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Analysis of mathematical critical thinking ability of students based on diagnostic assessment

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Abstract. The purposes of this study are 1) to describe the level of students' critical thinking ability in learning Complex Analysis based on the results of diagnostic assessments, 2) to identify errors experienced by students in solving problems that measure mathematical critical thinking ability in learning Complex Analysis based on the results of diagnostic assessment, and 3) to determine instructional therapy for students who experience errors in solving problems that measure mathematical critical thinking ability in learning Complex Analysis based on results diagnostic assessment. The research design uses a combination of quantitative and qualitative research, with simultaneous models not balanced. The population of the research was Mathematics Department students of FMIPA UNNES in 2017/2018 who had participated in complex analysis studies consisting of two classes with a population size of 68. The samples were randomly selected from two classes with a sample size of 33. The four subjects were selected from two levels. Critical and normal of each of the two students. Data were analysed quantitatively and qualitatively. The results of the study show that student learning outcomes on achieving critical mathematical thinking ability in complex analysis learning are complete. There are two levels of students' critical thinking ability, namely critical and normal levels. The results of the analysis of students' mathematical critical thinking ability, subject to a critical level, there are four indicators of questions that have not been reached. These indicators include aspects of assumptions, deductions, interpretations, and judging arguments. Subjects are of normal level, there are eleven indicators of questions that have not been reached. These indicators include all aspects of mathematical critical thinking ability namely, inferential, assumptions, deductions, interpretations, and judging arguments. The diagnosis of errors found that subjects with normal levels experienced errors of all types, namely errors of understanding, process, conclusion, transformation, and carelessness, as well as lack of material mastery. Subjects with critical levels experience misunderstanding and conclusions.

1. Introduction

Mathematical critical thinking ability is the result of learning cognitive aspects as well as other cognitive aspects whose achievement is measured, for example problem solving aspects. Therefore critical thinking ability can be achieved, trained, and given assignments through learning activities. This is in accordance with the results of a study which states Peter [1], that mathematical critical thinking ability can be improved by providing suitable tasks, and can be trained some researchers [1,2]. Likewise, higher-order thinking ability including critical thinking can be improved through suitable learning activities [3].

Several other studies [4] have been conducted related to various efforts to improve mathematical critical thinking ability, ability level analysis, and analysis of errors made by students in achieving mastery learning. The implications of the results of the analysis are important information as a follow-up to decide the next learning plan. However, it is not continued as instructional therapy for those who



have not yet completed their study rarely done by teachers or lecturers. Need to design assessment techniques to deal with this. For these reasons, critical thinking ability are very important for students in Indonesia.

Concerning the critical thinking ability of several experts [5,6] has presented conceptual definitions and descriptions of their aspects. However, in general the definition and aspects of critical thinking ability of each expert are different. In this study the aspects of mathematical critical thinking ability used are aspects of critical thinking according to Watson and Glaser [5] which include drawing conclusions, assumptions, deductions, interpretations, and evaluation of arguments. As a consideration in choosing aspects of mathematical critical thinking ability according to Watson and Glaser is the ease of arranging the measuring instruments used.

Assessments that use diagnostic test instruments are known as diagnostic assessments. The results of the diagnostic assessment can be used to identify the main problems faced by students that cause students not to achieve the specified learning achievement. The same was stated by experts, that the diagnostic assessment functions to analyse abilities, identify errors experienced by students and plan follow-up in the form of efforts to solve according to problems or errors that have been identified [7]. Diagnostic tests can be used to determine student weaknesses and strengths [8]. Therefore test results can be used as a basis for providing follow-up in the form of appropriate treatment and in accordance with the weaknesses of students. Diagnostic tests have the following characteristics: (1) designed to detect student learning difficulties; (2) developed based on an analysis of sources of error; (3) using form questions; and (4) accompanied by a follow-up plan in accordance with the identified difficulties.

Based on the results of studies by several researchers such as Herholdt and Sapire [9], Suyitno and Suyitno [10], Rohmah and Sutiyarso [11], students' errors in completing mathematics can be classified as follows: (1) reading errors, (2) misunderstanding, (3) transformation errors, (4) settlement process errors, and (5) errors draw a conclusion. This type of error is used as a basis for identifying. The analysis of these errors was observed from the student worksheets related to the questions given to him. Students' mistakes in solving problems can be done by looking at the completion steps made by students in completing the test questions.

The problem are 1) how are students' critical thinking ability in learning Complex Analysis based on the results of diagnostic assessments? 2) what are the mistakes experienced by students in solving problems that measure mathematical critical thinking ability in learning complex analysis based on the results of diagnostic assessments? 3) what follow-up can be given to students who make mistakes in solving problems that measure mathematical critical thinking ability in learning complex analysis based on the results of diagnostic assessments?

2. Methods

This research is a type combination research including qualitative and quantitative with concurrent embedded models. Qualitative is a major part of this research. Population of the research was students of Mathematics Department UNNES who had participated in complex analysis studies. The samples were randomly selected from two classes with a sample size of 33. The four subjects were selected from two levels of critical and normal. Each level was the two students. The instruments in the study were Diagnostic Tests of Mathematical Critical Thinking Ability (DTMCTA). Quantitative data is used to test student mastery learning. Mastery test uses the Minimum Mastery Criteria (MMC) 71 or B, and 75% mastery, with the reason for the Complex Analysis course. Interviews are used to ensure the diagnosis and causes of errors. Furthermore, qualitative data were analyzed using technical analysis that refers to the opinion of Miles and Huberman. Data analysis in this study was carried out through data reduction, data presentation, and conclusion drawing.

3. Result and Discussion

3.1 Quantitative analysis of mathematical critical thinking ability

Quantitative data analysis begins with a prerequisite test, which is the normality of data distribution. The results of the normality test showed that the data from DTMCTA on the sample came from the population with normal distribution. Mastery learning test was carried out, which includes 1) the average test of mathematical critical thinking ability is that students achieve the MMC score, which is 71 or the value of B, 2) the proportion test, namely the minimum number of students as the sample

size reaches the MMC value is 75%. Based on the results of the test that the average students' critical thinking ability in Complex Analysis learning achieved the MMC value of 71. The proportion test results showed that a minimum of 75% of the sample size in complex analysis learning had achieved the MMC score.

3.2 *Qualitative analysis of mathematical critical thinking ability*

In the qualitative analysis of students' critical thinking ability, based on the results of the indicator performance can be categorized into two levels, namely critical level and normal level. Includes a critical level if the value of critical thinking ability is greater or equal to the MMC, including the normal level if the score is less than MMC. Categorizing the results of measuring students' critical thinking ability in complex analysis learning is purposely made into two levels, with consideration of the ability of research subjects considered to be above the average ability in general. Therefore categorization is only divided into two levels namely critical and normal levels.

Four subjects were then selected consisting of two subjects M11, M12 with a critical level and two subjects M21, M22 with a normal level. Mathematical critical thinking ability measured in this study consist of three aspects, namely knowing assumptions, assessing arguments, and drawing conclusions. Aspects of drawing conclusions consist of three sub-aspects, namely inferential, deduction, and interpretation. The type of error used to diagnose failure to achieve students' mathematical critical thinking ability indicators includes: 1) misunderstanding, 2) transformation errors, 3) errors in the completion process, 4) miscalculation, and 5) carelessness mistakes.

3.3 *Mathematical critical thinking ability in critical level*

M11 subjects have a critical level whose test results reach the highest score of 95. There are 2 indicators of mathematical critical thinking ability that he has not achieved from the 18 indicators to be achieved. These indicators include aspects of conclusions. Based on the interpretation of student responses and in-depth interviews, the results of the diagnosis of M11 subjects experienced misunderstanding, information in drawing conclusions and also an element of carelessness. He has not yet completed the aspect of drawing conclusions, especially in the sub-aspects of deduction in mathematical critical thinking. Alternative instructional therapies for M11 subjects include the following. Need to practice solving problems related to drawing conclusions to increase understanding. Need to learn more about the logic associated with drawing conclusions.

The critical level M12 subject also has a test score of 87.5. There are 4 indicators with mathematical critical thinking ability that he has not achieved from the 18 indicators to be achieved. The four indicators include aspects of assumptions, drawing conclusions, and assessing arguments. Based on the interpretation of the question responses by students and in-depth interviews, the results of M12 subject diagnoses experienced misinformation in answering the problem of experiencing information errors, understanding. M12 subjects have not been completed on the assumption aspect and assess the mathematical critical thinking ability argument.

The answer to the subject of M11 for problem number 11, he chose the answer not in accordance with the reasons as can be seen in Figure 1. Based on the reasons written it seems that he was still unclear about the information contained in the problem, seen from the picture of the situation he presented. It could be that he has a wrong perception of the information about the problem. Based on the interview, it turns out he still does not understand the question given, so he draws the situation information from the problem based on the picture in the textbook he learned. It can be said that the M11 subject experienced misinformation in answering problem number 11, so that the picture of the information situation from the problem was wrong. As a result the answer to question number 11 is incorrect. M11 subjects in solving problem number 11 experienced errors in the elements of misunderstanding, misinformation, and process errors.

3.4 *Mathematical critical thinking ability in normal level*

M21 subjects with normal levels whose test results reach a value of 50. There are 10 indicators of mathematical critical thinking ability that he has not achieved from the 18 indicators available. These ten indicators, including all aspects of mathematical critical thinking ability, are recognizing assumptions, evaluating arguments, and drawing conclusions. Aspects of drawing conclusions include inferential, deduction, and interpretation.

As with the M21 subject, the M22 subject is of normal level whose test results reach a value of 45. There are 11 indicators of mathematical critical thinking ability that he has not achieved from the 18 indicators. These indicators include all aspects of mathematical critical thinking ability, namely knowing assumptions, assessing arguments, and drawing conclusions. Aspects of drawing conclusions include inferential, deduction, and interpretation.

The question number 7 indicator is as follows. Given a statement about integral trajectories, students can submit correct assumptions. The M21 Subject Answer to question number 7, he chose the wrong answer for the reasons as can be seen in Figure 2. Based on the reasons written by the M21 Subject he seems to assume that the assumptions needed are statements on assumptions that are of true value. It could be that he has the wrong perception of the instructions to solve the problem. Based on the interview, it turns out he really still does not understand about the instructions to solve the given problem. It could be said that the subject of M21 experienced misinformation in answering problem number 7, so the answer to the question was wrong. M21 subjects in solving problem number 7 experienced misinformation. The subject of M21 is not yet complete in the aspect of assuming mathematical critical thinking ability.

3.5 Error analysis of normal level subject

M21 subjects are normal level, there are 10 questions that fail to be answered correctly from the 18 questions tested. The ten questions that failed to be answered correctly covered all aspects of mathematical critical thinking ability, namely knowing assumptions, assessing arguments, and drawing conclusions. Aspects of drawing conclusions consist of inferential, deduction, and interpretation aspects. Furthermore, from the 10 questions that failed to be answered correctly, there were 4 questions that were not answered. In this case, the mastery of the material tested for M21 Subjects is lacking. All aspects of mathematical critical thinking ability have not been completed. All kinds of mistakes in solving problems he experienced, namely errors of understanding, transformation, process, drawing conclusions, and carelessness. M21 subjects need to follow remedial teaching.

Subjects of M22 are of normal level, there are 11 questions that fail to be answered correctly from the 18 questions tested. The eleven failed questions cover all aspects of mathematical critical thinking ability, namely knowing assumptions, assessing arguments, and drawing conclusions. Aspects of drawing conclusions consist of inferential, deduction, and interpretation aspects. Furthermore, from the 11 questions that failed to be answered correctly, there were 6 questions that were not answered. In this case, the mastery of the material tested for the M22 Subject is lacking. All aspects of mathematical critical thinking ability have not been completed. All kinds of mistakes in solving problems he experienced, namely errors of understanding, transformation, process, drawing conclusions, and carelessness. M22 subjects need to follow remedial teaching.

3.6 Follow-up learning

The results of error analysis experienced by the four research subjects can be divided into two categories, namely the light and heavy category errors. The error category in this study is adjusted to the mastery of learning outcomes. Subjects are said to experience minor errors if the subject is completely studied. Subjects are said to experience severe errors if the subject has not yet been completed.

Based on the results of the ability analysis and errors experienced by the research subjects, subjects were of critical level, they experienced mild errors, while subjects with normal levels experienced severe errors. Subjects who experience minor errors are given training questions or tasks related to indicators that have not been reached. Subjects who experience severe errors, are obliged to take re-learning, especially in aspects of mathematical critical thinking ability whose achievements are not optimal.

Like other high-level thinking ability, among others, the ability to think creatively, solve problems, etc., the ability to think critically mathematically can be grown or enhanced through learning. This is closely related to the results of research by Aizikovitsh-Udi and Amit [4]. That students' critical thinking ability can be influenced by life experiences and experiences in other mathematical domains. Means to grow or improve the ability to think critically mathematically is suitable if the learning material is mathematics, especially Complex Analysis.

In Learning, the achievement of critical thinking ability mathematically does not have to use specific learning models, but learning models that are commonly applied. It is precisely what needs to be emphasized in learning activities is the assessment activities. Assessment activities need to be designed including the process and learning outcomes. Process assessments related to achieving indicators of aspects of mathematical critical thinking ability through the learning process, can be operationally given examples or exercises that specifically measure aspects of critical thinking ability. Results assessment is related to learning outcomes after participating in learning activities within a certain period of time, measured by a specific measuring instrument that is a test of mathematical critical thinking ability.

What is more interesting is related to students' critical thinking ability, that these abilities are strongly influenced by affective abilities. For example, the results of Magno's research [12] that students' mathematical critical thinking ability are influenced by their metacognition abilities. Metacognition ability including affective abilities consist of 8 aspects: 1) declarative knowledge, 2) procedural knowledge, 3) conditional knowledge, 4) planning, 5) information management strategies, 6) monitoring, 7) strategy selection, and 8) learning evaluation.

Between aspects of the ability to think critically mathematically with aspects of theoretical metacognition ability if observed closely there is a close relationship. When someone will conclude a statement or assess the validity of an argument he needs the name of knowledge, it takes a strategy to conclude that all of these are aspects of metacognition. Affective aspects of learning outcomes are the accompanying effects of cognitive aspects of learning outcomes. Although affective learning outcomes are the effects of accompaniment, if the learning is well designed related to the affective aspects the results can increase faster.

It is time to carry out measurements of mathematical critical thinking ability in mathematics learning including Complex Analysis, because the core of mathematics learning is critical thinking [2]. It is not easy to compile instruments for measuring mathematical critical thinking ability, but do not be used as an excuse not to do, in fact the ability to think critically is a student learning outcome bill. Specific affective aspects of mathematics learning outcomes, greatly support the learning outcomes of specific cognitive aspects such as critical thinking ability. Specific affective aspects of mathematics learning outcomes such as mathematical disposition, mathematics belief, learning independence, metacognition and others. Empirically it has been proven by experts, among others, that they have succeeded in conducting research that links cognitive and specific affective learning outcomes such as those carried out by [3]. The results show that specific affective learning outcomes affect cognitive learning outcomes.

4. Conclusion

Based on the results of the study and discussion, the following conclusions were obtained.

1. Learning outcomes of students' critical thinking ability in complex analysis learning are complete. This means that at least 75% of the total number of students who take part in complex learning, their mathematical critical thinking ability have a minimum score of 71.
2. The results of the indicators for each subject at each level can be described as follows.
 - a. Subjects have a critical level, there are four indicators of questions that have not been reached. These indicators include aspects of assumptions, deductions, interpretations, and assessing arguments in mathematical critical thinking skills
 - b. Subjects are of normal level, there are eleven indicators of questions that have not been reached. The indicator includes all aspects of mathematical critical thinking skills namely, inferential, assumptions, deductions, interpretations, and judging arguments.
3. The errors experienced by each subject in completing the diagnostic test questions the ability to think critically mathematically in the study of complex analysis are as follows.
 - a. Error Analysis of Critical Level Subject
Errors experienced by the subject of critical level include types of misunderstanding, drawing conclusions, and carelessness. The main cause is weak in the logic of thinking, lack of mastery of concepts, and students are less careful in presenting the information contained in the problem.
 - b. Error Analysis of Normal Level Subject

Errors experienced by subjects of normal level include all types of errors, namely errors of understanding, process, transformation, conclusions and carelessness. The main cause is weak in the logic of thinking, mastery of the concept of material, and less thorough in presenting information - information that is in the problem.

4. Follow-up that can be given to students who make mistakes in solving problems that measure mathematical critical thinking skills in learning Complex Analysis based on the results of diagnostic assessment are as follows.
 - a. Follow-up for subjects of critical level as a result of their mistakes, they do not need re-learning enough to be given tasks related to indicators and aspects of mathematical critical thinking abilities that they have not achieved.
 - b. Follow-up for subjects of normal level as a result of their mistakes, needs re-learning, because all aspects of mathematical critical thinking ability have not been achieved.

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