

# On total vertex irregularity strength of generalized uniform cactus chain graphs with pendant vertices

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## On total vertex irregularity strength of generalized uniform cactus chain graphs with pendant vertices

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### Abstract

In this research, we use the concept of a vertex irregular total  $k$  labeling and  $tvs(G)$ . Specifically, we investigate  $tvs$  of generalized uniform cactus chain graphs  $C_r(C_n^{n-2})$  with  $(n-2)r$  pendant vertices and length  $r$ . The result is as follows:  $tvs(C_r(C_n^{n-2})) = \left\lceil \frac{2(n-2)r+3}{4} \right\rceil$  for  $n \geq 6$ .

**Subject Classification:** 05C78.

**Keywords:** Vertex irregular total  $k$ -labeling,  $Tvs$ , Uniform, Cactus chain.

### 1. Introduction

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Throughout this paper, we consider a graph  $G(V, E)$  with  $V = V(G)$  and  $E = E(G)$ . It is presumed that  $G$  is finite, simple, and undirected. "A total labeling of  $G$  is a function  $\lambda$  that assigns  $V \cup E$  into a set of integers,

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mentioned as labels" [9,13,17,18]. "The function  $\lambda$  is also called a vertex irregular total  $k$ -labeling if the weights  $wt_\lambda(u) \neq wt_\lambda(v)$  for  $u \neq v \in V$  where  $wt_\lambda(u) = \lambda(u) + \sum_{ux \in E(G)} \lambda(ux)$ . Moreover, a total vertex irregularity strength of  $G$ , symbolized by  $tvs(G)$ , is a minimum number  $k$  for which  $G$  has a vertex irregular total  $k$ -labeling" [2]. Lower bound of  $tvs$  of a graph  $G$  was proposed in [5]:  $\left\lceil \frac{(p+\delta)}{(\Delta+1)} \right\rceil \leq tvs(G) \leq p + \Delta - 2\delta + 1$ , where  $p$  is  $|V|$ ,  $\delta = \min\{d(v) \mid \forall v \in V\}$  and  $\Delta = \max\{d(u) \mid \forall u \in V\}$ . Furthermore, another lower bound was given by Anholcer, et al. in [3]. In addition,  $tvs$  of any connected graph which consists of  $n_i$  vertices having degree  $i$  for  $\delta \leq i \leq \Delta$  satisfies lower bound [14]:

$$tvs \geq \max \left\{ \left\lceil \frac{\delta(G) + n_{\delta(G)}}{\delta(G) + 1} \right\rceil, \left\lceil \frac{\delta(G) + n_{\delta(G)} + n_{\delta(G)+1}}{\delta(G) + 2} \right\rceil, \dots, \left\lceil \frac{\delta(G) + \sum_{i=\delta(G)}^{\Delta(G)} n_i}{\Delta(G) + 1} \right\rceil \right\} \quad (1)$$

Many scholars have invented  $tvs$  of many graph classes such as in [1,2,3,6,11,14], etc. More results on  $tvs$  of any graphs can be seen in [6] and [8]. The concept of cactus graph and some results on the cactus can be found in [4,7,9,12,19], etc. "A cactus graph is defined as a connected graph for which each edge lies in exactly one cycle. In other words, the cactus graph comprises several blocks where each block is either a cycle or an edge. If each block of the cactus graph has at most two cut-vertices and every two blocks share exactly one cut vertex, then we call it as a cactus chain graph" [7,12]. Some results related to  $tvs$  of cactus chain graphs have been invented in [15,16]. In this article, we examine  $tvs$  of generalized uniform cactus chain graphs  $C_r(C_n^{n-2})$  with  $(n - 2)r$  pendant vertices and length  $r$ .

**2. Main Result and Discussion**

We provide formulas for labels of vertices and edges and a formula for  $tvs$  of generalized uniform cactus chains  $C_r(C_n^{n-2})$ .

*2.1 Generalized uniform cactus chain graphs with pendant vertices*

Firstly, we present the concept of generalized uniform cactus chains having pendant vertices in Definition 2.1.1.

**Definition 2.1.1 :** "A cactus graph, in which each block is a cycle with the same size  $n$ , will be called as an  $n$ -uniform cactus graph for any natural

number  $n$ . If each cycle of the  $n$ -uniform cactus has at most two cut-vertices and each cut-vertex is shared by exactly two cycles, then it is called an  $n$ -uniform cactus chain graph. The number of cycles indicates the length of the cactus chain graph" [7,12]. Further, generalized uniform cactus chain graphs having pendant vertices of length  $r$ , denoted by  $C_r(C_n^{n-2})$ , are the  $n$ -uniform cactus chains in which each block is a cycle  $C_n$  connected with  $n-2$  pendant vertices. The vertex and edge sets of  $C_r(C_n^{n-2})$  are as follows:

$$V(C_r(C_n^{n-2})) = \{a_i, b_{1i}, b_{2i}, b_{3i}, b_{4i}, \dots, b_{pi}, c_{1i}, c_{2i}, c_{3i}, c_{4i}, \dots, c_{qi}, a_{i+1}\} \cup \{b'_{1i}, b'_{2i}, b'_{3i}, b'_{4i}, \dots, b'_{pi}, c'_{1i}, c'_{2i}, c'_{3i}, c'_{4i}, \dots, c'_{qi}\}$$

and

$$E(C_r(C_n^{n-2})) = \left\{ \begin{aligned} &a_i b_{1i}, b_{1i} b_{2i}, b_{2i} b_{3i}, b_{3i} b_{4i}, \dots, b_p a_{i+1}, a_i c_{1i}, c_{1i} c_{2i}, c_{2i} c_{3i}, c_{3i} c_{4i}, \dots, c_q a_{i+1}, \\ &b_{1i} b'_{1i}, b_{2i} b'_{2i}, b_{3i} b'_{3i}, b_{4i} b'_{4i}, \dots, b_p b'_p, c_{1i} c'_{1i}, c_{2i} c'_{2i}, c_{3i} c'_{3i}, c_{4i} c'_{4i}, \dots, c_q c'_q \end{aligned} \right\}$$

$\forall i = 1, 2, 3, \dots, r$ , where  $r$  indicates the length of the chain graphs and the indexes  $p$  and  $q$  are equal to  $\frac{n-2}{2}$  if  $n$  is even. Meanwhile,  $p = \lceil \frac{n-2}{2} \rceil$  and  $q = \lfloor \frac{n-2}{2} \rfloor$  for odd  $n$ .

2.2 Tvs of the generalized uniform cactus chain graphs

**Theorem 2.2.1 :** Given the generalized uniform cactus chain graphs  $C_r(C_n^{n-2})$  with  $(n-2)r$  pendant vertices and length  $r \geq 2$ . Then,  $tvs(C_r(C_n^{n-2})) = \lfloor \frac{2(n-2)r+3}{4} \rfloor, n \geq 6$ .

**Proof:** Let us consider the vertices  $b_{ji}$  and  $c_{li}$  with degree 1 for  $i = 1, 2, \dots, r$ , where the indexes of  $j$  and  $l$  are  $j = 1, 2, \dots, \frac{(n-2)}{2}; l = 1, 2, \dots, \frac{(n-2)}{2}$  when  $n$  is even. Whereas,  $j = 1, 2, \dots, \lfloor \frac{(n-2)}{2} \rfloor; l = 1, 2, \dots, \lfloor \frac{(n-2)}{2} \rfloor$  if  $n$  is odd. Further,  $b_{jp}, c_{li}$  are vertices of degree 3. Moreover, vertices  $a_1, a_{r+1}$  have degree 2; and  $a_{i+1}$  have degree 4 for  $i = 1, 2, \dots, r-1$ . According to Lower bound (1), we get

$$tvs(C_r(C_n^{n-2})) \geq \max \left\{ \left\lfloor \frac{1+(n-2)r}{2} \right\rfloor, \left\lfloor \frac{(n-2)r+3}{3} \right\rfloor, \left\lfloor \frac{(n-2)r+3+(n-2)r}{4} \right\rfloor, \left\lfloor \frac{2(n-2)r+3+(r-1)}{5} \right\rfloor \right\} \geq$$

$$\left\lceil \frac{2(n-2)r+3}{4} \right\rceil \tag{2}$$

To proof that  $k = \left\lceil \frac{2(n-2)r+3}{4} \right\rceil$  is the upper bound of the tvs, we establish a total  $k$ -labeling  $f: V \cup E \rightarrow \{1, 2, \dots, k\}$  and show that it is vertex irregular by means of two cases.

**Case 1 :** For  $n$  is odd ( $n \geq 7$ )

Case 1 is divided into two subcases as follows.

**Subcase 1.1 :** When the length  $r$  is odd.

In this subcase, we provide labels  $f(x)$  and  $f(e)$  for each  $x \in V(C_r(C_n^{n-2}))$  and  $e \in E(C_r(C_n^{n-2}))$  in Table 1.

**Table 1**  
Labels of vertices and edges of  $C_r(C_n^{n-2})$  for both  $n$  and  $r$  are odd

$x, e$	$f(x)$ and $f(e)$	The index $i = 1, 2, \dots, r$
$b_{\beta}$	$\left\lfloor \frac{n-2}{2} \right\rfloor i - \left\lfloor \left\lfloor \frac{n-2}{2} \right\rfloor - (j+1) \right\rfloor$ ,	$j = 1, 2, \dots, \left\lceil \frac{n-2}{2} \right\rceil$ (3)
$b_{\beta}'$	$\left\lfloor \frac{n-2}{2} \right\rfloor i - \left\lfloor \left\lfloor \frac{n-2}{2} \right\rfloor - j \right\rfloor$ ,	$j$ as in (3)
$c_{\beta}$	$\left\lfloor \frac{r}{2} \right\rfloor + \left\lfloor \frac{n-2}{2} \right\rfloor i - \left\lfloor \left\lfloor \frac{n-2}{2} \right\rfloor - (j+1) \right\rfloor$ ,	$j = 1, 2, \dots, \left\lceil \frac{n-2}{2} \right\rceil$ (4)
$c_{\beta}'$	$\left\lfloor \frac{r}{2} \right\rfloor + \left\lfloor \frac{n-2}{2} \right\rfloor i - \left\lfloor \left\lfloor \frac{n-2}{2} \right\rfloor - j \right\rfloor$ ,	$j$ as in (4)
$a_i$	$i$	
$a_{r+1}$	1	
$a_{\beta} b_{1i}$	$\left\lceil \frac{2(n-2)r+3}{4} \right\rceil$	$j$ equals to (3)
$b_{\beta} b_{(j+1)i}$	$\left\lceil \frac{2(n-2)r+3}{4} \right\rceil - 1$	$j = 1, 2, \dots, \left\lceil \frac{n-2}{2} \right\rceil - 1$
$b_{\left\lfloor \frac{n-2}{2} \right\rfloor} a_{i+1}$	$\left\lceil \frac{2(n-2)r+3}{4} \right\rceil - 1$	

Contd...

$a_i c_{1i}$	$\left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor$	
$c_j c_{(j+1)i}$	$\left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor - 1$	$j = 1, 2, \dots, \left\lfloor \frac{n-2}{2} \right\rfloor - 1$
$c_{\left\lfloor \frac{n-2}{2} \right\rfloor} a_{i+1}$	$\left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor - 1$	$i = 1, 2, \dots, r-1$
$c_{\left\lfloor \frac{n-2}{2} \right\rfloor} a_{r+1}$	$\left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor$	
$b_j b'_j$	$i$	$j$ as in (3)
$c_j c'_j$	$\left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor$	$j$ equals to (4)

Under labeling  $f$ , we observe the vertex weights:

$$wt(b_{j_i}) = 2 \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor + \left\lfloor \frac{n-2}{2} \right\rfloor i - \left\{ \left\lfloor \frac{n-2}{2} \right\rfloor - j \right\},$$

$$wt(b'_{j_i}) = \left\lfloor \frac{n-2}{2} \right\rfloor i - \left\{ \left\lfloor \frac{n-2}{2} \right\rfloor - (j+1) \right\}, 1 \leq i \leq r, j = 1, 2, \dots, \left\lfloor \frac{n-2}{2} \right\rfloor$$

$$wt(c'_{j_i}) = \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor + \left\lfloor \frac{r}{2} \right\rfloor + \left\lfloor \frac{n-2}{2} \right\rfloor i - \left\{ \left\lfloor \frac{n-2}{2} \right\rfloor - j \right\},$$

$$wt(c_{j_i}) = 3 \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor + \left\lfloor \frac{r}{2} \right\rfloor + \left\lfloor \frac{n-2}{2} \right\rfloor i - \left\{ \left\lfloor \frac{n-2}{2} \right\rfloor - (j-1) \right\},$$

$$1 \leq i \leq r, j = 1, 2, \dots, \left\lfloor \frac{n-2}{2} \right\rfloor$$

$$wt(a_1) = 2 \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor - 1,$$

$$wt(a_{r+1}) = 2 \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor,$$

$$wt(a_{i+1}) = 4 \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor + (i-3), 1 \leq i \leq r-1.$$

**Subcase 1.2 :** When the length  $r$  is even.

In this subcase, we give labels  $f(x)$  and  $f(e)$  for each  $x \in V(C_r(C_n^{n-2}))$  and  $e \in E(C_r(C_n^{n-2}))$  in Table 2.

**Table 2**  
Labels of vertices and edges of  $C_r(C_n^{n-2})$  for  $n$  is odd and  $r$  is even

$f(x)$ and $f(e)$	The index $i = 1, 2, \dots, r$
$f(b_{ji}) = \lfloor \frac{n-2}{2} \rfloor i - \lfloor \lfloor \frac{n-2}{2} \rfloor - (j+2) \rfloor$ ,	$j = 1, 2, \dots, \lfloor \frac{n-2}{2} \rfloor - 1$
$f(b_{\lfloor \frac{n-2}{2} \rfloor i}) = \lfloor \frac{n-2}{2} \rfloor i - \lfloor \lfloor \frac{n-2}{2} \rfloor - (\lfloor \frac{n-2}{2} \rfloor + 2) \rfloor$ ,	$i = 1, 2, \dots, r; r \geq 4$ $i = 1 (r = 2)$
$f(b_{\lfloor \frac{n-2}{2} \rfloor r}) = \lfloor \frac{n-2}{2} \rfloor r + 2$	$r = 2$
$f(b_{ji}') = \lfloor \frac{n-2}{2} \rfloor i - \lfloor \lfloor \frac{n-2}{2} \rfloor - j \rfloor$ ,	$j$ as in (3)
$f(c_{ji}) = \lfloor \frac{r}{2} \rfloor + \lfloor \frac{n-2}{2} \rfloor i - \lfloor \lfloor \frac{n-2}{2} \rfloor - (j+2) \rfloor$ ,	$j = 1, 2, \dots, \lfloor \frac{n-2}{2} \rfloor - 1$
$f(c_{\lfloor \frac{n-2}{2} \rfloor j}) = \lfloor \frac{r}{2} \rfloor + \lfloor \frac{n-2}{2} \rfloor i - \lfloor \lfloor \frac{n-2}{2} \rfloor - (\lfloor \frac{n-2}{2} \rfloor + 2) \rfloor$ ,	$i = 1, 2, \dots, r - 1$
$f(c_{\lfloor \frac{n-2}{2} \rfloor r}) = \lfloor \frac{r}{2} \rfloor + \lfloor \frac{n-2}{2} \rfloor r + 1$	
$f(c_{ji}') = \lfloor \frac{r}{2} \rfloor + \lfloor \frac{n-2}{2} \rfloor i - \lfloor \lfloor \frac{n-2}{2} \rfloor - j \rfloor$ ,	$j$ is equal to (4)
$f(a_1) = f(a_{r+1}) = 2$	$r \geq 4$
$f(a_{i+1}) = i + 3$	$i = 1, 2, \dots, r - 1; r \geq 4.$
$f(a_1) = 2; f(a_2) = 1$	$r = 2$
$f(a_i b_{1i}) = \lfloor \frac{2(n-2)r+3}{4} \rfloor - 1$ ,	
$f(b_{ji} b_{(j+1)i}) = \lfloor \frac{2(n-2)r+3}{4} \rfloor - 1$ ,	$j = 1, 2, \dots, \lfloor \frac{n-2}{2} \rfloor - 1$

Contd...



$f\left(b_{\lfloor \frac{n-2}{2} \rfloor} a_{i+1}\right) = \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor - 1,$	$i = 1, 2, \dots, r; r \geq 4$
$f\left(b_{\lfloor \frac{n-2}{2} \rfloor} a_{i+1}\right) = \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor - 1,$	$i = r - 1 = 1; r = 2$
$f\left(b_{\lfloor \frac{n-2}{2} \rfloor} a_{r+1}\right) = \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor,$	$r = 2$
$f(a_i c_{1i}) = \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor - 1,$	
$f(c_{\beta} c_{(j+1)\beta}) = \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor - 1,$	$j = 1, 2, \dots, \left\lfloor \frac{n-2}{2} \right\rfloor - 1$
$f\left(c_{\lfloor \frac{n-2}{2} \rfloor} a_{i+1}\right) = \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor - 1,$	$i = 1, 2, \dots, r - 1$
$f\left(c_{\lfloor \frac{n-2}{2} \rfloor} a_{r+1}\right) = \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor,$	
$f(b_{\beta} b_{\beta}') = i,$	$j$ is given in (3)
$f(c_{\beta} c_{\beta}') = \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor,$	$j$ is equal to (4)

We get the weights of vertices in the following way:

$$wt(b_{\beta}) = 2 \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor + \left\lfloor \frac{n-2}{2} \right\rfloor i - \left\{ \left\lfloor \frac{n-2}{2} \right\rfloor - (j+1) \right\},$$

$$wt(b_{\beta}') = \left\lfloor \frac{n-2}{2} \right\rfloor i - \left\{ \left\lfloor \frac{n-2}{2} \right\rfloor - (j+1) \right\}, 1 \leq i \leq r, j = 1, 2, \dots, \left\lfloor \frac{n-2}{2} \right\rfloor$$

$$wt(c_{\beta}') = \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor + \frac{r}{2} + \left\lfloor \frac{n-2}{2} \right\rfloor i - \left\{ \left\lfloor \frac{n-2}{2} \right\rfloor - j \right\},$$

$$wt(c_{\beta}) = 3 \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor + \frac{r}{2} + \left\lfloor \frac{n-2}{2} \right\rfloor i - \left\{ \left\lfloor \frac{n-2}{2} \right\rfloor - (j) \right\},$$

$$1 \leq i \leq r, j = 1, 2, \dots, \left\lfloor \frac{n-2}{2} \right\rfloor$$

$$wt(a_1) = 2 \left\lfloor \frac{2(n-2)r+3}{4} \right\rfloor,$$

$$wt(a_{r+1}) = 2 \left\lceil \frac{2(n-2)r+3}{4} \right\rceil + 1, wt(a_{i+1}) = 4 \left\lceil \frac{2(n-2)r+3}{4} \right\rceil + (i-1), 1 \leq i \leq r-1.$$

**Case 2 :** For  $n$  is even ( $n \geq 6$ ).

In this case, we construct vertex and edge labels as in Table 3.

**Table 3**  
Vertex and edge-labels of  $C_r(C_n^{n-2})$  for  $n$  is even.

$f(x)$ and $f(e)$	The index $1 \leq i \leq r$
$f(b_{\beta}) = \left(\frac{n-4}{2}\right)i + j - \left\lceil \left(\frac{n-4}{2}\right) - 2 \right\rceil,$	$1 \leq j \leq \frac{n-2}{2}$ (5)
$f(b_{\beta}') = \left(\frac{n-4}{2}\right)i + \left\lceil j - \left(\frac{n-4}{2}\right) \right\rceil,$	$j$ is equal to (5)
$f(c_{\beta}) = \left(\frac{n-2}{2}\right)i + \left\lceil j - \left(\frac{n-6}{2}\right) \right\rceil,$	$1 \leq i \leq r-1; 1 \leq j \leq \frac{n-2}{2} - 1$
$f(c_{\beta}') = \left(\frac{n-2}{2}\right)r + 1,$	$j$ is given in (5)
$f(c_{\beta}'') = \left(\frac{n-2}{2}\right)i + \left\lceil j - \left(\frac{n-2}{2}\right) \right\rceil,$	$j$ as in (5)
$f(a_{i+1}) = i + 3,$	$i = 1, 2, \dots, r-1$
$f(a_1) = f(a_{r+1}) = 2,$	
$f(a_{\beta}b_{11}) = \left\lceil \frac{2(n-2)r+3}{4} \right\rceil - 1,$	
$f(b_{\beta}b_{(j+1)1}) = \left\lceil \frac{2(n-2)r+3}{4} \right\rceil - 1,$	$1 \leq j \leq \frac{n-2}{2} - 1$
$f\left(b_{\frac{n-2}{2}i}a_{i+1}\right) = \left\lceil \frac{2(n-2)r+3}{4} \right\rceil - 1,$	
$f(a_{\beta}c_{11}) = \left\lceil \frac{2(n-2)r+3}{4} \right\rceil - 1,$	
$f(c_{\beta}c_{(j+1)1}) = \left\lceil \frac{2(n-2)r+3}{4} \right\rceil - 1,$	$1 \leq j \leq \frac{n-2}{2} - 1$

Contd...

$f\left(c_{\frac{n-2}{2}, a_{i+1}}\right) = \left\lceil \frac{2(n-2)r+3}{4} \right\rceil - 1,$	$i = 1, 2, \dots, r-1$
$f\left(c_{\frac{n-2}{2}, a_{r+1}}\right) = \left\lceil \frac{2(n-2)r+3}{4} \right\rceil,$	
$f(b_{\mu}, b_{\mu}') = i,$	$j$ is given in (5)
$f(c_{\mu}, c_{\mu}') = \left\lceil \frac{2(n-2)r+3}{4} \right\rceil$	$j$ equals to (5)

In this case, the weights of vertices are provided below:

$$wt(b_{\mu}) = 2 \left\lceil \frac{2(n-2)r+3}{4} \right\rceil + \frac{n-2}{2}i + j - \left(\frac{n-4}{2}\right),$$

$$wt(a_i) = 2 \left\lceil \frac{2(n-2)r+3}{4} \right\rceil,$$

$$wt(b_{\mu}') = \frac{n-2}{2}i + j - \left(\frac{n-4}{2}\right), 1 \leq i \leq r, j = 1, 2, \dots, \frac{n-2}{2}$$

$$wt(c_{\mu}') = \left\lceil \frac{(2(n-2)r+3)}{4} \right\rceil + \frac{n-2}{2}i + j - \frac{n-2}{2},$$

$$wt(c_{\mu}) = 3 \left\lceil \frac{(2(n-2)r+3)}{4} \right\rceil + \frac{n-2}{2}i + j - \frac{n-2}{2}, 1 \leq i \leq r, j = 1, 2, \dots, \frac{n-2}{2}$$

$$wt(a_{r+1}) = 2 \left\lceil \frac{(2(n-2)r+3)}{4} \right\rceil + 1,$$

$$wt(a_{i+1}) = 4 \left\lceil \frac{(2(n-2)r+3)}{4} \right\rceil + (i-1), 1 \leq i \leq r-1.$$

In Case 1 and Case 2, the minimum label of elements of  $C_r(C_n^{n-2})$  is 1 and the largest integer of vertex and edge labels is  $\left\lceil \frac{(2(n-2)r+3)}{4} \right\rceil$ . It is shown that the vertex weights are all different and it proves the upper bound  $tvs(C_r(C_n^{n-2})) \leq \left\lceil \frac{(2(n-2)r+3)}{4} \right\rceil$ . According to the Lower bound (2), it shows that  $tvs(C_r(C_n^{n-2})) = \left\lceil \frac{(2(n-2)r+3)}{4} \right\rceil, n \geq 6$ . The proof is complete. ■

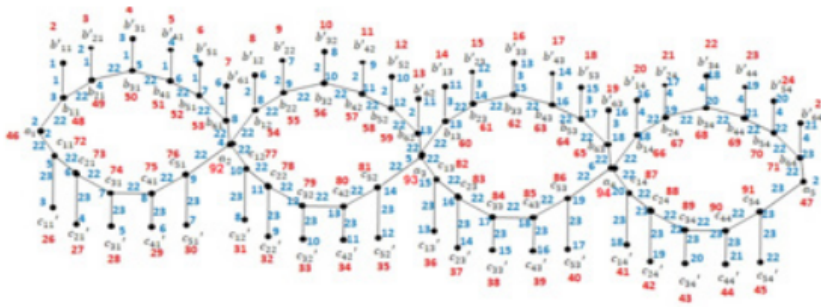


Fig. 1  
The pattern to get  $tv_s(C_4(C_{13}^{11})) = 23$ .

**Example 2.1.1 :** Figure 1 illustrates labels for vertices and edges such that  $tv_s(C_4(C_{13}^{11})) = 23$ .

**3. Conclusions**

The formula for  $tv_s$  of generalized uniform cactus chain graphs  $C_r(C_n^{n-2})$  with  $(n-2)r$  pendant vertices and length  $r$  has been proved in this article. We get  $tv_s(C_r(C_n^{n-2})) = \left\lceil \frac{2(n-2)r+3}{4} \right\rceil$  for  $n \geq 6$ . The patterns to get the vertex irregular total  $k$ -labeling were presented in the theorem. In upcoming research, we are interested to investigate  $tv_s$  or  $tes$  of generalized tadpole chain graphs. Also, we will examine the  $tv_s$  or  $tes$  using algorithmic approaches.

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