

On perturbation method of an oscillator single degree of freedom with mass that changes periodically and forced vibrations

S B Waluya* , I Rosyida, and M Kharis

Mathematics Department, Universitas Negeri Semarang, Semarang, Indonesia

*Corresponding author: s.b.waluya@mail.unnes.ac.id

Abstract. In this paper will be studied an Oscillator with Mass that changes periodically and forced vibration of a simple degree of freedom. This system related to the dynamics of cables such as stayed bridges, electricity, telephone, and others. Under normal conditions for all type of cables one would not expect galloping type of vibrations due to wind-forces. There is an exception concerns vibrations excited by a wind-field containing raindrops. As has been observed on scale models in wind-tunnels the raindrops that hit the inclined stay cable generate one or more rivulets on the surface of the cable. It will be studied in this paper an oscillator single degree of freedom with mass that changes periodically and forced vibrations. Small masses which periodically hitting and leaving to the oscillator with different velocity give a forced to the vibration. Because the small mass stay on the oscillator surface for some time then the effective mass of the oscillator will periodically vary in time. In this paper it will be studied the possibilities of the mass and the velocity of the masses which are hitting and leaving to the oscillator by using a perturbation method.

1. Introduction

The Den Hartog's criteria should be considerate for modeling of vibration from a cable bridge. This criterion takes into account a number of assumptions that can cause a galloping phenomenon that is vibrations with small frequencies and large amplitude. (see [1, 2, 3, 4]). System of differential equation becomes a mathematical model that describes vibration of the cables in vertical direction because of the galloping phenomenon. The characteristic of cables of cable-stayed bridges usually smooth polyurethane mantle and have a cross section which is nearly circular. In the rain season the water on the cable may induce aerodynamic instability resulting in vibrations with relatively large amplitudes. The instability mechanism for this type of vibrations is known and can be understood on the basis of quasi-steady modeling and analysis. The experimental validation of a new energy harvesting system based on the wake galloping phenomenon has been studied by Jo Jung and Woo Lee [5]. Babu and Prasad [1], Van Horssen, et.al, [2], and Waluya [3, 4] have studied oscillation of bridge cable because of wind induce vibration, while mechanism and characteristic of rain-induced vibration on high-voltage transmission line have been studied by Zhou et.al[6]. The dynamic of cable stayed bridges with a time periodic damping coefficient have been studied by Van Horssen, et.al [2] and Waluya [3, 4]. It has been observed in experiments regarding wires that live tend to produce one or more rivulet on the cable surface. The pressure distribution on the cable with respect to the direction of the wind flow (uniform) can be asymmetrical, producing a lift force perpendicular to the direction of the wind speed. If there is a water ridge, it may be blown off if the wind speed exceeds the critical



value. With different speeds forced vibration small mass that periodically hits and leaves the oscillator. Due to the small mass stays on the surface of the oscillator for some time, the effective mass of the oscillator will vary in time.

Many type of the oscillators have been studied by many researchers. Afzali, et.al [7] have studied Analysis of the Periodic Damping Coefficient Equation Based on Floquet Theory, Bayat, et.al [8] have studied Non-Linear Oscillation by series solution technique. On the separation of fast and slow motions in mechanical systems has been studied by Blekhman and Sorokin [9]. Analytical solution of strongly nonlinear Duffing oscillators has been studied by El-Naggar and Ismail [10]. Mechanism and characteristic of rain-induced vibration on high-voltage transmission line have been studied by Zhou et.al [6]. Von Wantoch et. Al [11] have studied Adaptive phasor control of a Duffing oscillator with unknown parameters. A novel method for the forced vibrations of nonlinear oscillators have been studied by En Du, et.al [12]. Ismail [13] has studied Analytical Technique for Solving Nonlinear Oscillators of the Motion of a Rigid Rod Rocking Bock and Tapered Beams. Many researcher have studied a various perturbation techniques to analyze nonlinear oscillators, such as a modified homotopy analysis method [14], Floquet Theory [7], Iteration Perturbation Method [15], Multiple-Scales Lindstedt-Poincaré Method [16], a modified homotopy analysis method [14], multiple time scales perturbation [2, 3, 4], and many others perturbation technique. In this paper will be studied vibration of an oscillator due to small masses which periodically changing with slow and high frequencies. In [3, 4] used the assumption that external forces were not taken into account in the system, while in this paper will be studied that the force energy give significant impact to the vibration.

This paper is organized as follows. In section 2 of this paper it will be shown the mathematical model. The analysis of the solution will be studied in section 3 of this paper. Finally in section 4 of this paper some conclusions will be drawn and some remarks will be made.

2. Mathematical Model

It will be derived in this section a mathematical model that describe an oscillator with forced for vibration of the cable. The model oscillator is a model in one degree of freedom that arises to the second order differential system with external forced. The Model Equation for Rain-Wind Induced Vibrations of a Prototype Oscillator is closely related to the quasi-steady approach as given in [2, 4]. The oscillator system is designed so that the oscillation (single degree of freedom) is only in the vertical direction. On the surface of cylinder there is a ridge that can bring the small amplitude oscillations. A quasi-steady approach is use to model the rain-wind forces oscillations with respective assumptions are known as galloping. The mathematical model can be given by [see 2, 3, 4]

$$M\ddot{y} = \dot{M}(w - \dot{y}) - k_y y + F_y, \quad (1)$$

where $M = M(t)$ the mass of the cylinder is, $w = w(t)$ is a velocity of the raindrops which hit and leave to the cylinder, $c_y > 0$ the damping coefficient of the mechanical structure and $k_y > 0$ the linear spring constant, \dot{y} represents the velocity of the system, F_y is forcing. By Transformation and scaling then we obtain

$$\frac{d}{dt} \left(1 - B(t) \frac{dy}{dt} \right) + y = - \frac{w}{\sqrt{M_0 k}} \frac{dB}{dt} + \frac{1}{k_y} F_y, \quad (2)$$

where $B(t) = \left(\frac{m(t)}{M_0} \right)$. In the next following section will be discussed the case of $B(t)$ is a periodic function, for instance $B(t) = A \sin(\omega t)$, $w = A \cos(\omega t)$, and let $\omega_0 = \sqrt{\frac{k}{M_0}}$, $F_y = A \cos(\omega t)$. In paper [3, 4] have been used the assumption that external forces were not taken into account in the

system while this paper will be studied for cases: (i) $A = O(\epsilon)$, $\omega = O(\epsilon)$ and (ii) $A = O(\epsilon)$, $\omega = O(1)$.

3. Solution of The Model

Case $A = O(\epsilon)$, $\omega = O(\epsilon)$.

In this case let $A = A_0\epsilon$ and $\omega = \alpha\epsilon$, $A_0 = 1$, $k_y = 1$. If we take initial value problem $y(0) = y_0$, $\dot{y}(0) = \dot{y}_0$ then from the equation (2) will be obtained

$$\begin{aligned} \epsilon^2 \sin(\epsilon at) \alpha \left(\frac{d}{dt}(y(t)) \right) + (1 - \epsilon \cos(\epsilon at)) \left(\frac{d^2}{dt^2}(y(t)) \right) + y(t) \\ + \frac{\epsilon^3 \cos(\epsilon at) \sin(\epsilon at) \alpha}{\sqrt{M_0 k}} - \epsilon \cos(\epsilon at) = 0 \end{aligned} \quad (3)$$

Letting $t = t$, $\epsilon t = t_1$,

$$y(t) = y_0(t, t_1) + \epsilon y_1(t, t_1) + \epsilon^2 y_2(t, t_1) + \dots$$

Substitute the series into equation (9) and collect the terms in power series in ϵ then it will be obtain the respectively problems in power series in ϵ for order $O(1)$ problem. Substitute the series into equation (3) and collect the terms in power series in ϵ then it will be obtain the respectively problems in power series in ϵ for order $O(1)$ problem is

$$\begin{cases} \frac{\partial^2 y_0}{\partial t^2} + y_0 = 0, \\ y_0(0,0) = y_0, \frac{\partial y_0}{\partial t_1}(0,0) = \dot{y}_0, \end{cases} \quad (4)$$

Order $O(\epsilon)$ –problem the becomes

$$\begin{cases} \frac{\partial^2 y_1}{\partial t^2} + y_1 + 2 \frac{\partial^2 y_0}{\partial t \partial t_1} - \cos(\epsilon at) - A_0 \cos(\epsilon at) \frac{\partial^2 y_0}{\partial t^2} = 0, \\ y_1(0,0) = 0, \frac{\partial y_1}{\partial t}(0,0) + \frac{\partial y_0}{\partial t_1}(0,0) = 0, \end{cases} \quad (5)$$

Solution for order $O(1)$ problem equation (4) can be given by

$$y_0(t, t_1) = a_0(t_1) \sin t + b_0(t_1) \cos t, \quad (6)$$

with $a_0(0) = \dot{y}_0$, $b_0(0) = y_0$. Substituting equation (6) into equation $O(\epsilon)$ problem (5) and removing secular term, will be obtained the system differential in a_0 and b_0 . After solving the system then for the initial values $a_0(0) = 1$, $b_0(0) = 0$ then will be obtained

$$a_0(t_1) = \cos\left(\frac{\sin(\alpha t_1)}{2\alpha}\right), \quad b_0(t_1) = \sin\left(\frac{\sin(\alpha t_1)}{2\alpha}\right) \quad (7)$$

Because all the terms are linear then solution for order $O(\epsilon^2)$ will be bounded, so, the approximation solution for equation problem (8) can be given up to $O(\epsilon)$, that is

$$y(t, t_1) = a_0(t_1) \sin t + b_0(t_1) \cos t,$$

with $a_0(t_1)$ dan $b_0(t_1)$ are given in equation (7).

Plot of the solution approximation and numerical result by using Runge-Kutta method for an initial conditions and set of parameters can be given in Figure (1).

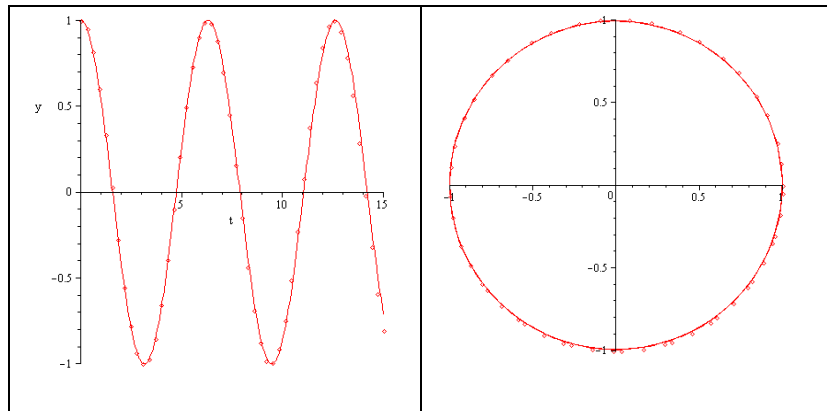


Figure 1. Plot solution and phase portrait approximation versus numeric for $A_0 = 1, \alpha = 1, \varepsilon = 0.01, y(0) = 1, \dot{y}(0) = 0$ Straight line for solution approximation and dot points for numeric.

From Figure (1) it can be seen that by using the multiple time scale perturbation method can be obtained the approximation solution of the model is almost periodic.

$$\text{Case } A = O(\varepsilon), \omega = O(1).$$

In this case let $A = A_0\varepsilon$, dan $\omega = \alpha, y(0) = y_0, \dot{y}(0) = \dot{y}_0$. Let $A_0 = 1, k_y = 1, M_0 = 1, k = 1, \alpha = 1$. By using two time multiple time scale, let $t = t, \varepsilon t = t_1$,

$$y(t) = y_0(t, t_1) + \varepsilon y_1(t, t_1) + \varepsilon^2 y_2(t, t_1) + \dots$$

Substitute the series into equation (9) and collect the terms in power series in ε then it will be obtain the respectively problems in power series in ε for order $O(1)$ problem is

$$\begin{cases} \frac{\partial^2 y_0}{\partial t^2} + y_0 = 0, \\ y_0(0,0) = y_0, \frac{\partial y_0}{\partial t_1}(0,0) = \dot{y}_0, \end{cases} \quad (8)$$

Order $O(\varepsilon)$ –problem the becomes

$$\begin{cases} \frac{\partial^2 y_1}{\partial t^2} + y_1 + 2 \frac{\partial^2 y_0}{\partial t \partial t_1} - \cos(t) - \cos(t) \frac{\partial^2 y_0}{\partial t^2} + \sin(t) \frac{\partial y_0}{\partial t} + \sin(t) \cos(t) = 0, \\ y_1(0,0) = 0, \frac{\partial y_1}{\partial t}(0,0) + \frac{\partial y_0}{\partial t_1}(0,0) = 0, \end{cases} \quad (9)$$

Solution for equation (8) problem is

$$y_0(t, t_1) = a_0(t_1) \sin t + b_0(t_1) \cos t, \quad (10)$$

with $a_0(0) = \dot{y}_0, b_0(0) = y_0$. Substitute y_0 equation (10) into equation (9) then will be obtained

$$\frac{\partial^2 y_1}{\partial t^2} + y_1 + \left(2 \frac{da_0}{dt_1} - 1\right) \cos t + \left(-2 \frac{db_0}{dt_1}\right) \sin t + a_0(t_1) \sin(2t) + b_0(t_1) \cos(2t) + \frac{1}{2} \sin(2t) = 0 \quad (11)$$

Removing secular term in equation (11), will be obtained

$$\begin{cases} 2 \frac{da_0}{dt_1} - 1 = 0, \\ -2 \frac{db_0}{dt_1} = 0. \end{cases} \quad (12)$$

Solving problem in equation (12) and will be obtained. For the initial values $a_0(0) = 1, b_0(0) = 0$ then we obtain

$$a_0(t_1) = \frac{1}{2}t_1 + 1, \quad b_0(t_1) = 0 \quad (13)$$

Because all the terms are linear then solution for order $O(\varepsilon^2)$ will be bounded, so, the approximation solution for equation problem (2) can be given up to $O(\varepsilon)$, that is

$$y(t, t_1) = \frac{1}{2}\varepsilon t \sin(t) + \sin(t)$$

Plot of the solution approximation and numerical result by using Runge-Kutta method for an initial condition and set of parameters can be given in Figure (2).

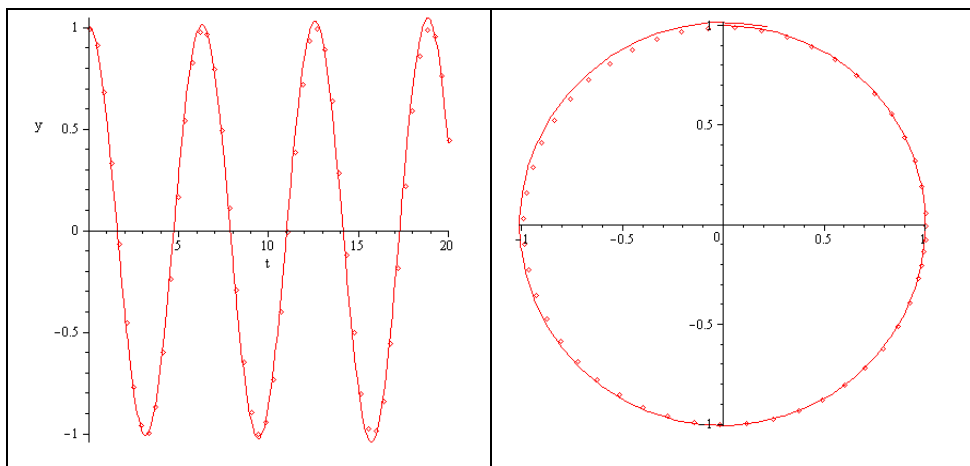


Figure 2. Plot solution and phase portrait approximation versus numeric for $A_0 = 1, \alpha = 1, \varepsilon = 0.01, y(0) = 1, \dot{y}(0) = 0$

From Figure (2) it can be seen that the approximation of the solution by using perturbation technique also similar with the solution by numerical calculation, however the solution is not periodic solution. For this case there are no periodic solutions.

4. Conclusion

In the previous section has been derived an oscillator with mass that changes periodically and forced vibration of a simple degree of freedom. This system related to the dynamics of cables from stayed

bridges. The mathematical model is a differential equation that describes a vibration of cables in vertical direction. Those vibrations are assumed in the vertical direction due to the galloping phenomena, that is vibrations with small frequencies and large amplitude. Based on the analysis can be concluded that for case $A = O(\epsilon)$, $\omega = O(\epsilon)$ using the multiple time scale perturbation method can be obtained the approximation solution of the model is almost periodic. This result is comparable with the result by Runge-Kutte technique. For the case $A = O(\epsilon)$, $\omega = O(1)$ the approximation of the solution by using perturbation technique also similar with the solution by numerical calculation, however the solution is not periodic solution. For this case there are no periodic solutions.

Acknowledgement.

This research is supported by Hibah Pengembangan Keilmuan UNNES, contract No. 47.18.4/UN37/PPK.3.1/2018, 18 April 2018.

References

- [1] Babu R and Prasad R 2017 *Int. Res. J. Eng. Technol. (IRJET)* **04**(1)
- [2] Van Horssen W T, Pischansky O V, and Dubbeldam J L A 2010 *J. Sound Vib* **329** pp 538-569.
- [3] Waluya S B 2010 *Int. J. Math, Sci, Engg, Appls, (IJMSEA)* **4**(2) pp 385-394.
- [4] Waluya S B 2012 *Int. J. Math, Sci, Engg, Appls, (IJMSEA)* **6**(1) pp 249-261.
- [5] Jo Jung H and Woo Lee S 2011 *Smart Mater, Struct.* **20**
- [6] Zhou C, Liu Y, and Rui X 2012 *J. Mech. Sci. Technol.* **26**(8) pp 2505~2510
- [7] Afzali F, Acar G D, and Feeny B F 2017 *ASME 2017 Int. Des. Eng. Tech. Conf. Comput. Inf. Eng. Conf. 8: 29th Conf. Mech. Vib. Noise Clevel. Ohio USA*, August 6–9
- [8] Bayat M, Pakar I, Barari A, Nejad M G, and Motevalli M 2014 *Obuda Univ. e-Bull.* **4**(1)
- [9] Blekhman I I and Sorokin V S 2010 *J. Sound Vib* **329** pp 4936-4949
- [10] El-Naggar A M and Ismail G M 2016 *Alex. Eng. J.* **55** pp 1581–1585
- [11] Von Wantoch T, Rock H, Koschmieder F, and Benecke W 2014 *Prepr. 19th World Congr. Int. Fed. Autom. Control Cape Town S. Afr.* August 24-29
- [12] En Du H, Kang Er G, and IU V P 2018 *5th Jt. Int. Conf. Multibody Syst. Dyn.* (Lisboa, Portugal ,June 24 – 28)
- [13] Ismail G M 2016 *J. Appl. Comput. Mech.* **2**(1) pp 29-34
- [14] Zurigat M, Momani S, and Alawneh A 2012 *Stud, Univ, Babe,s-Bolyai Math* **57**(4) pp 579–588
- [15] El-Naggar A M and Ismail G M 2014 *Inf. Sci. Lett.* **3**(3) pp 91-95
- [16] En Du H, Kang Er G, and IU V P 2017 *ENOC 2017* (Budapest, Hungary, June 25-30)