The dynamics of the basic model for the zika epidemic

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Abstract. Zika disease began to get the world spotlight when in 2016 around 1-1.5 million people were infected with the Zika virus in Brazil and 4000 babies were born Microcephaly. Indonesia is a region that is potentially attacked by Zika virus outbreaks. This study discusses the mathematical model for the spread of Zika disease in humans is associated with the population of spreading vectors which in this case are mosquitoes. The model used is a deterministic model in the form of a compartment model in sub-population classes. The purpose of this research is to build a mathematical model, analyze the stability point, and arrange a simulation of a mathematical model.

1. Introduction

Zika disease is an infectious disease caused by the Zika virus which is derived from a type of flavivirus that is similar to the dengue virus. This virus is transmitted through the bite of the Aedes sp. Zika virus (ZIKV) is originally discovered in a captive monkey living in the Zika Forest of Uganda, Africa, in 1947 [1]. Symptoms arising from this viral infection include fever, freckled skin, headaches, joint pain, muscle aches, and inflammation of the conjunctiva. Symptoms of this disease appear for 2 to 7 days [2]. The Zika virus became a worldwide concern after Brazilian health authorities discovered a link between pregnant women infected with the Zika virus and the birth of a microcephaly baby. Microcephaly is a condition in which babies have small heads and incomplete brain development [3]. In addition, the Zika virus is indicated to cause Guillain-Barre syndrome which is an acute inflammation that can cause nerve cell damage.

The spread of the Zika virus was first identified in Uganda [4]. The Zika virus was reported to have spread in Yap, Micronesia and French Polynesia in 2007 [5-6]. In October 2013, the Zika virus spread in Polynesia, France [7]. Early in 2015, the Zika virus was reportedly detected in Brazilian territory [8]. In 2016 the Zika virus has also spread in Singapore.

In Indonesia, the Eijkman Molecular Biology Institute has reported the presence of the Zika virus to the Ministry of Health. The agency noted that there were five cases of the Zika virus in Indonesia, namely: (1) in 1981 there was reported one patient at Tegalyoso Hospital, Klaten, Central Java; (2) in 1983 there were reported six out of 71 samples in Lombok, NTB; (3) in 2013 it was reported that a female tourist from Australia tested positive for the Zika virus after nine days living in Jakarta; (4) in 2015 it was reported that a tourist from Australia was infected with the Zika virus after being bitten by a monkey in Bali; and (5) in 2015-2016 a patient in Jambi Province tested positive for the Zika virus. The spread of the Zika virus in Indonesia is still relatively low, but the potential spread of the Zika disease needs to be watched out and needs to be anticipated. At present, a vaccine to cure the Zika virus has not been found. Treatment that can be done is still supportive such as adequate rest, consuming enough water to prevent dehydration, and taking fever or pain relievers [2].

2. Methods

The research started by doing literacy study. In this step, the fact and some assumptions from various scientific literacy are collected. After that, some assumptions added to complete the facts. The next step was building and analyzing the mathematics model. In this step, the mathematics model was constructed and analyzed to determine the equilibrium points and their stability. The next step was making simulations with parameters value got from another paper. In this paper, we did't give simulations.

3. Result and Discussion

In this paper, we used recruitment-death to the Dynamic of human and mosquito population. Transfer diagram of Zika epidemic was given in Figure 1.

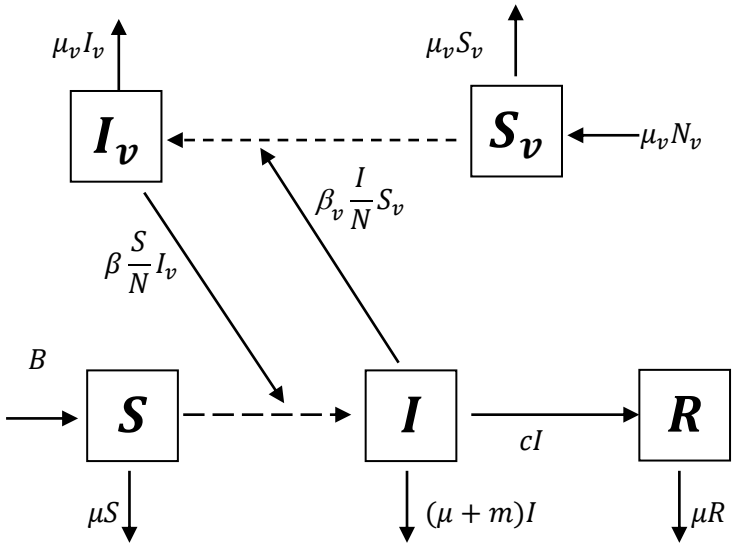

Figure 1. Transfer diagram of Zika epidemic

Where N, S, I, R respectively are the human population, the number of susceptible human, the number of infected human, and the number of recovered human. For mosquito, N_v, S_v, I_v respectively are the mosquito population, the number of susceptible mosquito, and the number of infected mosquito.

The meaning of parameters: B means recruitment rate of human population, μ means the natural death rate in human, β means the probability of virus transfer from infected mosquito to susceptible human, c means the recovery rate of infected human, q means the recovery rate of treated human, and m means the death rate caused of infected. μ_v means the birth rate which is assumed same with the natural death rate in mosquito, β_v means the probability of virus transfer from infected human to susceptible mosquito. From Fig. 1, we construct the System (1).

$$\begin{aligned}
&\frac{dS_v}{dt} = \mu_v N_v - \mu_v S_v - \beta_v \frac{I}{N} S_v \\
&\frac{dI_v}{dt} = \beta_v \frac{I}{N} S_v - \mu_v I_v \\
&S_v + I_v = N_v \\
&\frac{dS}{dt} = B - \mu S - \beta \frac{S}{N} I_v \\
&\frac{dI}{dt} = \beta \frac{S}{N} I_v - (\mu + m + c) I \\
&\frac{dR}{dt} = cI - \mu R \\
&S + I + T + R = N
\end{aligned} \tag{1}$$

From System (1), we get $\frac{dN_v}{dt} = 0 \Leftrightarrow N_v = K, K \in \mathbb{R}$ and $\frac{dN}{dt} = \frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = B - \mu N - mI$.

Hence, we get System (2).

$$\begin{aligned} \frac{dI_v}{dt} &= \beta_v \frac{I}{N}(K - I_v) - \mu_v I_v \\ \frac{dN}{dt} &= B - \mu N - mI \\ \frac{dI}{dt} &= \beta \frac{N-I-R}{N} I_v - (\mu + m + c)I \\ \frac{dR}{dt} &= cI - \mu R \end{aligned} \qquad (2)$$

Clear that $\frac{dN}{dt} = 0 \Leftrightarrow N = \frac{B-mI}{\mu} \leq \frac{B}{\mu}$.

Hence, we can determine the domain of System (2) on region

$D = \left\{(I_v, N, I, R) \in R_4^+ \mid 0 \leq I_v \leq K, 0 \leq I + T \leq N \leq \frac{B}{\mu}, N > 0\right\}$ where R_4^+ is non negative region subset R_4.

Lemma 1

1) If $\frac{\beta K}{B} > 1$ then $R_0 = \frac{\beta_v}{\mu_v} . \frac{\mu}{\mu+m+c}$.
2) If $\frac{\beta K}{B} < 1$ then $R_0 = \frac{\beta K}{B} . \frac{\beta_v}{\mu_v} . \frac{\mu}{\mu+m+c}$.

Lemma 1 will be used in determining the existence of equilibrium points such that in theorem 2.

Theorem 2

Let R_0 in Lemma 1 and $P_0 = (I_v, N, I, R) = \left(0, \frac{B}{\mu}, 0, 0\right)$ dan $P_1 = \left(\hat{I}_v, \hat{N}, \hat{I}, \hat{R}\right)$ dengan

$\hat{I}_v = \frac{\beta_v . K . \hat{I}}{\frac{\mu_v(B-m.\hat{I})}{\mu} + \beta_v . \hat{I}}$, $\hat{N} = \frac{B-m\hat{I}}{\mu}$, $\hat{R} = \frac{c\hat{I}}{\mu}$, and $\hat{I} = \frac{-a_1 - \sqrt{{a_1}^2 - 4.a_0.a_2}}{2.a_0}$

where $a_0 = {\mu_v}^2 m^2 (\mu + m + c)\left(\frac{\beta_v}{\mu_v} . \frac{\mu}{m} - 1\right)$,

$a_1 = B{\mu_v}^2 m(\mu + m + c)\left(1 - \frac{\beta K}{B} . \frac{\beta_v}{\mu_v} . \frac{\mu}{\mu+m+c}\right)$

$+B{\mu_v}^2 m(\mu + m + c)\left(1 - \frac{\beta_v}{\mu_v} . \frac{\mu}{m}\right) - \beta . \beta_v . K\mu_v . \mu(\mu + c)$, and

$a_2 = B^2 {\mu_v}^2 (\mu + m + c)\left(\frac{\beta K}{B} . \frac{\beta_v}{\mu_v} . \frac{\mu}{\mu+m+c} - 1\right)$.

1) If $R_0 < 1$ then System (2) only have one equilibrium point P_0.
2) If $R_0 > 1$ then System (2) has two equilibrium points i.e. P_0 and P_1.

Proof:

The equilibrium points were determined by making $\frac{dI_v}{dt}$, $\frac{dN}{dt}$, $\frac{dI}{dt}$, and $\frac{dR}{dt}$ in System (2) equal to zero. We got the System (3)

$$\begin{aligned} \beta_v \frac{I}{N}(K - I_v) - \mu_v I_v &= 0 \\ B - \mu N - mI &= 0 \\ \beta \frac{N-I-R}{N} I_v - (\mu + m + c)I &= 0 \\ cI - \mu R &= 0 \end{aligned} \qquad (3)$$

From the second and fourth equations, we got $N = \frac{B-mI}{\mu}$ dan $R = \frac{cI}{\mu}$. Both were substituted to the first equation then we got

$$I_v = \frac{\beta_v.K.I}{\mu_v.N + \beta_v.I} = \frac{\beta_v.K.I}{\frac{\mu_v(B-m.I)}{\mu} + \beta_v.I} = \frac{\beta_v.\mu.K.I}{\mu_v(B-m.I) + \beta_v.\mu.I}.$$

Substitute all value to the third equation, then we got

$$\frac{I}{\mu_v(B-mI)[\mu_v B + \mu.I(\beta_v - \mu_v)]}[a_0.I^2 + a_1.I + a_2] = 0 \qquad (4)$$

where

$$a_0 = {\mu_v}^2 m^2(\mu + m + c)\left(\frac{\beta_v}{\mu_v}.\frac{\mu}{m} - 1\right),$$

$$a_1 = B{\mu_v}^2 m(\mu + m + c)\left(1 - \frac{\beta K}{B}.\frac{\beta_v}{\mu_v}.\frac{\mu}{\mu + m + c}\right)$$

$$+B{\mu_v}^2 m(\mu + m + c)\left(1 - \frac{\beta_v}{\mu_v}.\frac{\mu}{m}\right) - \beta.\beta_v.K.\mu_v.\mu(\mu + c)$$

$$a_2 = B^2{\mu_v}^2(\mu + m + c)\left(\frac{\beta K}{B}.\frac{\beta_v}{\mu_v}.\frac{\mu}{\mu + m + c} - 1\right)$$

The equation (4) gave solutions $I = 0$ and $I \neq 0$.

If $I = 0$ then we get free disease equilibrium point $P_0 = (I_v, N, I, R) = \left(0, \frac{B}{\mu}, 0, 0\right)$.

For $I \neq 0$, we will find $I > 0$.

Clear that if $N > 0$ and $I \neq \frac{\mu_v B}{\mu_v m - \beta_v \mu}$, then $\mu_v(B - mI)[\mu_v B + \mu.I(\beta_v - \mu_v)] \neq 0$. Hence, to determine I in equation (4), we can solve only the equation $a_0.I^2 + a_1.I + a_2 = 0$.

By Lemma 1, we got that if $R_0 > 1$ then $a_0 > 0$, $a_1 < 0$, dan $a_2 > 0$.

Hence, $a_0.I^2 + a_1.I + a_2 = 0$ has two solutions i.e.

$I_1 = \frac{-a_1 - \sqrt{{a_1}^2 - 4.a_0.a_2}}{2.a_0}$ and $I_2 = \frac{-a_1 + \sqrt{{a_1}^2 - 4.a_0.a_2}}{2.a_0}$.

Clear that ${a_1}^2 - 4.a_0.a_2$

$$= B^2{\mu_v}^4 m^2(\mu + m + c)^2\left(\frac{\beta_v}{\mu_v}.\frac{\mu}{m} - \frac{\beta K}{B}.\frac{\beta_v}{\mu_v}.\frac{\mu}{\mu + m + c}\right)^2 + [\beta.\beta_v.K.\mu_v.\mu(\mu + c)]^2$$

$$+2.\beta.\beta_v.K.\mu_v.\mu(\mu + c).B{\mu_v}^2 m(\mu + m + c)\left[\left(\frac{\beta K}{B}.\frac{\beta_v}{\mu_v}.\frac{\mu}{\mu + m + c} - 1\right) + \left(\frac{\beta_v}{\mu_v}.\frac{\mu}{m} - 1\right)\right].$$

Hence ${a_1}^2 - 4.a_0.a_2 > 0$ if $R_0 > 1$ where R_0 in Lemma 1.

Hence $I_1 = \frac{-a_1 - \sqrt{{a_1}^2 - 4.a_0.a_2}}{2.a_0} > 0$ and $I_2 = \frac{-a_1 + \sqrt{{a_1}^2 - 4.a_0.a_2}}{2.a_0} > 0$.

We will show that $I_1 < \frac{B}{m}$.

Clear that $\frac{B}{m} - I_1 = \frac{B}{m} - \frac{-a_1 - \sqrt{{a_1}^2 - 4.a_0.a_2}}{2.a_0} = \frac{2.B.a_0 + m.a_1 + m.\sqrt{{a_1}^2 - 4.a_0.a_2}}{2.m.a_0}$.

To determine the sign of $\frac{B}{m} - I_1$, we exactly can check the sign of $2.B.a_0 + m.a_1 + m.\sqrt{{a_1}^2 - 4.a_0.a_2}$.

Clear that $2.B.a_0 + m.a_1 + m.\sqrt{{a_1}^2 - 4.a_0.a_2}$

$$= -B.{\mu_v}^2 m^2(\mu + m + c)\left(\frac{\beta K}{B}.\frac{\beta_v}{\mu_v}\frac{\mu}{\mu + m + c} - \frac{\beta_v}{\mu_v}.\frac{\mu}{m}\right) - \beta.\beta_v.K.\mu_v.\mu.m(\mu + c)$$

$$+\left[B^2.{\mu_v}^4 m^4(\mu + m + c)^2\left(\frac{\beta K}{B}.\frac{\beta_v}{\mu_v}.\frac{\mu}{\mu + m + c} - \frac{\beta_v}{\mu_v}.\frac{\mu}{m}\right)^2 + \beta^2.{\beta_v}^2.K^2.\mu_v^2.\mu^2.m^2(\mu + c)^2\right.$$

$$+2.\beta.\beta_v.K.\mu_v.\mu(\mu+c).B{\mu_v}^2m^3(\mu+m+c)\left[\left(\frac{\beta K}{B}.\frac{\beta_v}{\mu_v}.\frac{\mu}{\mu+m+c}-1\right)+\left(\frac{\beta_v}{\mu_v}.\frac{\mu}{m}-1\right)\right]\Bigg]^{\frac{1}{2}}.$$

From the calculating process above we get $2.B.a_0+m.a_1+m.\sqrt{{a_1}^2-4.a_0.a_2}>0$

So $\frac{B}{m}-I_1>0$ or $I_1<\frac{B}{m}$. So $\frac{B}{m}-I_1>0\Leftrightarrow B-mI_1>0\Leftrightarrow\frac{B-mI_1}{\mu}>0\Leftrightarrow N>0.$

Hence $I_1\in D$.

With a similar process, we can show $I_2>\frac{B}{m}$. We get $B-mI_2<0\Leftrightarrow\frac{B-mI_1}{\mu}<0\Leftrightarrow N<0.$

Hence $I_2\notin D$.

The stability theorem of the free disease equilibrium point was given in Theorem 3.

Theorem 3

Let R_0 in Lemma 1 and $P_0=(I_v,N,I,R)=\left(0,\frac{B}{\mu},0,0\right)$

1) If $\frac{\beta K}{B}<1$ and $R_0<1$ the P_0 is locally asymtotical stable.
2) if $R_0>1$ then P_0 is unstable.

The stability theorem of the endemic equilibrium point was given in Theorem 4.

Theorem 4

Let R_0 in Lemma 1 and $P_1=(I_v,N,I,R)$ was defined in Theorem 2.

if $\frac{B\mu_v}{\mu(\mu_v+\mu+c)+\mu_v(\mu+m+c)}<1$, $[(\mu+m+c)+(\mu+\mu_v)][(\mu+m+c)(\mu+\mu_v)+\mu\,\mu_v]\geq 1$, and $R_0>1$ then P_1 is locally asymptotically stable.

Proof:

The characteristic equation of jacobian matrices of P_1 is

$$\frac{(x+\mu_v)(x+\mu)}{{N_2}^3\left(\mu_vN_2+\beta_v\hat{I}\mu\right)}(c_0x^3+c_1x^2+c_2x+c_3)=0 \qquad (5)$$

Where

$N_2=B-m\hat{I}$

$c_0=c_3={N_2}^3\mu_v\left(\mu_vN_2+\beta_v\,\hat{I}\,\mu\right)$

$c_1={N_2}^3\mu_v\left(\mu_vN_2+\beta_v\,\hat{I}\,\mu\right)[(\mu+m+c)+(\mu+\mu_v)]+\beta_v\,\hat{I}\,\mu\,\mu_v\left(\mu_vN_2+\beta_v\hat{I}\mu\right)$
$+\beta\beta_v\,K\mu_v\,\hat{I}\,\mu^2{N_2}^2$

$c_2={N_2}^3\mu_v\left(\mu_vN_2+\beta_v\,\hat{I}\,\mu\right)[(\mu+m+c)(\mu+\mu_v)+\mu\,\mu_v]$
$+{N_2}^2\mu_v\,\beta_v\,\hat{I}\,\mu\left(\mu_vN_2+\beta_v\,\hat{I}\,\mu\right)(2\mu+m+c)+N_2\beta\,\mu^2\,\beta_v\,K\mu_v\,\hat{I}^2\,[m(\mu+c)+\beta_v\mu]$
$+{N_2}^2\,\beta\,\mu\,\beta_v\,K\mu_v\left[\hat{I}\,(\mu+\mu_v)(\mu+c)+\hat{I}\,\mu\,\mu_v-N_2\mu_v\right]$

From equation (5) we got two eigen values i.e. $x_1=-\mu_v<0$ and $x_2=-\mu<0$.

We will show that $c_0x^3+c_1x^2+c_2x+c_3=0$ has roots with negative real part by referring Routh Hurwitz criteria i.e.

(1) $c_0>0$, $c_1>0$, $c_2>0$, and $c_3>0$
(2) $c_1c_2-c_0c_3>0$

Clear that $c_0=c_3>0$ dan $c_1>0$.

Clear that $c_2>0$ if and only if $\hat{I}\,(\mu+\mu_v)(\mu+c)+\hat{I}\,\mu\,\mu_v-N_2\mu_v>0$.

Clear that $\hat{I}\,(\mu+\mu_v)(\mu+c)+\hat{I}\,\mu\,\mu_v-\left(B-m\hat{I}\right)\mu_v>0$

$$\Leftrightarrow \hat{I}>\frac{B\mu_v}{\mu(\mu_v+\mu+c)+\mu_v(\mu+m+c)}.$$

So $c_2 > 0$ if and only if $\hat{I} > \frac{B\mu_v}{\mu(\mu_v+\mu+c)+\mu_v(\mu+m+c)}$.
Because $\hat{I}$ means the number of infected humans then $\hat{I} > 1$ Hence $c_2 > 0$ if and only if $\frac{B\mu_v}{\mu(\mu_v+\mu+c)+\mu_v(\mu+m+c)} < 1$.
Clear that $c_1c_2 - c_0c_3 =$
$= \left(N_2{}^3\mu_v(\mu_vN_2 + \beta_v\hat{I}\,\mu)\right)^2\left[[(\mu+m+c)+(\mu+\mu_v)][(\mu+m+c)(\mu+\mu_v)+\mu\,\mu_v]-1\right]$
$+\{N_2{}^3\mu_v(\mu_vN_2+\beta_v\hat{I}\,\mu)[(\mu+m+c)(\mu+\mu_v)+\mu\,\mu_v]\}[\beta_v\hat{I}\,\mu\,\mu_v(\mu_vN_2+\beta_v\hat{I}\mu)+\beta\beta_v K\mu_v\hat{I}\,\mu^2N_2{}^2]$
$\{[N_2{}^3\mu_v(\mu_vN_2+\beta_v\hat{I}\,\mu)[(\mu+m+c)+(\mu+\mu_v)]+\beta_v\hat{I}\,\mu\,\mu_v(\mu_vN_2+\beta_v\hat{I}\mu)+\beta\beta_v K\mu_v\hat{I}\,\mu^2N_2{}^2\}$
$\{N_2{}^2\mu_v\,\beta_v\hat{I}\,\mu(\mu_vN_2+\beta_v\hat{I}\,\mu)(2\mu+m+c)+N_2\beta\,\mu^2\,\beta_v K\mu_v\hat{I}^2\,[m(\mu+c)+\beta_v\mu]$
$+N_2{}^2\,\beta\,\mu\,\beta_v K\mu_v[\hat{I}\,(\mu+\mu_v)(\mu+c)+\hat{I}\,\mu\,\mu_v - N_2\mu_v]\}$
Hence $c_1c_2 - c_0c_3 > 0$ if $[(\mu+m+c)+(\mu+\mu_v)][(\mu+m+c)(\mu+\mu_v)+\mu\,\mu_v] \geq 1$
Hence, using Routh Hurwitz criteria we can conclude that $c_0x^3 + c_1x^2 + c_2x + c_3 = 0$ has roots with a negative real part.

4. Conclusion

From the analysis above, we get the dynamic of the mathematics model of Zika epidemic. These dynamic of model are connected with Reproduction number. The reproduction number can be used to determine whether the epidemic outbreak or vanish.

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