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by Mulyono Matematika

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The Spatial Thinking Process of the Field-Independent Students based on Action-Process-Object-Schema Theory

Henry Suryo Bintoro* Universitas Negeri Semarang, INDONESIA
Y.L. Sukestiyarno Universitas Negeri Semarang, INDONESIA
Mulyono Universitas Negeri Semarang, INDONESIA
Walid Universitas Negeri Semarang, INDONESIA

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Abstract: Spatial thinking has roles to facilitate learners to remember, understand, reason, and communicate objects and the connections among objects that are represented in space. This research aims to analyze the spatial thinking process of students in constructing new knowledge seen from the field-independent cognitive style learners based on Action-Process-Object-Schema (APOS) theory. APOS theory is used to explore spatial thinking processes which consist of mental structures of action, process, object, and schema. This research is qualitative research with an exploratory method. It provided the students' opportunity to solve problems alternately until the method found the most appropriate subjects for the research objectives. The subjects were 2 students of Mathematics Education in the fourth semester of Universitas Muria Kudus Indonesia. The data collection techniques were started by distributing the validated and reliable spatial thinking questions, the cognitive style question, and the interview. The applied data analysis consisted of data reduction, presentation, and conclusion. The findings showed (1) spatial thinking process of holistic-external representation typed learners were indicated by the representative thinking element, abstract-illustrative figure expression to communicate and complete the tasks correctly, (2) spatial thinking process of the holistic-internal representation typed learners were indicated by the representative means, having ideas, connecting with the previous knowledge in the forms of symbols and numbers, and finding the final results correctly although incomplete.

Keywords: *APOS theory, cognitive style, field-independent, spatial thinking.*

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Introduction

Understanding geometric materials require high visualization because of their abstract nature. Learning geometry requires excellent understanding and conceptual reasoning to avoid difficulties. It is in line with (Gal, 2019). He found that students' difficulties to learn geometry dealt with the gaps of geometrical thinking, hindrances to understand and reason the concept, or misunderstanding and visual perception limitation. The finding was in line with the fact. It showed low geometrical learning results (Bintoro et al., 2021; Schindler & Lilienthal, 2019; Zuliana et al., 2020).

The difficulties could be traced so that further appropriate revision to overcome could be promoted. It can be done by analyzing the students' thinking processes. The thinking process refers to analyzing and interpreting skills on objects. It is an important aspect of education (Sapti et al., 2019). Learning activities always deal with the thinking process. It is because the occurring process while learning requires a mental process of the brain (Khusna, 2020). It is in line with (Muhtarom et al., 2017). They argue that the thinking process is an activity that occurs inside of humans' brains. Understanding the students' thinking processes to solve problems is important for a lecturer. It allows a lecturer to notice the state and the error types of the students. Thus, he could design appropriate learning for them.

There are some thinking process types. This research dealt with the spatial thinking process because the students had difficulties in geometry. The spatial thinking process consists of three elements: spatial concept, representative mean, and reasoning process (National Research Council, 2005). The results showed that the correlation of spatial thinking process and the students' skills to solve geometrical problems (Tikhomirova, 2017). According to (Cheng & Mix, 2014), spatial thinking is important for mathematics education. In various contexts, spatial thinking is an important skill to

* **Corresponding author:**
Henry Suryo Bintoro, Universitas Negeri Semarang, Doctoral Students of Graduate School, Indonesia. ✉ henrysuryo18@students.unnes.ac.id

solve mathematics problems (Yuda, 2011). Based on the explanation, the researchers analyzed the spatial thinking process on the geometrical material.

The skill is connected with common-mental skills to learn and apply knowledge to manipulate the environment and the skills in an abstract manner (Putri et al., 2019). Spatial skill is not a single construct unit. It consists of some dimensions or aspects. According to (Shawky et al., 2020), spatial skill consists of visualization, spatial relation, spatial orientation, and mental rotation. According to (Ramful et al., 2017), spatial skill has three dimensions: mental rotation, spatial orientation, and spatial visualization. (Yilmaz, 2009) Explains the spatial skill aspects into three matters: spatial visualization, spatial relation, and spatial orientation. According to (Maier, 1998), the indicators of spatial skills are such as spatial perception, spatial visualization, thought rotation, spatial relation, and spatial orientation. In this research, the spatial skill indicators are observing the geometric object from various perspectives, imagining the object from the internal-part changes, rotating the object, identifying the spatial realization connection of the object, and constructing the pattern in a space based on the geometrical object.

There are several ways to trace the thinking process. In this research, it was traced with APOS theory. The APOS theory explains the mental structures of thinking. They are action, process, object, scheme, and mental mechanism. The mental mechanism consists of interiorization, coordination, reversal encapsulation, de-encapsulation, and thematization. APOS theory is a model to describe how mathematics concept is learned and used to explain how an individual develops this understanding mentally about the mathematics concept (Arnon et al., 2014). The theory application in learning could construct the students' understanding (Inglis, 2015).

Geometrical problems are not only caused by many concepts and their abstractness. However, they could be influenced by different perceptions, individual characters, thinking skills, or cognitive styles while the students are learning or engage the problems. (Junarti et al., 2020) found the causes of the learning difficulties were due to cognitive hindrances. It means the differences in perceptions and thinking activities in solving problems are probably caused by the cognitive style differences.

Cognitive style is defined as a stable dimension to describe the individual consistency to process information from various tasks. It represents the relatively stable heuristic of an individual to process the information from his environment (Singer et al., 2017). The cognitive style also refers to the preferred method to select, understand, and process new information based on reality. It shows the surrounding conceptual fields influence the individual perception (Cataloglu & Ates, 2014). According to (Chen et al., 2019; Masalimova et al., 2019), cognitive style is grouped into two types: field-independent and field-dependent types. The field-independent cognitive style tends to promote an analytical approach. The style tends to analyze the arranged field or apply a certain structure on a field when the field is unorganized. The field-independent typed learners tend to have personal objectives and empowerment. They have excellent task mastery skills and they do not limit their learning only to their closest environment and the given materials. However, they could broaden their experience to a wider area (Khodadady & Zeynali, 2012; Motahari & Norouzi, 2015; Muhammad et al., 2015). This research was focused on the field-independent cognitive style. The reason was an individual with this style had higher spatial skills than the field-dependent cognitive style type. (Cataloglu & Ates, 2014) found that the field-independent typed learners' outcomes were higher than the field-dependent typed learners. It is important to find out the students' spatial thinking process in constructing their geometrical concept understanding correctly. The reason is - it will facilitate the lecturer to provide a learning recommendation.

The geometrical problems and the individual cognitive style differences require an excellent spatial thinking process to solve the problems. The excellent spatial thinking process could be identified with the cognitive styles. An individual's cognitive styles could be used as a reference to solve geometrical problems. There were correlations between cognitive styles and spatial skills (Boccia et al., 2017; Erkan Yazici, 2017; Hanifah et al., 2018; Nori & Giusberti, 2006).

Based on the previous findings, there had not been any theory that discussed the spatial thinking process of students based on the APOS theory to solve geometrical problems. This research reviewed to obtain the detailed processes of students' spatial thinking to solve the problems based on cognitive style, especially the field-independent type style. This research aims to analyze the students' spatial thinking process with field-independent type in constructing the new knowledge. It was used to propose a revision to learn a geometrical material based on the cognitive style.

Methodology

This is qualitative research with an exploratory method. It provided the students' opportunity to solve problems alternately until the method found the most appropriate subjects for the research objectives. The researcher explored the students until he found a subject with holistic-external representation type and holistic-internal representation type.

Research Goal

This research aims to review the students' spatial thinking skills of the field-independent students in solving geometrical problems based on the APOS theory. This study specifically analyzes students' spatial thinking processes

with the types of holistic-external representation and holistic-internal representation. The students' spatial thinking processes were traced using the APOS theoretical framework.

The Study Group and Data Collection

This research was carried out in Universitas Muria Kudus Indonesia, Mathematics Education study program, with the subjects from the fourth-semester students. There were 27 students as the subjects. They were grouped into three groups with the embedded figure test: field-dependent typed students (FD), field-independent typed students (FI), and field-intermediate typed students (N). Then, they were given spatial skill tests with materials about coordinate, line, and field.

Based on the collected data, there were 12 subjects grouped into FI students. From these 12 FI subjects, there were six persons selected by considering the answer variety, uniqueness, and communication skills to be interviewed. Then, the researcher selected two out of six students to be interviewed to discuss the occurring spatial thinking process. The reason for selecting two subjects was based on the five-indicator fulfillments. The students could meet the three elements of spatial thinking especially the differences in the applied thinking process representation types. Then, the subjects were labeled into S1 and S2. The validity and reliability of this study used triangulation of data sources. This is done by comparing the data from the assignment of spatial questions with the results of in-depth interviews to produce the truth.

Analyzing of Data

The data analysis of this research was qualitative. The researchers analyzed the written and spoken answers to obtain the spatial thinking process type of the students based on the APOS theory. The stages to analyze the data were transcribing the data, reducing the data, creating an abstraction, arranging each part of the data, grouping the data by creating coding, creating the types of thinking process based on the APOS theory, and concluding. The results of the assignment of spatial questions and in-depth interviews were analyzed using these steps to find the type of student's spatial thinking process with a field-independent type of cognitive style.

Findings / Results

Here are the research results dealing with the students' spatial thinking skills in solving the problems based on the APOS theory.

The Spatial Thinking Process of S1 Subject with Holistic-External Representation Type

The mental action structure of the S1 subject was revealed via interview. It was by reading the questions. The S1 subject promoted interiorization by identifying the problem components into spoken forms, such as a box for a cat to rest, the intention of the father and Arfa, the box for a hamster to rest owned by the little brother, the distance between the red field until the yellow field, the distance between H to ACG field, and the distance from P to line AG.

Here are the interiorization results based on the interview.

R: What did you do the first time you were given the questions?

S1: I tried to understand the given and questioned elements, sir. After that, I tried to find the solution stages.

In terms of mental process structure, S1 promoted the coordination process into a written task form. Here are the S1's coordination results in the form of written tasks.

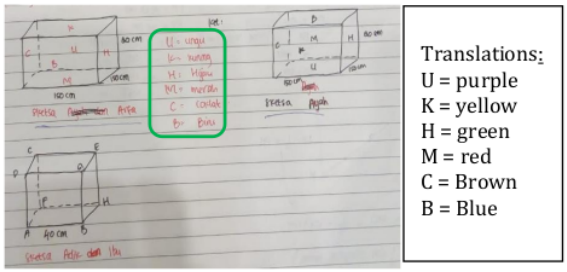


Figure 1. The S1's Coordination Results in the Forms of Written Tasks by presupposing the Box Sides

Then, S1 promoted a coordination process by interviewing. Here are the coordination results based on the interview.

R: What did you think after you obtained the information from the questions?

S1: I created a presupposition of the box and the cube sides, sir. I did it since it would make it easy to solve the questions and it was brief. By creating presupposition, it would make me easy to create the sketch. It made me easy to understand the given and questioned elements and solve the questions.

Based on Figure 1 and the interview results, S1 promoted the coordination process in the form of written tasks. He did this by creating a presupposition for the sides of the three boxes. The S1 subject started to understand and adjust the regulation to follow. Then, he put the size of the boxes by converting the units. It was important to avoid miscalculation. Then, he understood the box arrangement based on the question's demands. Therefore, he could obtain the boxes for the cat and hamster. It showed that S1 did spatial thinking by understanding the concept correctly.

Dealing with the mental process, S1 promoted the coordination process in the form of the written tasks by sketching the two-dimensional figure that contained two lines or requested distances. S1 drew the rectangle that had two triangles and a triangle to determine the distance to the line while determining the distance of the point to the field. It showed that S1 did spatial thinking by applying external representation correctly.

S1 did a reversal process during his mental structure process. It was done by remembering the formula, Pythagoras theorem, and formula of comparing two triangles. Both formulas were re-coordinated with the interiorized components. Here are the S1's reversal results in the form of written tasks

Figure 2. The S1's Reversal Results in the Form of Written Tasks by Remembering the Pythagoras Theorem Formula

The reversal process was continued. It could be traced via interview.

R: What did you think to determine the distance from P point to AG line?

S1: I used the Pythagoras theorem formula to find the distances. The theorem was used to find the sides of the connecting liens with the P point to the line, AG. The Pythagoras theorem says the square of the hypotenuse is equal to the sum of the right side squares.

Based on Figure 2 and the interview results, S1 promoted the reversal process in the form of written tasks. He did this by using the Pythagoras theorem. S1 subject made a right-triangle with lines that were the distances. Then, he applied the theorem on some right-triangles until he found the line length that was the distance of point P toward line AG. It meant that S1 remembered the theorem and he could re-coordinate with the interiorized components, dealing with the sketched figures. He could also determine the line length that was the distance.

S1 could promote the encapsulation by bringing the process into objects in the form of written and spoken tasks during the object mental structure stage. The encapsulation was in the form of written tasks. He calculated the distance between two fields by determining a line as the distance. From the sketch of the father and Arfa's box figures, the positions of the red and yellow plywoods were parallel but they had different sizes. The distance of the father's box was 1 m while Arfa's box was 80 cm. Thus, the shortest distance was Arfa's box. Here are the S1's encapsulation results in the form of written tasks

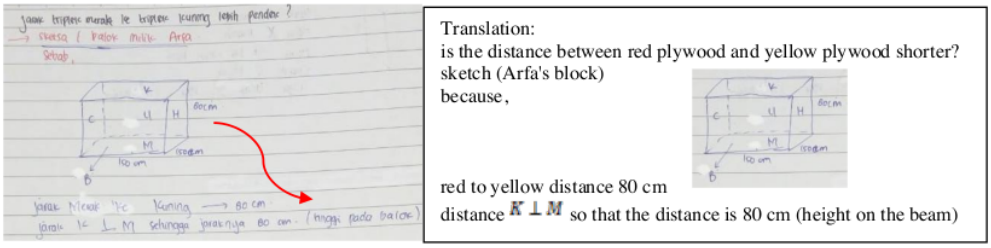


Figure 3. The Encapsulation Result of S1 in the Form of Written Tasks in Determining the Distances between the Fields

Here are the mechanisms of mental encapsulation based on the interview results.

R: In your opinion, whose cat box had the shortest distance from the red and yellow plywoods? Please tell!

S1: The distance of the red to yellow plywoods of Arfa's box was as high as the bar, 80 cm. The distance of the red to yellow plywoods of father's box was 150 cm as the width of the bar, 150 cm. Therefore, the shortest distance was found in Arfa's box, from the red to the yellow plywood distance.

The object mental structure of S1 showed the de-encapsulation process. S1 elaborated or deconstructed the previous knowledge about the distances of a certain point toward a field and the distances of a point toward a line. Then, he obtained an understanding that the distance was the line length from the point and it was perpendicular on the field. The point was also perpendicular to the line. S1 had a mistake at the beginning while he was applying the line length that was the distance (including the distance of a point toward the line). After elaborating on the two-dimensional figure, the obtained line length was not accurate. It was because he assumed the triangle was a right triangle. Thus, S1 finished it by applying the formula of the two-triangle congruence. S1 noticed that the main triangle was not a right-triangle made up from the connection of the three-obtained sides. Here are the S1's de-encapsulation results in the form of written tasks

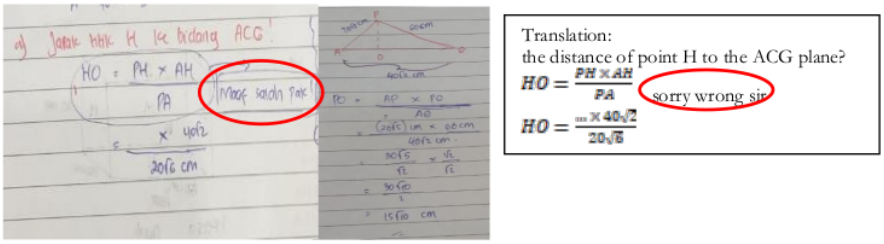


Figure 4. The De-encapsulation Result of S1 in the Form of Written Tasks in Determining the Distances between the Fields

Here are the mechanisms of mental de-encapsulation based on the interview results.

R: What did you think to determine the distance from H point to ACG field and the P point to AG line? Please tell!

S1: The distance was the perpendicular line toward the field or the line. The distance of P to the line AG was obtained by the formula of two-triangle congruence.

R: Are you sure?

S1: Hmmm. I see. The big triangle was not a right triangle. It does not meet the Pythagoras requirements. Thus, it could not be applied with a congruency feature to solve it. Sorry, sir. I did a mistake.

Based on Figure 5 and the interview results, S1 promoted re-coordination in the form of written and spoken tasks. The coordination in the form of written tasks was done by connecting the interiorized components. S1 created a two-dimensional sketch to create the requested distance. The triangle was any triangle because the sides were unknown. The requested distance was the height of the triangle. The subject made a presupposition from the base and created a theorem concept of Pythagoras from those two right-triangles inside of the any-triangle. From the presupposition, the length of the triangle base until the height of the triangle (the requested length) could be obtained. It showed that S1 did spatial thinking by applying external representation correctly. Here are the coordination results in the form of written tasks.

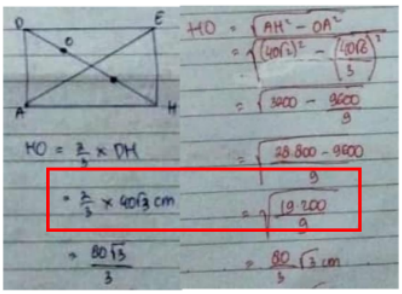


Figure 5. The S1's Coordination Results in the Form of Written Tasks by Comparing the Two Right-Triangles

Here are the investigated mental mechanisms of coordination based on the interview results.

R: What did you think to determine the distance from H point to ACG field? Please tell!

S1: I found the distance by looking at the H point. It had to be perpendicular to the part of the ACG field. From the figure, I thought to create a sketch. It would facilitate me to count it. Then, the solution could be done by comparing the two triangles or comparing the space diagonal of the property.

Based on Figure 6 and the interview, S1 could promote the encapsulation by bringing the process into objects in the form of written and spoken tasks during the object mental structure stage. The encapsulation was in the form of written tasks. He calculated the distance of a point toward the field by determining a line as the distance. S1 could obtain the length that was the distance of the comparison results of the two triangles with the same height. The height was the length of the requested distance. It showed that S1 did spatial thinking by a correct reasoning process. Here are the S1's encapsulation results in the form of written tasks

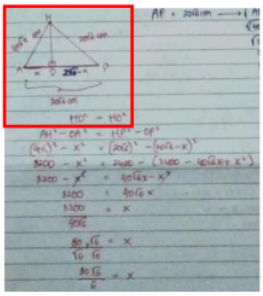


Figure 6. The S1's Encapsulation Results in the Form of Written Tasks by Comparing the Two Right-Triangles and Comparing the Spatial Diagonals

Here are the mechanisms of mental encapsulation based on the interview results.

> *R: What did you think to determine the distance from H point to ACG field?*
>
> *S1: I used two methods to find it. First, I compared the two triangles. They had the same heights. I obtained the distance of point H to the ACG field after equalizing both triangles' heights. I did it by directly comparing the spatial diagonals in which the distance is $\frac{2}{3}$ from the space diagonal. The second method was to check my first answer.*

In the mental structure of the schema, S1 did thematization to orally reveal and generalize the segments that were the distances from the field to another field, from a point to a field, and from a point to a line. The subject drew a sketch then determined the segment that was the distance. The shortest segment from a point, a line, or a field was obtained by drawing a perpendicular line toward the point, line, and the intended field. The segment was re-described in a two-dimensional figure in the form of a triangle. From the Pythagoras theorem concept and comparison of two triangles, S1 could determine the length of the segment that was the requested distance accurately. It showed that S1 did spatial thinking by a correct reasoning process. Here are the oral investigation results via an interview with S1.

> *R: After you estimated the shortest distance, did you know the concept of the shortest distance? Please tell me!*
>
> *S1: In my opinion, the concept of the shortest distance of a point, line, or field on geometry is obtained by drawing a perpendicular line toward the intended line, point, or field.*

It showed that S1 did a spoken thematization by creating a generalization. Here is the spatial thinking process analysis by promoting the spatial concept, reasoning process, and external representation correctly to solve the problems based on the APOS theory shown in Figure 7.

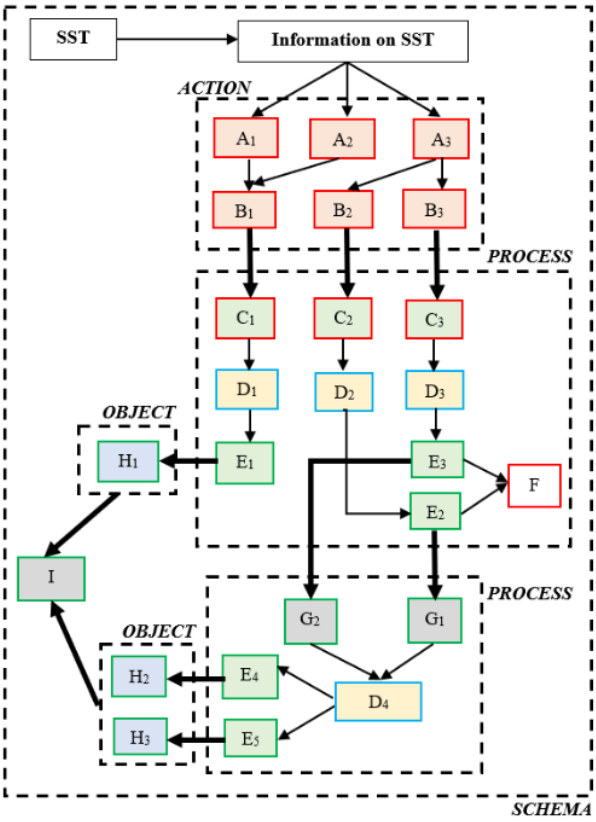


Figure 7. The Analysis of S1's Spatial Thinking Process with Holistic-External Representation Type

Table 1. The Remarks of the Notes in Figure 7

Graphic Codes	Remarks	Graphic Codes	Remarks
SST	Spatial Skill Task	F	Remembering Pythagoras Formula
A_1	Identifying the father's box	G_1	Elaborating and deconstructing the knowledge about a distance of a point toward a field.
A_2	Identifying Arfa's box	G_2	Elaborating and deconstructing the knowledge about a distance of a point toward a line.
A_3	Identifying the brother's box	H_1	Determining the distance of a field to a field.
B_1	Identifying the distance of red toward yellow plywoods	H_2	Determining the distance of a point to a field.
B_2	Identifying the distance of point H to ACG field.	H_3	Determining the distance of a point to a line
B_3	Identifying the distance of point P to AG line	I	The distance is the shortest segment of a point, line, or field on a geometry that is obtained by drawing a perpendicular line toward the intended line, point, or field.
C_1	Creating a presupposition of the father's box content		Spatial Skill Task
C_2	Creating a presupposition of Arfa's box content		The spatial concept
C_3	Creating a presupposition of brother's box content		The representation tools

Table 1. Continued

Graphic Codes	Remarks	Graphic Codes	Remarks
D_1	Sketching the father's box		Reasoning process
D_2	Sketching Arfa's box		Observing the geometrical object from various points of view
D_3	Sketching the brother's box		Imagining the geometrical object from the internal part changes
D_4	Sketching a triangle that contained the distance		Rotating the geometrical object
E_1	Connecting the lines to get the distance from the field to another field correctly		Identifying the connection of spatial realization of the geometrical object
E_2	Connecting the lines to get the distance from a point to another field incorrectly		Constructing a pattern inside of the geometrical object
E_3	Connecting the lines to get the distance from a point to another point incorrectly	→	Promoting a mental mechanism
E_4	Connecting the lines to determine the distance of a point to a field by using two right-triangles correctly	⟹	Moving to another mental structure
E_5	Connecting the lines to determine the distance of a point to a line by using two right-triangles correctly		

The Spatial Thinking Process of S2 Subject with Holistic-Internal Representation Type

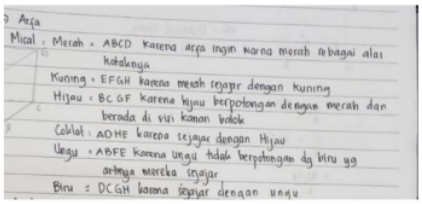
The mental action structure of the S2 subject was revealed via interview. It was by reading the questions. The subject did interiorization by identifying the problem components orally, such as the color of the six sides of a cat box demanded by the father and Arfa, the size of the box, the shape of the hamster box of the brother, and the length of the segment as the distance.

Here are the interiorization results based on the interview.

R: What did you do the first time you were given the questions?

S2: Reading and understanding the information then understanding the connection with the question to be solved.

In terms of mental process structure, S2 promoted the coordination process into a written task form. Here are the S2's coordination results in the form of written tasks.



Translation:
Arfa: Red = ABCD because Arfa wants red as the base of the box
Yellow = EFGH because red is parallel to yellow
Green = BCGF because green intersects red and is on the right side of the block
Brown = ADHE because it is parallel to green
Purple = ABFE because purple doesn't intersect with blue which means they are parallel
Blue = DCGH because it is parallel to purple

Figure 8. The S2's Coordination Results in the Forms of Written Tasks by Presupposing the Box Sides

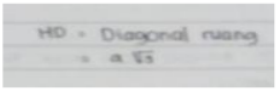
Then, S2 promoted a coordination process by interviewing. Here are the coordination results based on the interview.

R: What did you think after you obtained the information from the questions?

S2: What I thought after finding the information of the question was how to work and solve the question. It was about finding the requested distance.

Based on Figure 8 and the interview result, S2 did a coordination process in the form of written tasks. He did it by creating a presupposition about the plywood colors with the name of the rectangle, for example, "red=ABCD". The subject started to understand the question then created a presupposition of the six sides of the plywood boxes for the cat and the hamster. The subject created a presupposition of all sides without drawing the sketch completely. It showed that the subject did spatial thinking process by understanding the concept correctly and did internal representation.

S2 did a reversal process during his mental structure process. It was done by remembering the formula of space diagonal length in a cube. The formula was re-coordinated with the interiorized components. Here are the S2's reversal results in the form of written tasks



Translation:
HD = the length of the diagonal of the space
$= a\sqrt{3}$

Figure 9. The S2's Reversal Results in the Form of Written Tasks by Remembering the Space Diagonal Length Formula in a Cube

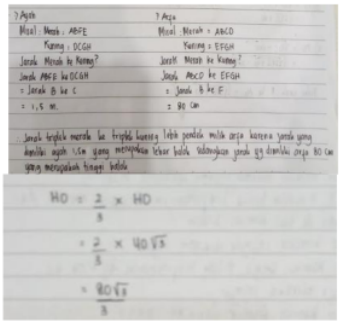
The reversal process was continued. It could be traced via interview.

R: What information did you use to determine the distance from H point to ACG field?

S2: I used the space diagonal length formula in a cube. Then, I found the distance of point H to the ACG field with a ratio of 2/3 of the space diagonal.

Based on Figure 9 and the interview results, S2 promoted the reversal process in the form of written tasks. He did this by using the space diagonal formula in a cube. The subjects applied their previous knowledge that the point H to the field ACG was $\frac{2}{3}$ of the cube-space diagonal. It meant that S2 remembered the formula and he could re-coordinate with the interiorized components. He could also determine the line length that was the distance.

S2 could promote the encapsulation by bringing the process into objects in the form of written and spoken tasks during the object mental structure stage. The encapsulation was in the form of written tasks. He calculated the distance of a field toward another field and a point to a field by determining a line as the distance. From the presupposition, algebraically, the subject could determine the distance of the field to the other field correctly. The length of the segment was the distance from point H toward the ACG field. It was also answered correctly without a complete calculation process. The subject only used the comparison process of the cube-space diagonal. Here are the S2's encapsulation results in the form of written tasks



Translation:

Father	Arfa
e.g., Red = ABFE	e.g., Red = ABCD
Yellow = DCGH	Yellow = EFGH
Red to yellow distance?	Red to yellow distance?
Distance ABFE to DCGH	Distance ABCD to EFGH
= Distance B to C	= Distance B to f
= 1.5 m	= 80 cm

The distance between the red plywood and the yellow plywood is shorter than Arfa's because the distance that Father has is 1.5 m which is the width of the beam, while the distance owned by Arfa is 80 cm which is the height of the

Figure 10. The Encapsulation Result of S2 in the Form of Written Tasks in Determining the Distances from a Point to other Field

Here are the mechanisms of mental encapsulation based on the interview results.

R: In your opinion, whose cat box had the shortest distance from the red and yellow plywoods? Please tell!

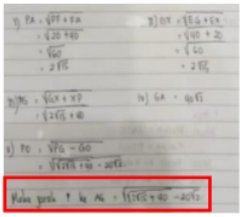
S2: Arfa's cat box had the shortest distance between the red and yellow plywoods. The distance was 80 cm while the distance of the father's box was 1.5 m.

R: In your opinion, how is the distance of point H to the ACG field?

S2: The distance was two-third of the cube space diagonal. It obtained a value of $(80\sqrt{3})/3$.

The object mental structure of S2 showed the de-encapsulation process. S2 elaborated or deconstructed the previous knowledge about the distances of a certain point toward a field. Then, he obtained an understanding that the distance was the segment length from the point and it was perpendicular on the field. The point was also perpendicular to the line. The subject had answered completely without sketching. However, he had a mistake in his Pythagoras formula. The subject wrote the formula without squaring the sides so the obtained result did not make sense. S2 thought again

about his mistake so he could find the mistake and redo to apply the formula. Here are the S2's de-encapsulation results in the form of written tasks



Translation:
Then the distance P to AG =
$\sqrt{\sqrt{2\sqrt{15}+40}-20\sqrt{2}}$

Figure 11. The De-encapsulation Result of S2 in the Form of Written Tasks in Determining the Distances from a Point to a Line

Here are the mechanisms of mental de-encapsulation based on the interview results.

R: In your opinion, how is the distance of point P to the ACG line?

S2: the distance of point P to the AG line was $\sqrt{(\sqrt{(2\sqrt{15+40})-20\sqrt{2}}}$. It was based on the estimation that point P was perpendicular to the AG line and located in the middle of the AG line.

R: Are you sure?

S2: Wait a moment, sir. How did it turn so? I see. I did a mistake, sir. I had not squared the values in this Pythagoras formula.

Based on Figure 12 and the interview results, S2 promoted re-coordination in the form of written and spoken tasks. The re-coordination of the subject was done by reconnecting the lines that were the distance between the point and the line. He did it by applying the Pythagoras theorem accurately. The subject did reversal by remembering the Pythagoras theorem formula because he incorrectly wrote the formula. The subject did not re-draw but directly calculated the distance of the point toward the line. He did it by calculating the segment lengths that corresponded with the segment lines that were the distance. It showed that S2 did spatial thinking by applying the internal representation correctly. He thought of the ideas and connected the problem components with the previous knowledge. Thus, he could arrange the problem-solving strategy. The internal representation could not be observed visually. It could not also be assessed directly because it was an individual's mental activity to think. Here are the coordination results in the form of written tasks.

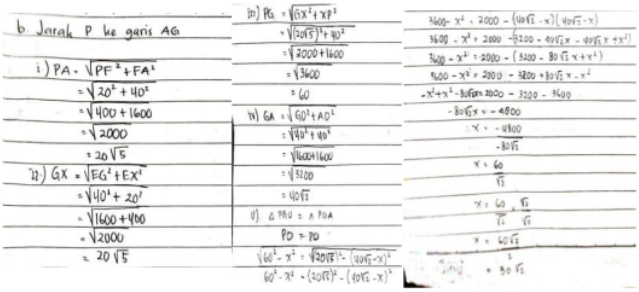


Figure 12. The Coordination Result of S2 in the Form of Written Tasks in Determining the Distances from a Point to a Line

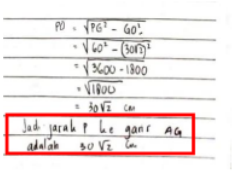
Here is the investigated coordination that contained internal representation based on the interview results.

R: What did you think to determine the distance from P point to AG line? Please tell!

S2: I wrote again the solution stages of what I did previously because I did a mistake while writing the formula. I did it by comparing the two-right triangles on a triangle that contained two segments that were the distance. It was by creating a presupposition of the base distribution. After obtaining the length of a part of the base, the triangle height could be found. The height of the triangle was the distance of point P toward the AG line.

Based on Figure 13 and the interview, S2 could promote the encapsulation by bringing the process into objects in the form of written and spoken tasks during the object mental structure stage. The encapsulation was in the form of written tasks. He calculated the distance between the point toward the line by determining a line as the distance. S2 could obtain the length that was the distance of the comparison results of the two triangles with the same height. The

height was the length of the requested distance. It showed that S2 did spatial thinking by a correct reasoning process. Here are the S2's encapsulation results in the form of written tasks



Translation:
So the distance P to the line AG is
$30\sqrt{2}$ cm

Figure 13. The S2's Encapsulation Results in the Form of Written Tasks by Comparing the Two Right-Triangles

Here are the investigated coordination based on the interview results.

R: What did you think to determine the distance from H point to AG line?

S2: The distance of point P toward the AG line was found by the assistance of a triangle. It was done by combining the points P, A, and G. Then, I found the distance of the points, from P to A, P to G, and G to A. then, after finding the distances, the distance of point P to line AG was done with the PAG triangle.

In the mental structure of the schema, S2 did thematization to orally reveal and generalize the segments that were the distances from the field to another field, from a point to a field, and from a point to a line. The subject could create the presupposition algebraically without drawing in detail. He directly created the lines that were the distances of geometry in general. Then, he determined the distance by finding a line crossing the path and minimized the path construction. The Pythagoras theorem concept and comparison of two triangles could determine the distance. Thus, S2 could determine the length of the segment that was the requested distance accurately. It showed that S2 did spatial thinking by a correct reasoning process. Here are the oral investigation results via an interview with S2.

R: After you estimated the shortest distance, did you know the concept of the shortest distance? Please tell me!

S2: The shortest distance concept was to find the distance that crossed the path by minimizing the path construction. It was commonly known as the shortest distance.

It showed that S2 did a spoken thematization by creating a generalization. Here is the spatial thinking process analysis by promoting the spatial concept, reasoning process, and external representation correctly to solve the problems based on the APOS theory shown in Figure 14.

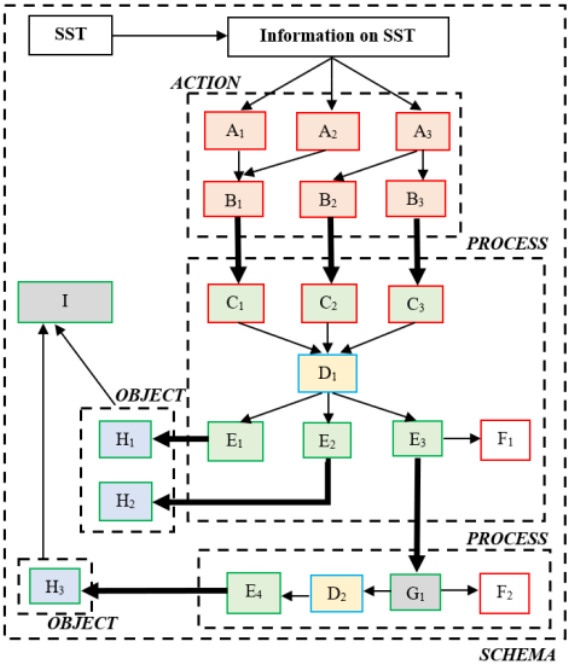


Figure 14. The Analysis of S2's Spatial Thinking Process with Holistic-Internal Representation Type

Table 2. The Remarks of the Notes in Figure 14

Graphic Codes	Remarks	Graphic Codes	Remarks
SST	Spatial Skill Task	F_2	Remembering Pythagoras Formula
A_1	Identifying the size and shape of the father's box	G	Elaborating and deconstructing the knowledge about a distance of a point toward a line.
A_2	Identifying the size and shape of Arfa's box	H_1	Determining the distance of a field to a field.
A_3	Identifying the size and shape of the brother's box	H_2	Determining the distance of a point to a field.
B_1	Identifying the distance of red toward yellow plywoods	H_3	Determining the distance of a point to a line
B_2	Identifying the distance of point H to ACG field.	I	Determining the distance by finding a line crossing the path and minimized the path construction
B_3	Identifying the distance of point P to AG line		Spatial Skill Task
C_1	Creating a presupposition of the father's box content		The spatial concept
C_2	Creating a presupposition of Arfa's box content		The representation tools
C_3	Creating a presupposition of brother's box content		Reasoning process
D_1	Thinking the ideas and connecting the background knowledge to find the distance		Observing the geometrical object from various points of view
D_2	Thinking the ideas and connecting the background knowledge to find the distance by applying two right-triangles		Imagining the geometrical object from the internal part changes
E_1	Connecting the lines to get the distance from the field to another field correctly		Rotating the geometrical object
E_2	Connecting the lines to get the distance from the field to another field correctly		Identifying the connection of spatial realization of the geometrical object
E_3	Connecting the lines to get the distance from a point to another point incorrectly		Constructing a pattern inside of the geometrical object
E_4	Connecting the lines to determine the distance of a point to a field by using two right-triangles correctly		Promoting a mental mechanism
F_1	Remembering the formula of a cube-space diagonal		Moving to another mental structure

Discussion

The students had holistic-external and holistic-internal representation types in terms of action mental structures. It was found after interviewing the students about the given questions and their written tasks by writing the problem components of the question. (Arnon et al., 2014) argue that an external action has explicit and integrated external transformation stages. They are guided by external instructions. The subjects did the mental mechanism of interiorization by identifying the problems to find the solution. It was done by identifying the box forms of Ayah, Arfa, and the brother; the distance from a field to another field, a point to a field, and a point to a line. Interiorization is a mechanism that allows the mental shift (Arnon et al., 2014). In this second stage, the subject only met one indicator, the spatial skill by imagining the geometry object of the internal part changes. Thus, it meant they had imagined the intended geometry object. The second subject could meet the spatial concept of spatial thinking element accurately. It meant he could use the knowledge to identify problems, arrange problems, finding answers, and communicating the solution. (Michel & Hof, 2013) argue that spatial concept was the conceptual and analytical frameworks in which the data could be integrated, connected, and arranged to be a complete problem.

In the mental structure of the process, the holistic-external representation type coordinated by creating a presupposition of a box side demanded by the father and Arfa with color and separating one of the box sides demanded by the brother with four corner points. Then, he determined the sketch of the boxes demanded by the father, Arfa, and the brother. The sketch result was processed in terms of the connecting lines. They were the distances between one field to another field, from a point to the field, and the point to the line. The holistic-internal representation type

coordinated by creating a presupposition of a box side demanded by the father and Arfa with color and separating one of the box sides demanded by the brother with four corner points. By only thinking about the idea, the subject promoted coordination by processing the line connections. They were the distances between one field to another field, from a point to the field, and the point to the line.

In this mental coordination stage, the coordination, both subjects met three spatial thinking indicators. They were identifying the connection of spatial realization of the geometrical object; rotating the geometrical object, and constructing a pattern inside of the geometrical object. Excellent spatial thinking could improve the visualization skills of the students toward geometrical objects (Patahuddin et al., 2020). The coordination mechanism is required to construct several objects (Arnon et al., 2014). The holistic-external representation typed students interpreted by creating sketches to facilitate them finding the distance. The students drew the geometry and the segments that were the distances correctly. The holistic-internal representation typed students represented by thinking the ideas and connecting the knowledge to solve problems without sketching. Representation was used to facilitate learners to remember, understand, reason, and communicate objects and the connections among objects that are represented in a space (Zwartjes, 2018).

While promoting the mental process structure, both subjects met the spatial elements. They were the spatial concept, the representation tools, and the reasoning process. In this case, the holistic-external representation students promoted the external representation. It was meant to facilitate solving the abstract question and make it into the concrete matter by drawing the sketch. The students drew the two-dimensional figures to explain the three-dimensional figures. It meant they could rotate the objects to explain the structures and the functions. (Fiantika, 2016) argue that external representation covers figures, sounds, and movements to describe and communicate an object. The external representation could be observed concretely in the form of Cartesian coordinate, diagram, figure, algebraic expression, and real-number line (Aziz & Kurniasih, 2019). The use of external representation could facilitate students to solve problems and prevent misunderstanding. Thus, the external representation was important for the spatial thinking process (Handhika et al., 2015; Ijirana & Mansyur, 2020; Zhang & Wang, 2005). On the other hand, the holistic-internal representation students showed the internal representation process. It could be seen from the produced written tasks. He did it without sketching. Based on the interview, they expressed the ideas by thinking and connecting the ideas with the previous knowledge. Thus, they could get the correct answers. The students did process by thinking and reasoning. (Crollen et al., 2017) found that previous knowledge influenced an individual's internal representation. According to (Fiantika, 2021), internal representation is a mental process inside of the human mind. An individual with the internal representation understood something from a point of view. This person then connected several visual data to solve the problems (Atit et al., 2020).

Both subjects did a reversal mental mechanism. It was found from the produced written tasks and the interview results. It showed that both subjects recalled the previous knowledge about Pythagoras theorem formula for the right triangles. The formula was used to process the line connections. They were the distances between one field to another field, from a point to the field, and the point to the line. It was applied to determine the segment length that was the distance. It meant the formula was important to promote the next thinking stage. Besides remembering the Pythagoras formula, the holistic-internal representation learners also recalled the space diagonal formula and its comparison. It was used to work on the questions with internal representation. The reversal mental mechanism supported and was needed to create a new process to solve problems (Arnon et al., 2014).

Both subjects did encapsulation by determining the segment lengths that were the distances between the yellow and red plywood fields as demanded by the father and Arfa during the object mental structure stage. Thus, they could determine the shortest distance that was demanded by Arfa. The holistic-internal representation students also encapsulated by determining the segment line that was the distance of point H to ACG field. He also compared the space diagonal. The encapsulation occurred when an individual applied the action into the process. It was by looking at the dynamic structure process as the static structure in which the action was applied (Arnon et al., 2014). While encapsulating, both subjects met two indicators of spatial thinking skills. They were observing the geometrical object from various points of view and reasoning process. The reasoning process provided ways to interpret, manipulate, and explain the information structurally (National Research Council, 2005). The results showed that the reasoning process correlated the abstraction, numerical scheme, and integrated geometry. They led to the success of thinking the geometrical problems (Seah & Horne, 2020). The holistic-internal representation typed students could determine the distance of a field to a field accurately. However, they could not accurately determine the distance of a point to a field and a point to a line. Thus, they did the de-encapsulation. On the other hand, the holistic-external representation students could directly determine the distance of a field to another field and a point to a field. However, they could not accurately determine the distance of a point to a line. Thus, the student conducted the de-encapsulation.

In the mental mechanism, the holistic-external representation typed students de-encapsulated by elaborating and deconstructing their previous knowledge about the distance of a point to a field or the distance of a point to a line based on one of the obtained results. The students assumed that the triangle containing the segments were the right-triangles. After obtaining all sides, the triangles did not meet the Pythagoras theorem. On the other hand, the holistic-internal representation typed students de-encapsulated by elaborating and deconstructing their previous knowledge about the

distance of a point to a line with the incorrectly obtained results. He did a mistake to write the Pythagoras formula. He was not squaring all sides so the result did not make sense. It showed that by applying de-encapsulation, an individual could return to a process that showed off the object (Arnon et al., 2014).

The holistic-external representation typed students re-structured the mental process. It was by promoting the mental coordination mechanism. The student sketched more specifically by drawing the any-triangle with a height that was the requested distance. The student created a presupposition that there were two right-triangles inside the any-triangle. Thus, the students could compare by applying the Pythagoras theorem formula. Then, the student re-encapsulated by determining the line length that was the distance from a point to a field and a point to a line. He did it by comparing the two triangles with the height was the correct distance. The holistic-internal representation students structured the mental process. He did by promoting mental coordination mechanism by thinking and connecting the ideas with the previous knowledge. Thus, he could complete the task by writing the corrected formula. Then, the students encapsulated by determining the line length that was the distance of a point to a line correctly. The students could process accurately but the writing of the formula still had a mistake. Thus, he had to revise the writing.

Both students' types had met the indicators of identifying the connection of spatial realization of the geometrical object, constructing a pattern inside of the geometrical object, and rotating the geometrical object. The differences of the holistic-external representation typed student met the spatial thinking process of reasoning and external representation. The student, without identifying the problems, directly expressed the ideas into figures; promoted the analysis, and wrote the answer solution correctly. The differences of the holistic-internal representation typed student met the spatial thinking process of spatial concept, reasoning process, and internal representation. The student recalled then thought about the ideas without drawing the sketch. Then, he analyzed and determine the correct solution answer.

The mental scheme structure of the holistic-external representation student was shown while he was doing the thematization. He generalized the segment that was the line by drawing the sketch. Then, he determined the segment that was the distance. It was the shortest distance from a line, point, or field obtained by drawing the line perpendicularly from the intended point, line, or field. The holistic-internal representation students did thematization by generalizing the shortest distance concept. It was by determining the distance by finding a line crossing the path and minimized the path construction. The construction scheme functioned as the achieved object by applying a mental mechanism, thematization. Thus, this mechanism allowed students to apply the transformation into the mental scheme structure (Arnon et al., 2014).

The mental structure scheme of both student types met all spatial thinking indicators. They were observing the geometric object from various perspectives, imagining the object from the internal-part changes, rotating the object, identifying the spatial realization connection of the object, and constructing the pattern in a space based on the geometrical object. It meant both types met the spatial indicators completely or holistically. Dealing with the spatial thinking elements, both students met the three elements of spatial thinking skills. They were spatial concept, representation, and reasoning process. The differences were the holistic-external representation typed students did external representations while the holistic-internal representation typed students did the interpretation internally.

Both types were included in the field-dependent cognitive style. They had excellent language mastery. The style tends to understand a matter globally. Students with this style are interested in interacting with other people. They prefer to be guided, to rely on external references, and to depend on the given materials previously while learning. It meant the students did the spatial thinking process completely or holistically by applying five spatial thinking skill indicators and three spatial elements. The holistic-external representation typed students did external interpretation by mastering the language excellently and interpreted the questions into detailed sketches, from the geometry until the two-dimensional figure with the questioned distance. Thus, he could solve the problems immediately and completely. On the other hand, the holistic-internal representation typed students did the interpretation internally by solving the problems with mental activity. The students only thought and connected the previous knowledge into symbols and numbers. Thus, they could find the results although they were not complete.

Thus, both types had different spatial thinking processes, but they had the same results. It was noticed that the holistic-internal representation typed student was less complete than the holistic-external representation typed student. The field-independent students were strongly correlated to the various cognitive process that required cognitive re-structure. It was to create a stable mental representation environment and to influence spatial skills (Boccia et al., 2017). An individual with a field-independent cognitive style could separate an element from the whole unit. Thus, he could arrange new information completely (Hanifah et al., 2018). An individual with a field-independent cognitive style tended to analyze, elaborate, and differ the components. However, they still did it based on the relevant knowledge they had (Naurzalina et al., 2015). The cognitive styles would provide importantly portrays for lecturers about how students obtained and managed the information (Chasanah et al., 2020).

Conclusion

Based on the results and discussion, it could be firstly concluded that spatial thinking process with holistic-external representation typed students could solve the spatial problems completely. It was done by meeting all five indicators of

spatial thinking indicators and three elements of spatial thinking with holistic-external representation type. The holistic-external representation type was indicated by revealing the abstract matter into figures. Thus, it could be communicated and solved completely and correctly. The holistic-external representation could realize the forms mentally. They could be observed and interpreted as a structured system. Secondly, the holistic-internal representation typed students could solve the spatial problems completely. It was done by meeting all five indicators of spatial thinking indicators and three elements of spatial thinking with holistic-internal representation type. The holistic internal representation was indicated by thinking and connecting the ideas and the previous knowledge with the abstract problems without being drawn. The student could imagine the problems so he could answer although it was not complete it was correct. This type required identification, analysis, and interpretation stages. The students' thinking process in solving the problems was done with the APOS theory. This theory could construct a concept to be revealed in an oral and spoken task.

Recommendations

Based on the research results, there are several recommendations. First, lecturers should always control the construction process of students' spatial thinking, this is intended to minimize student errors in uncovering geometric problems in spatial thinking. Second, to realize the spatial thinking process correctly and completely, a learning approach that is oriented towards the elements of spatial thinking external representation should be carried out. Third, further researchers can use the APOS theory to explore various students' mathematical thinking processes so that the flow of the thinking process will be obtained correctly and completely.

Limitations

This research is limited to subjects who have a field-independent type of cognitive style and spatial thinking in solving geometry problems traced based on APOS theory. Thus, it is still very open to conduct research related to spatial thinking in the form of other problems with various course characteristics, such as field-dependent type cognitive style, neutral type cognitive style, and learning style. Spatial thought processes can also be explored using the theory of assimilation and accommodation.

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Authorship Contribution Statement

Bintoro: Concept planner, designer, analyst, and scriptwriter. Sukestiyarno: Reviewer, supervisor, and validator. Mulyono: Reviewer, script critical reviser. Walid: Reviewer, the data analysis/interpreter

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