects took 6 students of State Senior High School 1 of Ungaran of Semarang Regency during this research, which the Research Team appointed and State Senior High School 1 of Ungaran. Data analysis includes data reduction, data

exposure, data interpretation, and concluding. Data obtained from test results, intensive interviews, and triangulation.

2.2. Techniques of Data Analysis and Interpretation

[15] argued that activities in qualitative data analysis were carried out interactively and lasted until completion. Data analysis activities include data reduction, data display, data interpretation, and conclusion drawing/verification. When interpreting the data, the researcher analyzed the results of student work after working on Open-Ended Problems, conducting interviews with research subjects, and conducting triangulation.

2.3. Category of Development of Higher Mathematical Thinking Based on Scaffolding

Table 1. Higher Mathematical Thinking (HMT) of Leveling of the Teacher Scaffolding

No	Component of Scaffolding Level based on Solutions of Open-Ended Problem	Category of HMT
1.	The student can reach level 4	Very Excellent
2.	Students can only reach level 3.	Excellent
3.	Students can only reach level 2.	Medium
4.	Students can only reach level 1.	Less
5.	Student failed.	Very Less

3. Results and Discussions

Following the problems, objectives achieved, and the methods, the results of this study are as follows. The results of the analysis of open-ended problems that can be used to develop Higher Mathematical Thinking for Senior High School students are as follows. (1) Open-ended problems that have more than one correct answer. (2) Open-ended problems that have more than one correct solution algorithm. (3) These results indicate that if a high school math teacher wants to cultivate Higher Mathematical Thinking, then the teacher must make open-ended problems.

The teacher gives the results of the analysis of the use of Open-Ended Problems after students Scaffolding levels for the disclosure of the development of High Mathematical Thinking for Senior High School students. (1) The teacher had previously made Open-Ended Problems and was consulted with the Research Team. (2) Before the High Mathematical Thinking competency test, the teacher needs to provides scaffolding to students following [16] previous research findings, namely classically: (1) The teacher provides routine problems exercises. Students practice solving problems based on methods or examples from the teacher. (2) The teacher begins to provide examples of how to work on Open-Ended Problems. Then students practice solving is based on the teacher's method or example. (3) The teacher begins to provide Open-Ended Problems. Then students' practice solving is based on the methods or examples the teacher provides. Teachers help students who experience difficulties proportionally. (4) The teacher provides Open-Ended Problems. Then the students solve them without the help of the teacher.

It turns out that after the teacher gave the students scaffolding, then the result of the six Research Subjects: The competency of High Mathematics Thinking was in the Very Good category, there was 1 student. There were 3 students in the Very Good category. There were 2 students in the Medium category. None of the Research Subjects of this study were categorized as Less and Very Less. The second-year research results are clearly in line with [16] initial research at the same high school, namely SMAN 1 Ungaran, to grow Higher Mathematical Thinking, then: (1) Students need to be trained in Open-Ended Problems by the teacher. (2) The results of student exercises need to be corrected by the teacher. (3) The teacher needs to provide scaffolding according to the level of difficulty of the students. After the research recommendations in 2019 were carried out for research in 2020, the competence of

Higher Mathematical Thinking students of SMAN 1 Ungaran would be better than in 2019. In 2020, no Research Subject had Higher Mathematical Thinking competence with Less or Very Less.

Based on the theory of [17] and according to [18], to develop the competence of Higher Mathematical Thinking, open-ended problems are needed to develop Higher Mathematical Thinking for Senior High School students. The types are (1) Open-ended problems that have more than one correct answer and (2) Open-ended problems that have more than one correct solution algorithm.

To obtain good open-ended problems related to qualitative approach action, the teacher has discussed the open-ended problems she made with the research team. Furthermore, by referring to the research of [19] and [16], in practice, the teacher also needs and has implemented scaffolding leveling according to previous research results. So, if a Senior High School math teacher wants to grows Higher Mathematical Thinking, then the teacher concerned must be able to make Open-Ended Problems well. In the implementation of scaffolding leveling by the teacher to grow Higher Mathematical Thinking, [20] and [21] provide learning theories that teachers need and must do, are as follows: (1) At level 1, the teacher provides subject matter according to the content of the Lesson Plan, the teacher provides math problems as practice and solutions for students. Then students learn or practice solving problems based on methods or examples from the teacher. The teacher is the primary resource person and facilitator. (2) At level 2, the teacher begins to provide examples of working on open-ended problems. Then students are asked to practice solving open-ended problems based on the teacher's example. (3) At level 3, the teacher gives open-ended problems. Then students practice solving them. In this case, the teacher proportionally helps students who experience difficulties. (4) At level 4, the teacher gives open-ended problems. Then students solve them without the teacher's help. By providing scaffolding levels before the Competency Test is carried out through Open-Ended Problems, the Higher Mathematical Thinking competence of Senior High School students grows faster. This can be seen in the results of research in this second year [22].

In entering the 21st-century era, Higher Mathematical Thinking is needed. If a mathematics teacher knows the students' level of Higher Mathematical Thinking ability, the teacher can provide students' assistance. Regarding the teacher's assistance in guiding students, if the teacher knows their students' level of ability in Higher Mathematical Thinking, teacher assistance to students will be appropriate and valuable. By knowing the level of ability of their students in Higher Mathematical Thinking, teachers can adjust and differentiate how they provide the assistance provided. For intelligent students, the assistance will undoubtedly be different for less intelligent students. Likewise, for students who have moderate Mathematical Thinking skills.

Based on the process of assistance that must be provided to students, [23] also wrote that the assistance provided by teachers to students who have low Mathematical Thinking abilities needs to be distinguished from those who are high or moderate. Teacher assistance must be comprehensive.

Teacher assistance to students who have Mathematical Thinking skills needs serious attention from the teacher. Higher Mathematical Thinking is an essential factor that determines high school student's success in participating in mathematics learning. Students' mastery of mathematics itself is essential if students want to continue their studies.

Therefore, teachers' assistance to students through scaffolding following the level of students' Higher Mathematical Thinking ability needs to be done. This is following the opinion [24] that teachers' role in guiding and providing assistance to students in working on open-ended problems is essential.

4. Conclusions

The results of the analysis of open-ended problems that can be used to develop Higher Mathematical Thinking for Senior High School students are as follows. (1) Open-ended problems which have more than one correct answer. (2) Open-ended problems that have more than one correct solution algorithm. So, if a Senior High School math teacher wants to cultivate Higher Mathematical Thinking, then the teacher concerned must be able to make open-ended problems. The Research Subject of this study, the Higher Mathematical Thinking competency, was in the Very Good category. There were 1 student, 3

students in the Good category, 2 students in the Medium category. There are none of the Research Subjects of this study categorized as Less and Very Less.

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