Far East Journal of Mathematical Sciences (FJMS)

Published Online: August 2016
http://dx.doi.org/10.17654/MS100050727
Volume 100, Number 5, 2016, Pages 727-746 **ISSN: 0972-0871**

A DYNAMIC S-I-P MODEL WITH DISEASE IN THE PREY POPULATION AND HOLLING TYPE II FUNCTIONAL RESPONSE

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Abstract

In this paper, a modified S-I-P of two species predator-prey interactions model is discussed. The S-I-P system consists of three differential equations that represent sub-populations growth of susceptible-preys (S), infected-preys (I) and predators (P). In prey population there is deadly disease transmission rated with Holling type II response characteristic of predator to catched prey. The analysis result showed that the system has six equilibrium points. There are always two points which are unstable in any condition. Routh-Hurwitz criteria are used to analyze the stability of the equilibrium points. Numerical simulations are carried out to demonstrate the results obtained.

Received: March 14, 2016; Accepted: May 8, 2016
2010 Mathematics Subject Classification: 34D23, 34C25, 37N25, 92D25.
Keywords and phrases: Holling type II, Routh-Hurwitz, epidemic, equilibrium, predator-prey.
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Communicated by K. K. Azad

1. Introduction

Ecology is the field of science that studies ecosystem. Food chain is an important item in ecology, which contains at least two kinds of species called predators and preys. Jorgensen [9] has studied food chain in predator-prey mathematical model. In the epidemic population when a disease occurs at a frequency higher than that is expected, it is said to be *epidemic* (see [6]). A localized epidemic may be referred to as an outbreak. Hence, in various types of mathematical models we happened to know the existence of epidemic in the population studied by Guerrant et al. [6]. Classic epidemic models split the population into the two classes: susceptible and infected. Susceptible class consists of population which can be easily infected and the infected class consists of those capable to move infection. Joydif and Sharma [10] have studied the SI model with population $N = S + I$, where S and I represent population of susceptible and infected classes, respectively.

The interaction system in the ecosystem that describes a physical phenomenon is a predator-prey interaction system, in which preys are eaten and predators are fed has been studied by Du et al. [5]. Predator-prey system is one of the kind of system that is a combination of the two populations, namely predators and preys. Interaction between these two populations is very important because the survival of species depends on the environment around them. The balance is achieved if the population of predators and preys interact according to their size and percentage (see [7]).

Predator-prey model widely used is the model that consists of two different species in which one of the two provides food to another. Predator-prey model was first introduced by Lotka in 1925 and Volterra in 1926, so that this model is called Lotka-Volterra model (see [17]). Holling in 1950 introduced the functional response for the predator-prey model. The functional response in ecology, i.e., the amount of food eaten by the predator population is a function of the density of food has been studied by Hunsicker et al. [8]. Lotka-Volterra model does not take into account the time needed by a predator to digest the food. One modification is made with the introduction of a response function in predator-prey model interaction. The amount of food eaten by predators is a function of density of food. This model has been studied by Brauer and Chaves [1]. In this paper, we use used the response function type II. In the type II response function, a predator has the characteristics to search a prey actively. Wolf is an example.

Das et al. [3] modified epidemic predator-prey model consisting of three species that are prey species, intermediate predators, and top predator in cases of epidemic diseases with the prey population used to response Holling type II. In this case, the existence of the rate of infections is responsible for the stability around the equilibrium points. Kooi et al. [11] have studied predator-prey model of two species appearing only in cases of predators in the population. The model based upon the function of a predatory behavior is hunting mechanism type II Holling response. In the analysis of the model, it was also obtained that the system was more stable with the increase of the rate of infection disease. Chattopadhyay and Arino [2] have also studied an epidemic predator-prey model consisting of two species in which predators use hunting prey following the type II Holling response function. The modified epidemic predator-prey mathematical model where there is the spread of disease in the population prey follows the law of action of the simple and hunting mechanism way predators type II Holling response function. Mathematical model consists of three equations, namely of the rate of population growth susceptible-prey, the rate of population growth infected-prey, and the rate of population growth predatory. Analysis of the model was conducted by determining equilibrium points and their stability. Numerical simulations were performed to support the results of the analysis (see [13, 14]).

This paper is organized as follows. In Section 2, the construction of mathematical model is discussed, and equilibrium points are found in Section 3. In Section 4, the stability analysis is the focal point. Numerical simulations of a modified predator-prey model within epidemic in prey population using type II Holling response function are carried out in Section 5 to demonstrate the result obtained by using Maple software. Finally, in Section 6, conclusions are drawn.

2. Mathematical Model

The mathematical model was restricted by some assumptions. The assumptions used in the S-I-P two species interaction predator-prey model with the type II Holling functional response are listed below:

1. The growth-rate of prey population has a pattern of growth logistics because of ecosystems carrying-capacity.

2. Disease only infects prey population and the infected prey cannot be cured or made immune.

3. Prey food supplies are limited, so there are competitions among prey population to obtain food.

4. Predator food supplies depend on prey population.

5. Prey population declines and population of predators increase at the time of the occurrence of preys-predators interaction.

6. In the interaction, predator only consumes infected-prey.

7. In the ecosystems, there is only a kind of prey for consumption by predator.

8. Preys respond presence of a predator, so the predator needs time to catch a prey (based on type II Holling functional response).

A modified S-I-P prey-predator model with Holling type II functional response can be described by

$$\begin{cases} \dfrac{dS}{dt} = rS\left(1 - \dfrac{S}{K}\right) - \beta SI, \\ \dfrac{dI}{dt} = rI\left(1 - \dfrac{2S + I}{K}\right) + \beta SI - \dfrac{aIP}{b + I} - cI, \\ \dfrac{dP}{dt} = \mu \dfrac{aIP}{b + I} - dP, \end{cases} \tag{2.1}$$

where $x = \frac{S}{K}$, $y = \frac{I}{K}$, $Z = \frac{P}{K}$, $T = rt > 0$, $A = \frac{a}{r} > 0$, $B = \frac{b}{K} > 0$, $C = \frac{c}{r} > 0$, $D = \frac{d}{r} > 0$, and $m = \frac{\beta K}{r} > 0$.

In the system (2.1), the parameter $S = Kx$ describes the susceptible-prey sub-population density with the carrying-capacity K, the parameter $I = Ky$ describes the infected-prey population density with carrying-capacity K, the parameter $P = Kz$ shows the predator population density with carrying-capacity K which is interacting with the prey population, $T = rt$ shows the time, $A = \frac{a}{r}$ describes the decrease in number of the prey population caused by the interaction of the prey and the predator populations, $B = \frac{b}{K}$ describes the saturation rate of the predator, $\beta = \frac{mr}{K}$ describes transmission rate of the infectious disease in the prey population, each $c = Cr$ and $d = Dr$ are natural death rates of the prey and the predator, the effort applied to harvest the prey population with the influence of the surroundings and μ describes that the growth rate of the predator population. System (2.1) can be rewritten as follows:

$$\begin{cases} \dfrac{dx}{dT} = x(1 - x^2) - mxy, \\ \dfrac{dy}{dT} = y(1 - 2x - y) + mxy - \dfrac{Ayz}{B + y} - Cy, \\ \dfrac{dz}{dT} = \mu\left(\dfrac{Ayz}{B + y}\right) - Dz, \end{cases} \tag{2.2}$$

with $x(0) > 0$, $y(0) > 0$ and $z(0) > 0$.

3. Equilibrium

Theorem 3.1. (1) *With no condition, the system* (2.2) *has two equilibrium points, namely* $E_0(0, 0, 0)$ *and* $E_1(1, 0, 0)$.

(2) *If* $C < 1$, *then the system* (2.2) *has three equilibrium points, namely* $E_0(0, 0, 0)$, $E_1(1, 0, 0)$, *and* $E_2(0, y_1, 0)$.

(3) *If* $C < 1$, *and* $m(1 - C) < 1 < 1 + C < m$, *then the system* (2.2) *has four equilibrium points, namely* $E_0(0, 0, 0)$, $E_1(1, 0, 0)$, $E_2(0, y_1, 0)$, *and* $E_3(x_2, y_2, 0)$.

(4) *If* $C < 1,$ $\mu A > D$ *and* $(1 - C)(\mu A - D) > BD,$ *then the system* (2.2) *has four equilibrium points, namely* $E_0(0, 0, 0),$ $E_1(1, 0, 0),$ $E_2(0, y_1, 0),$ *and* $E_4(0, y^\star, z_1^\star).$

(5) *If* $C < 1,$ $\mu A > D$ *and* $mBD < \mu A - D,$ *then the system* (2.2) *has four equilibrium points, namely* $E_0(0, 0, 0),$ $E_1(1, 0, 0),$ $E_2(0, y_1, 0),$ *and* $E_5(x^\star, y^\star, z_2^\star).$

Proof. To obtain the equilibrium points, we consider

$$\frac{dx}{dT} = 0, \quad \frac{dy}{dT} = 0 \quad \text{and} \quad \frac{dz}{dT} = 0.$$

From (2.2), we have

$$\begin{cases} x(1 - x^2) - mxy = 0, \\ y = (1 - 2x - y) + mxy - \dfrac{Ayz}{B + y} - Cy = 0, \\ \mu\left(\dfrac{Ayz}{B + y}\right) - Dz = 0. \end{cases} \tag{3.1}$$

From the third equation of (3.1), it follows that $\mu\left(\frac{Ayz}{B + y}\right) - Dz = 0 \Leftrightarrow z = 0 \vee y^\star = \frac{BD}{\mu A - D}.$ If $z = 0,$ from the second equation of the system (3.1), we obtain

$$y(1 - 2x - y) + mxy - \frac{Ayz}{B + y} - Cy = 0 \Leftrightarrow y = 0 \vee y = 0.$$

Substituting $y = 0$ and $z = 0$ into the first equation of the system (3.1), it follows that

$$(x - x^2) - mxy = 0 \Leftrightarrow x = 0 \vee x = 1.$$

Hence, for $y = 0$ and $z = 0,$ the equilibrium points are $E_0(0, 0, 0)$ and $E_1(1, 0, 0).$ If $y \neq 0,$ substitute $y = (m - 2)x + 1 - C$ and $z = 0$ into the first equation of the system (3.1) which provides

$$(x - x^2) - mxy = 0 \Leftrightarrow x = 0 \vee x_2 = \frac{1 - m(1 - C)}{1 + m(m - 2)} = \frac{1 - m(1 - C)}{(m - 1)^2}.$$

For $x = 0$, we obtain $y_1 = 1 - C$. It is clear that the initial condition $0 < C < 1$ implies $y_1 > 0$. Hence, if $C < 1$, then the equilibrium point is $E_2(0, y_1, 0)$.

For $x \neq 0$, substitute $x_2 = \frac{1 - m(1 - C)}{(m - 1)^2}$ into $y = (m - 2)x + 1 - C \Leftrightarrow y_2 = \frac{(m - 1 - C)}{(m - 1)^2}$. Because $y > 0$ and $C > 0$, $m > 1 + C$. Furthermore, because $x > 0$, $m(1 - C) < 1$. Hence, if $m(1 - C) < 1 < 1 + C < m$, then the equilibrium points is $E_3(x_2, y_2, 0)$.

If $z \neq 0$, substitute $y^\star = \frac{BD}{\mu A - D}$ into the second equality of the system (3.1) and obtain $y(1 - 2x - y) + mxy - \frac{Ayz}{B + y} - Cy = 0$. Because $y^\star = \frac{BD}{\mu A - D}$, $z = B\left[(1 - C) - (2 - m)x - \frac{BD}{\mu A - D}\right]\left(\frac{\mu}{\mu A - D}\right)$. Substitute $y^\star = \frac{BD}{\mu A - D}$ into the first equation of the system (3.1) to have

$$(x - x^2) - mxy = 0 \Leftrightarrow x[(1 - x) - my] = 0 \Leftrightarrow x = 0 \vee x^\star = 1 - \frac{mBD}{\mu A - D}.$$

If $x = 0$, then

$$z = B\left[(1 - C) - (2 - m)x - \frac{BD}{\mu A - D}\right]\left(\frac{\mu}{\mu A - D}\right),$$

and hence

$$z_1^\star = \frac{\mu B[(1 - C)(\mu A - D) - BD]}{(\mu A - D)^2}.$$

Because $y > 0$ and $z > 0$, $\frac{BD}{\mu A - D} > 0$ and $(1 - C)(\mu A - D) > BD > 0$. It is clear that $C < 1$. Hence, if $\mu A > D$ and $(1 - C)(\mu A - D) > BD$, then $E_4(0, y^\star, z_1^\star)$ is an equilibrium point of the system (3.1). If $x \neq 0$, then substitute

$$z = B\left[(1 - C) - (2 - m)x - \frac{BD}{\mu A - D}\right]\left(\frac{\mu}{\mu A - D}\right)$$

to get

$$z_2^\star = \mu B\left[\frac{(1 - C)(\mu A - D) - (m - 1)^2 BD}{(\mu A - D)^2}\right].$$

Because $x > 0$, $y > 0$ and $z > 0$,

$$0 < \frac{mBD}{\mu A - D} < 1, \quad 0 < \frac{BD}{\mu A - D}$$

and

$$\mu B\left[\frac{(1 - C)(\mu A - D) - (m - 1)^2 BD}{(\mu A - D)^2}\right] > 0.$$

Thus, $BD > 0$, $m > 0$ and $\mu B > 0$. Hence, if

$$0 < mBD < \mu A - D$$

and

$$(m - C - 1)(\mu A - D) > (m - 1)^2 BD,$$

then $E_5(x^\star, y^\star, z_2^\star)$ is an equilibrium point of the system (3.1). □

4. Stability Analysis

Theorem 4.1. *If* E_0 *and* E_1 *are the equilibrium points of the system* (2.2), *then*

(1) E_0 *is always unstable for any conditions,*

(2) E_1 *is an unstable saddle point if* $m > C + 1$ *and a stable node point if* $m < C + 1$.

Proof. Jacobian of the equilibrium point $E_0(0, 0, 0)$ is

$$J(E_0) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - C & 0 \\ 0 & 0 & -D \end{pmatrix}.$$

The eigenvalues of the characteristic equation $J(E_0)$ are $\lambda_1 = 1$, $\lambda_2 = 1 - C$ and $\lambda_3 = -D$. Hence, the equilibrium point $E_0(0, 0, 0)$ is an unstable saddle point. Jacobian of the equilibrium point $E_1(1, 0, 0)$ is

$$J(E_1) = \begin{pmatrix} -1 & -m & 0 \\ 0 & -1 + m - C & 0 \\ 0 & 0 & -D \end{pmatrix}.$$

The eigenvalues of the characteristic equation $J(E_1)$ are $\lambda_1 = -1$, $\lambda_2 = m - C - 1$ and $\lambda_3 = -D$. Hence, the equilibrium point $E_1(1, 0, 0)$ is an unstable saddle point if $m > C + 1$ and a stable node point if $m < C + 1$. □

Theorem 4.2. *If* E_2 *and* E_3 *are the equilibrium points of the system* (2.2), $C < 1$, $\mu A > D$, $m(1 - C) < 1 < 1 + C < m$, $(1 - C)(\mu A - D) > BD$, *then*

(1) E_2 *is a stable node point,*

(2) E_3 *is a stable node point if* $1 + \sqrt{2} < m$ *and* $(\mu A - D)(m - 1 - C) > BD(m - 1)^2$.

Proof. Jacobian of the equilibrium point $E_2(0, y_1, 0)$ with $y_1 = 1 - C$ is

$$J(E_2) = \begin{pmatrix} 1 - m(1-C) & 0 & 0 \\ (m-2)(1-C) & 3(1-C) & \left(-\frac{A(1-C)}{B+1-C}\right) \\ 0 & 0 & \mu\frac{A(1-C)}{B+1-C} - D \end{pmatrix}.$$

The eigenvalues of the characteristic equation $J(E_2)$ are $\lambda_1 = 1 - m(1-C)$, $\lambda_2 = 3(1-C)$ and $\lambda_3 = \frac{A(1-C)}{B+1-C} - D.$ Because $y_1 = 1 - C > 0$, λ_1 and λ_2 are always positive and λ_3 is negative if $\frac{\mu A(1-C) - D(B+1-C)}{B+1-C} < 0.$ Hence, the equilibrium point $E_2(0, y_1, 0)$ is an unstable saddle point if $\mu A(1-C) < D(B+1-C)$ and stable node point when $\mu A(1-C) > D(B+1-C)$.

Jacobian of the equilibrium point $E_3(x_2, y_2, 0)$ with $x_2 = \frac{1-m(1-C)}{1+m(m-2)} = \frac{1-m(1-C)}{(m-1)^2}$ and $y_2 = \frac{(m-1-C)}{(m-1)^2}$ is

$$J(E_3) = \begin{pmatrix} K_1 & K_2 & 0 \\ K_3 & K_4 & K_5 \\ 0 & K_6 & 0 \end{pmatrix},$$

with

$$K_1 = \frac{(m-1) - mC}{(m-1)^2}, \quad K_2 = \frac{m^2(1-C) - m}{(m-1)^2},$$

$$K_3 = \frac{m^2 - (3+C)m + 2 + 2C}{(m-1)^2}, \quad K_4 = \frac{m-1-C}{(m-1)^2}, \quad K_5 = -\frac{A[(m-1)-C]}{B+(m-1-C)}$$

and

$$K_6 = \mu\frac{A[(m-1)-C]}{B(m-1)^2 + (m-1-C)} - D.$$

The characteristic equation of $J(E_3)$ is

$$\lambda^3 - \lambda^2(K_1 + K_4) + \lambda(K_1K_4 - K_2K_3 - K_5K_6) + K_1K_5K_6 = 0.$$

If $E_3(x_2, y_2, 0)$ is an equilibrium point, then $(K_1 + K_4) > 0$. If $1 + \sqrt{2} < m$ and $(\mu A - D)(m - 1 - C) > BD(m-1)^2$, then we obtain

$$K_1K_4 - K_2K_3 - K_5K_6 > 0, \quad K_1K_5K_6 > 0.$$

Thus,

$$(K_1 + K_4) > 0, \quad K_1K_4 - K_2K_3 - K_5K_6 > 0, \quad K_1K_5K_6 > 0$$

and

$$(K_1 + K_4)(K_1K_4 - K_2K_3 - K_5K_6) - K_1K_2K_5 > 0.$$

Based on Routh-Hurwitz criterion, if $1 + \sqrt{2} < m$ and $(\mu A - D)(m - 1 - C) > BD(m-1)^2$, then all real parts of the eigenvalues are negative. So, the equilibrium point $E_3(x_2, y_2, 0)$ is a stable point. □

Theorem 4.3. *If E_4 is the equilibrium point of the system* (2.2), $C < 1$, $\mu A > D$ *and* $(1 - C)(\mu A - D) > BD$, *then* E_4 *is a stable point if* $D < 1$, $B + C > 1$ *and* $\mu A - D < mBD$.

Proof. Jacobian of the equilibrium point $E_4(0, y^\star, z_1^\star)$ with

$$y^\star = \frac{BD}{\mu A - D}, \quad z_1^\star = \frac{\mu B[(1 - C)(\mu A - D) - BD]}{(\mu A - D)^2}$$

is

$$J(E_4) = \begin{pmatrix} L_1 & 0 & 0 \\ L_2 & L_3 & L_4 \\ 0 & L_5 & 0 \end{pmatrix},$$

with

$$L_1 = 1 - \frac{mBD}{(\mu A - D)}, \quad K_2 = \frac{(m-2)BD}{(\mu A - D)}, \quad L_3 = \frac{1-(C+B)D}{\mu A}, \quad L_4 = -\frac{D}{\mu}$$

and

$$L_5 = (1-D)\left(\frac{(1-C)(\mu A - D) - BD}{A(\mu A - D)}\right).$$

The characteristic equation of $J(E_4)$ is

$$\lambda^3 - \lambda^2(L_3 + L_1) + \lambda(L_1L_3 - L_4L_5) + L_1L_4L_5 = 0.$$

If $D < 1$ and $\mu A - D < mBD$, then $L_3 + L_1 > 0$ and $L_1L_4L_5 > 0$. If $D < 1$, $B + C > 1$ and $\mu A - D < mBD$, then $L_1L_3 - L_4L_5 > 0$ and $(L_3 + L_1) \cdot (L_1L_3 - L_4L_5) - L_1L_4L_5 > 0$. Thus,

$$L_3 + L_1 > 0, \quad L_1L_3 - L_4L_5 > 0, \quad L_1L_4L_5 > 0$$

and

$$(L_3 + L_1)(L_1L_3 - L_4L_5) - L_1L_4L_5 > 0.$$

Based on Routh-Hurwitz criterion, if $D < 1$, $B + C > 1$ and $\mu A - D < mBD$, then all real parts of the eigenvalues are negative. So, the equilibrium point $E_4(0, y^\star, z_1^\star)$ is a stable point. □

Theorem 4.4. *If* E_5 *is the equilibrium point of the system* (2.2), $\mu A > D$, $mBD < \mu A - D$ *and* $(m-1-C)(\mu A - D) > BD(m-1)^2$, *then* E_5 *is a stable point if* $(m-1-C)(\mu A - D) < \mu AB$, $m > 2$ *and* $\mu B < 1$.

Proof. Jacobian of the equilibrium point $E_5(x^\star, y^\star, z_2^\star)$ with

$$x^\star = 1 - \frac{mBD}{\mu A - D}, \quad y^\star = \frac{BD}{\mu A - D}$$

and

$$z_2^{\star} = \mu B\left[\frac{(1-C)(\mu A - D) - (m-1)^2 BD}{(\mu A - D)^2}\right]$$

is

$$J(E_5) = \begin{pmatrix} M_1 & M_2 & 0 \\ M_3 & M_4 & M_5 \\ 0 & M_6 & 0 \end{pmatrix},$$

with

$$M_1 = \frac{(mBD)}{\mu A - D} - 1, \quad M_2 = m\left(\frac{mBD}{\mu A - D} - 1\right), \quad M_3 = (m-2)\frac{BD}{\mu A - D},$$

$$M_4 = \frac{[(m-C-1)(\mu A - D) - (m-1)^2 BD]D}{\mu(\mu A - D)} - \frac{\mu ABD}{\mu(\mu A(\mu A - D))},$$

$$M_5 = -\frac{-D}{\mu}$$

and

$$M_6 = \mu\,\frac{[(m-1) - C](\mu A - D) - (m-1)^2 BD}{A}.$$

The characteristic equation of $J(E_5)$ is

$$\lambda^3 - \lambda^2(M_1 + M_4) + \lambda(M_1M_4 - M_2M_3 - M_5M_6) + M_1M_5M_6 = 0.$$

If $E_5(x^{\star}, y^{\star}, z_2^{\star})$ is an equilibrium point, then $M_1M_5M_6 > 0$. Also, $(M_1 + M_4) > 0$ if $(m - C - 1)(\mu A - D) < \mu AB$. Thus,

$$M_1M_4 - M_2M_3 - M_5M_6 = M_1(M_4 - mM_2) - M_5M_6,$$

if $m > 2$ and $(m - C - 1)(\mu A - D) < \mu AB$ and then

$$M_1(M_4 - mM_2) - M_5M_6 > 0.$$

Furthermore,

$$(M_1 + M_4)(M_1M_4 - M_2M_3 - M_5M_6) - M_1M_5M_6 > 0$$

if $m > 2$, $(m - C - 1)(\mu A - D) < \mu AB$ and $\mu B < 1$.

Therefore,

$$M_1 + M_4 > 0, \quad M_1M_4 - M_2M_3 - M_5M_6 > 0, \quad M_1M_5M_6 > 0$$

and

$$(M_1 + M_4)(M_1M_4 - M_2M_3 - M_5M_6) - M_1M_5M_6 > 0.$$

Based on Routh-Hurwitz criterion, if $m > 2$, $(m - C - 1)(\mu A - D) < \mu AB$ and $\mu B < 1$, then all real parts of the eigenvalues are negative. So, the equilibrium point $E_5(x^\star, y^\star, z_2^\star)$ is a stable point. □

5. Numerical Simulations

Dynamics of the predator-prey population can be shown as the curve of the solution field that describes a population of prey and predator during certain period. The numerical simulations based on the parameters are provided in the following Table 1.

Table 1. Equilibrium points, bounded parameters of the existence of the equilibrium point, parameter values and stability of the equilibrium point

Equilibrium points	M	A	B	C	D	μ	Stability
E_0	1,5	1	0,4	0,7	0,2	0,5	Unstable
E_1 case 1	2,21	0,85	0,6	0,7	0,2	0,5	Unstable
E_1 case 2	1,5	1	0,4	0,7	0,2	0,5	Stable
E_2	2,21	0,85	0,6	0,7	0,2	0,5	Unstable
E_3	2,21	0,85	0,6	0,41	0,2	0,5	Stable
E_4	3,1	0,85	0,6	0,41	0,2	0,5	Stable
E_5	2,1	0,8	0,4	0,3	0,2	0,5	Stable

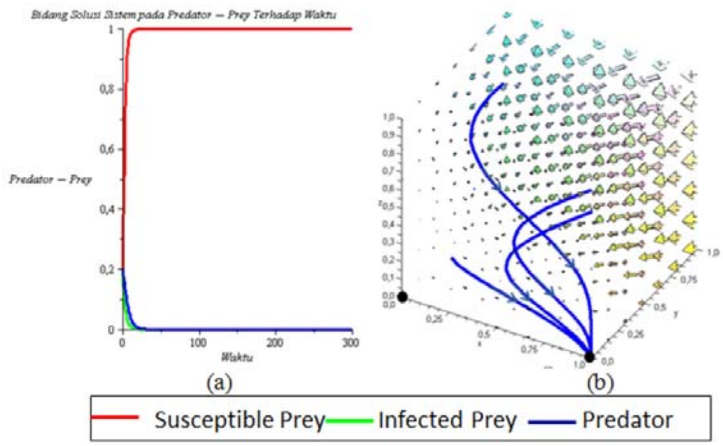

Figure 1. The solution field of predator-prey system and the phase portrait of predator-prey system at the equilibrium point E_0, E_1, and the equilibrium E_2 in Case 2, $m < C + 1$.

Figure 1 is phase portrait for system (3.1) which shows that the equilibrium point E_0 is an unstable node. In this case, we can say that both prey and predator population densities are unstable. In Case 1, the equilibrium E_1 is an unstable node. Furthermore, Figure 1 shows the phase portrait for the equilibrium point E_2 in Case 1 which indicates that the equilibrium point E_2 is a saddle point.

Figure 1(a) shows that in the beginning the infected-prey population and the predator population declined and then become extinct. While growth rate of the susceptible-prey population rose significantly and simultaneously with the decline of the infected-prey population until the extinct. This condition occurred because the pace of the spread of diseases is not large enough and predators do not get enough preys which caused the extinction of the predator population.

Figure 1(b) shows the point of equilibrium E_0 is a saddle point, because it is clearly visible that all the trajectories in the direction space around are avoiding the equilibrium point E_0, thus the equilibrium E_0 is unstable. This condition provides the stability of each population number. If $m > C + 1$, then the equilibrium point E_1 is a node point, and hence E_1 is unstable.

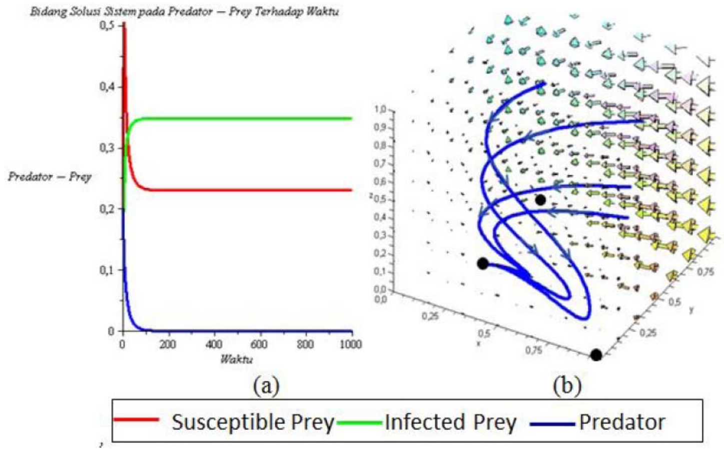

Figure 2. Phase portrait for the equilibrium points $E_1, E_2,$ and E_3 for $m > C + 1,\ m > 1 + \sqrt{2},$ and $(\mu A - D)(m - 1 - C) > BD(m - 1)^2.$

Figure 2(a) shows that in the early stage infected and susceptible prey grow significantly. When the total number of infected-prey comes close to 35%, then the population growth rate becomes constant. The number of experienced susceptible-prey decline to 25% and becomes stable at the position after representing a significant increase of 50% in the period up to 200 time unit. In the same period, the predator population declined and afterwards approaches to extinction.

Figure 2(b) indicates the equilibrium point E_1 with $m > C + 1$, $m > 1 + \sqrt{2}$, and $E_2(0,\ 0.3,\ 0)$ is the saddle point. All the trajectories in the direction space avoid the point E_1 and E_2, and then E_1 and E_2 become stable. Furthermore, the equilibrium point $E_3(0.547162,\ 0.348337,\ 0)$ is a spiral stable because all the trajectories in the direction space approach the equilibrium point $E_3(0.547162,\ 0.348337,\ 0)$. So the equilibrium point E_3 is stable.

Figure 3(a) shows that in the beginning the infected-prey sub-population significantly rose and the sub-population growth becomes stable into a constant rate after reaching more than 50% at the period 100 up to 200 time units. While the susceptible-prey population approaches to extinction. It is caused due to the disease spread rate in the prey population, so the predator populations decline from the very beginning, and then becomes constant in the value between 5% up to 10%.

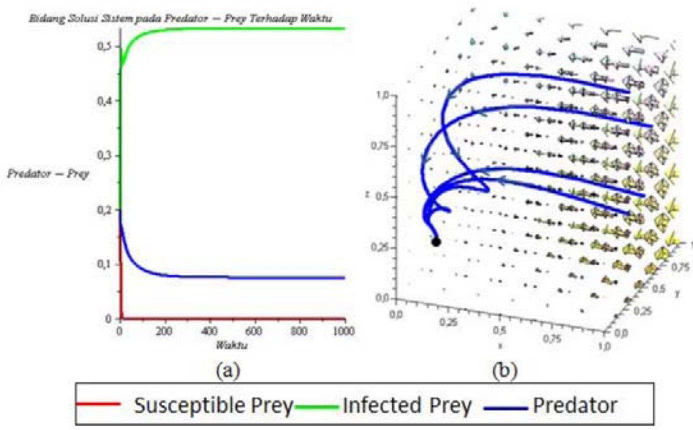

Figure 3. Phase portrait equilibrium E_4 for $D < 1$, $B + C > 1$ and $\mu A - D < mBD$.

This condition caused due to survival of the prey population only around 50% of the total.

Figure 3(b) indicates that the equilibrium point $E_4(0;\ 0,5333;\ 0,0755)$, is the node point. Clearly, all the trajectories in the direction space are towards the equilibrium point $E_4(0;\ 0,5333;\ 0,0755)$, and hence the point E_4 is stable if $D < 1$, $B + C > 1$ and $\mu A - D < mBD$.

Figure 4(a) shows that in the beginning the infected-prey and the susceptible prey population rose significantly when the total number of infected-prey populations declined in the range of 60% when $t < 500$ of time units. Then the rate of population growth being constant is not subjected to increase or decrease in the value of 40%. These conditions continued until drive t time unit. Susceptible-prey population rose in the range of 20%, and then became stable at this value because of the growth rate constant throughout. In the same period the predator population decreased, and then rose and became stable at about 15%.

Figure 4(b) indicates that the equilibrium point $E_5(0.16; 0.4; 0, 316)$ is a node point. We conclude that the equilibrium point $E_5(0.16; 0.4; 0, 316)$ is stable if $(m - C - 1)(\mu A - D) < \mu AB$, $m > 2$ and $\mu B < 1$.

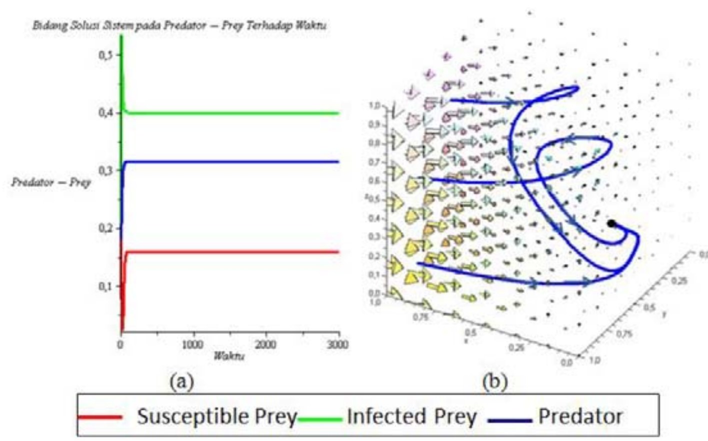

Figure 4. Phase portrait equilibrium point E_5 for $(m - C - 1)(\mu A - D) < \mu AB$, $m > 2$ and $\mu B < 1$.

6. Conclusions

In Section 2, a modified S-I-P predator-prey model with type II Holling functional response has been constructed. The six equilibrium points have been obtained from the mathematical model described by (2.2) in Section 3. In Section 4, stability analysis of the equilibrium points is carried out. Routh-Hurwitz criteria are used to discuss the stability of the critical points. The numerical simulations for some parameters have been carried out in Section 5 to show the stability of the equilibrium points.

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