The Profile of Structure Sense in Abstract Algebra Instruction in an Indonesian Mathematics Education

by Nurkaromah Dwidayati

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Junarti *	**Y. L. Sukestiyarno**	**Mulyono**	**Nur Karomah Dwidayati**
Universitas Negeri Semarang, INDONESIA	Universitas Negeri Semarang, INDONESIA	Universitas Negeri Semarang, INDONESIA	Universitas Negeri Semarang, INDONESIA

Abstract: The structure sense is a part that must be learned in order to help understand and construct connection in abstract algebra. This study aimed at building the pattern of a structure sense as a profile of the structure sense in group property. Using a qualitative study, the structure sense of group property was explored through lecturing activity of abstract algebra course from two individual assignments given to the students. The students who could provide the best answers from the first and second individual assignments were chosen to be the respondents. The data from the second assignment, then, was analyzed through presentation, interpretation, coding, making a pattern, leveling and continued with clarification through an interview. The results of the study show that there were six patterns of structure sense answers and five levels of structure senses made by the students as the profile of structure sense. The conclusion is the inability to recognize the structure of the set elements, operation notations, and binary operation properties is one of the causes of the constraints in structuring the proof construction of the group. Thus, a thinking of mathematics connection is needed in structure understanding as a connection between symbol in learning and the symbol of abstract.

Keywords: *Structure sense, group property, element structure.*

Introduction

A proof is the center of Mathematics. The difficulties in teaching and learning proof are internationally recognized (Miyazaki, et. al. 2017). Whereas, Basturk (2010) asserts that evidence and proof are considered to be the most important mathematical activities by mathematicians and mathematical educators, so it is reasonable that in abstract algebra, a proof is the most important part of learning. Algebra and algebraic structures are important aspects of algebraic thinking for students of secondary schools and universities (Wasserman, 2014). Understanding of abstract algebra can influence the teaching of structure in early algebra (Wasserman, 2017) so that the proof structure is the part that must be possessed by a Mathematics pre-service teacher like the students of Mathematics Education Study Program as a teacher candidate in secondary school.

Students' difficulties with algebraic structures are partly due to a lack of understanding of structural ideas in arithmetic (Linchevski & Livneh, as cited in Novotna & Hoch, 2008, p. 94) or because the components of algebraic structure in high schools are several components of algebraic structures in university (Novotna & Hoch, 2008) in which the students still do not have enough understanding so that it becomes a cause of trouble. Some studies on the structure sense that have been studied for algebra in high school (Hoch & Dreyfus, 2006), for algebra in university (Novotna, Stehlikova, & Maureen, 2006; Novotna & Hoch, 2008; Wasserman, 2016; Oktac, 2016), like a study that includes the theory of algebraic structures, specifically Groups, Ring and Field by Durand-Guerrier, Hausberger & Spitalas (2015) and other difficulties due to the lack of consistency of the concept in the construction of evidence (Alcock et al., 2011), so it is reasonable that this study is of particular concern for further investigation.

Why do the students experience difficulties in abstract algebra? This question is answered in Novotna & Hoch's (2008) study which revealed that students' difficulties in algebraic structures (abstract algebra) are partly due to a lack of understanding of structural ideas in arithmetic. According to Wasserman (2014), the idea of the basics of algebraic structure in arithmetic properties is important, because it is used to assume certain properties about addition and

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* **Corresponding author:**
Junarti, Universitas Negeri Semarang, Doctoral Program, Mathematics Education Program, Indonesia. ✉ junarti@ikippgribojonegoro.ac.id

multiplication. The arithmetic properties are known that they can form the axiomatic foundation of the algebraic structure needed, even in the early stages of algebraic reasoning by manipulating and solving equations (Wasserman, 2014). The notion of algebraic structures in human thought is a cognitive process involved in the conceptualization of abstract algebraic structures (Hausberger, 2015).

Understanding of the structure affects the performance of evidence (Stylianou, Chae, & Blanton, 2006), as in the study of Wasserman (2014) which introduces an approach to introducing group structures and fields through the arithmetic properties needed to solve simple equations for the combined interests of these axioms as an algebraic rationale which is discussed through an example of an abstract group. Exploring the potential aspects of abstract algebra for algebraic teaching in schools (Wasserman, 2016) is a part of the steps taken to help introduce the structure of evidence. Another of the steps taken to help introduce the structure of evidence is through the structure of the set as a basis for understanding the structure related to the properties or ideas that are more abstract (Wasserman, 2016).

Teaching early algebra recorded in memory will control the students' initial thinking in facing abstract algebra learning. As Collins and Frank (2013), state that cognitive control can facilitate reconnaissance as a learning strategy in providing potentially hidden structures, such as abstract rules or assignment sets needed for cognitive control. So that when students are faced with problems of set forms, binary operations, and group properties, they will be provoked to do so using arithmetic properties. The arithmetic traits of individuals have a good understanding of structural needs for algebraic reasoning (Wasserman, 2014) as an initial part that supports the development of a group property structure.

The structures associated with arithmetic in abstract circles are identified (Hausberger, 2017) and on the other hand the interaction between arithmetic operations and extension of number sets is often the source of structure in the set itself, this makes the knowledge potentially useful for conveying ideas in the development of Mathematics (Wasserman, 2016). In addition, the structure obtained by using definitions is accepted but does not always capture a more fundamental idea that there must be a fixed basis to classify (Inglis & Alcock, 2012) a structure.

Some components of the structure of algebra in university level are analogous to the components of algebraic structures in high schools (Novotna & Hoch, 2008) so that the structure in abstract algebra is influenced by students' experience in high school. Structural development in assignments also supports students in developing their understanding of structure as in the Schuler-Meyer's (2017) study conducted on distributive law using structural mapping. And through the use of examples of closure property, associative, identities and inverses in the section group property helps facilitate understanding of the structure sense. Although scholars highlighted that algebraic property starts with arithmetic and as an algebraic concept that is more than a technique to solve (Schuler-Meyer, 2017), every difficulty they display will be at the relational level (connection) of the notion of structure (Novotna & Hoch, 2008) and structures related to previous structures.

The term 'structure' is widely used and most people think that they do not need to explain what it means (Novotna & Hoch, 2008), but recognizing the structure used in each related structural change such as relationships between set elements or abstraction processes in understanding concepts is important. In different contexts, the term 'structure' means different things to different people (for example, Dreyfus & Eisenberg, 1996; Hoch & Dreyfus, 2004) and sometimes have different meanings.

The structure in mathematics can be seen as a broad view analysis of the way in which an entity consists of its parts and its analysis describes the system of connections or relationships between component parts (Hoch & Dreyfus, 2005). Algebraic structures will also be defined in terms of form and sequence (Hoch & Dreyfus, 2005). So that the appearance or external form expresses an internal order and internal sequence which is determined by the relationship between quantity and operation in which it is a component part of the structure (Hoch & Dreyfus, 2005). Meanwhile, the structure sense can be described that it also depends on the level of studies involved, and it is different between studying operations at the secondary school level and at the university level (Oktac, 2016; Hoch & Dreyfus, 2005). According to Hoch and Dreyfus (2005), the structure sense for high school algebra can be described as a collection of capabilities including 1) seeing algebraic expressions or sentences as entities, 2) recognizing algebraic expressions or sentences as previously fulfilled structures, 3) dividing entities into sub-structures, 4) recognizing reciprocal relationships between structures, 5) recognizing which manipulations are possible for done, and 6) recognizing which manipulations are useful to do.

The term algebraic structure is used in abstract algebra and can be understood to consist of a closed set under one or more operations which can include several axioms (Novotna & Hoch, 2008). Axioms, algebraic operations, and sets in each group have structures consist of different concepts and are integrated into one unit. To write evidence as well as build evidence construction, students need to gain further insight into the conception of evidence (Stylianou, Chae & Blanton, 2006). The conception of evidence in abstract algebra which includes the structure of evidence and the structure of concepts contained in a set, algebraic operations, and related axioms.

When students are asked to produce evidence, they tend to make empirical-numerical evidence (Stylianou, Blanton & Rotou, 2015) and students cannot release evidence through arithmetic and inductivity. Constructing evidence and evaluation of construction parts that help illuminate aspects of the evidence (Stylianides & Stylianides, 2009) are

interrelated to construct evidence. Besides, the structure they examine includes the sequence of operations in arithmetic expressions. In this study, the researchers want to explore the inability of students to construct evidence in abstract algebra which is assumed to have no structure sense in set elements, binary operations, and group property.

The structure sense used as an identification framework in this study is the sense of the structure of Novotna & Hoch (2008), which divides the sense of structure of algebra in university into 1) SSE: Structure Sense as applied to set elements and Notion of Binary Operation (Structure Sense as Applied to Elements of Sets and the Notion of Binary Operation); 2) SSP: Structure sense as applied to Binary Operating Properties (Sense as Applied to Properties of Binary Operations).

Previous studies on the structure sense of algebra in universities is dealing with algebraic expressions or equations that relate to (1) eight structure senses of abstract algebra (Novotna & Hoch, 2008) and (2) nine structure senses of binary operations in abstract algebra at universities (Novotna, et.al., 2006; Oktac 2016). However, the two studies did not perform the levelling. Whereas, this study adapts eight structure senses (Novotna & Hoch, 2008) which are used as a framework for the introduction of structure sense in group axioms and the structure sense was levelled based on the analysis of students' answers patterns. A preliminary study that has been done shows the results of the inability of students to explore the elements of the set which are stated on the terms of membership, but if the set is presented by listing the elements, the students are able to determine the results of the operations. An introduction to the structure sense of set elements is pivotal when students encounter binary operations with various set expressions. When students are able to recognize the set elements well, they will succeed in carrying out the binary operation process. Thus, the introduction of structure sense helps the understanding of group definitions consisting of four axioms (closed, associative, identity, and inverse).

From the explanation above, this study examines the following questions:

1. What are the SSE-SSP profiles that were built by students in group property?
2. To what extent do the SSE-SSP patterns that are built to be a level of structure in understanding group concepts?

Methodology

This research is a qualitative research with a case study design. The case observed in this study is the profile of students' structure sense in recognizing group axioms which are presented in the levelling form based on the analysis of students' answer patterns by following Novotna and Hoch's (2008) framework. This case study design was used since the objective of this study is building the pattern of a structure sense as a profile of the structure sense in group property.

Research Goal

Research in to find out the structure profile applied to set elements and binary operating notations built by students especially in operations (operations: +, -, x, :, operations on matrices, operation of function composition) and on some sets (which are limited to set of numbers, set of matrices, set of shapes, and set of functions) to show closure property in the Group.

Research Participants and Data Collection

The participants of this study were 6 students selected based on the pattern of answers and then categorized into five levels so that there is one student as a respondent who fits the category of the level-4, and there is also one student as a respondent who fits the category of the level-3 (because there was only one students in those categories). At the category of level-2, there are 2 out of 7 students were selected, and at the category of level-1, there are 2 out of 20 students were selected. The participants were chosen based on their communication skills and willingness to be interviewed.

The participants of this study were the students of the fifth-semester of Mathematics Education of IKIP PGRI Bojonegoro, East Java, Indonesia, who took abstract algebra courses in which the teaching emphasized the importance of structure in abstract algebra. Using a qualitative study, the structure sense of group property was explored through lecturing activity and two individual assignments for seven meetings of the learning process. The results of the first assignment were used for another related study. The results of the first assignment were used for preliminary study a part of the assignment paper of a doctoral program. While the second assignment was used to find out the profile of SSE-SSP to build patterns of answers and level in the structure sense built by students based on Novotna and Hoch's (2008) framework. From those patterns, then, the leveling was arranged based on the category of the SSE-SSP stages. Then, the extent to which the SSE-SSP patterns built to be a level of structure in understanding group concepts was analyzed later on.

The criteria of Novotna's (2008) structure sense in the group property are as follows.

1. SSE: Structure Sense as Applied to Elements of Sets and the Notion of Binary Operation

Students are said to display SSE if they can:

a. (SSE-1) Recognize a binary operation in familiar structures
b. (SSE-2) Recognize a binary operation in non-familiar structures
c. (SSE-3) See elements of the set as objects to be manipulated, and understand the closure property
2. SSP: Structure Sense as Applied to Properties of Binary Operations

Students are said to display SSP if they can:

a. (SSP-1) Understand identity element in terms of its definition (abstractly)
b. (SSP-2) See the relationship between identity and inverse elements
c. (SSP-3) Use one property as a supporting tool for the easier treatment of another: (e.g. commutativity for identity element, commutativity for the inverse element, commutativity for associativity)
d. (SSP-4) Keep the quality and order of quantifiers

The first assignment is presented in table 1. The time allocation given was 120 minutes with 9 questions which emphasized on constructing the structure of set elements and binary operations notation so that by giving four categories of sets and several operations, it can categorize the profile of the structure formed to be read well.

Table 1. Assignment-1 Problem

Show whether the following set meets the closure property of the specified operation!

1. $(N,+),(N,-),(N,x),(N,:)$ with N = set of natural numbers
2. $(Z,+),(Z,-),(Z,x),(Z,:)$ with Z = set of integers
3. $(Q,+),(Q,-),(Q,x),(Q,:)$ with Q = set of rational numbers
4. $(R,+),(R,-),(R,x),(R,:)$ with R = set of real numbers
5. Let $M = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a,b,c,d \in are\ real\ numbers\right\}$, show that $(M,+)$ fulfills the closure property!
6. Let $M = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a,b,c,d \in are\ real\ numbers, ad - bc = 1\right\}$, show that (M,x) fulfills the closure property!
7. Let $M\,M = \left\{\begin{bmatrix} 1 & w \\ 0 & 1 \end{bmatrix} \mid w \in are\ integers\right\}$, show that (M,x) fulfills the closure property!
8. G = {f_1, f_2, f_3, f_4, f_5, f_6} with the function composition operation and $f_1(x) = x$; $f_2(x) = \frac{1}{x}$; $f_3(x) = 1 - x$; $f_4(x) = \frac{1}{1-x}$; $f_5(x) = \frac{x}{x-1}$; $f_6(x) = \frac{x-1}{x}$. Show that (G, o) fulfills the closure property!
9. T = {x | x = p√q, p integers and q natural numbers}. Show that (T, x) fulfills the closure property!

The SSE-SSP profiles were examined from the results of assignment 2 about the structure of group property which include closed axioms, associative, identity elements, and inverse elements. The assignment 1 questions were taken one question for assignment 2, with the hope that by giving the same set, there would be improvements in the structure that had been previously made.

Table 2. Assignment 2 Problem

G = {x | x = a√b, a is integer and b is natural number} with the addition operation. Prove that G is a group!

From the results of the two assignments, it was selected the students whose work's results were quite good, then the answer was chosen based on which has the seven of Novotna's (2008) SSE-SSP criteria, then the interview was conducted to clarify the students' answers and as triangulation. The interview was recorded through video.

The data yielded form the result of the task from one respondent at level-3 was validated by triangulation method by comparing the task results and interview results, while the data of two respondents from level-2 and two respondents from level-1 were triangulated with source validation i.e., through respondent's partner at the same level. Video interviews were conducted by lecturers with five respondents to observe gestures as a form of seriousness of answers. Example: the questions used in the interview were structured questions (for example: from the student answers written "2 + 3 = 5, then 5 ∈Z"), namely: 1) Why do you write elements of a set of integers with 2 and 3 only?; 2) is the use of two elements, 2 and 3 only able to represent all elements in the set of integers ?; 3) How do you get the set of integer elements, so that you can represent all elements of the set of integers ?; 4) How many elements do you have to choose to be able to show as proof of the closed nature of the "+" binary operation of the set of integers?

Analyzing of Data

The results of the first assignment were described, interpreted, coded and the best answers were selected (However, it was used for another related study). There were six students who provided the best answers from the first assignment. The six students who provided the best answers from the first assignment were further evaluated the results of their second assignment. From the second assignment, after the coding process, the answers which have Novotna's (2008) SSE-SSP criteria were selected, and it was chosen only one student as the respondent. Furthermore, the results of second assignment were classified into answer patterns. From the answer patterns, it was constructed into six levels of answers. The highest level was the level of the Novotna's (2008) SSE-SSP criteria. Then, the extent to which the structure built by students was obtained from the results of interpretation through the levels formed from the entire structure of students' answers. To guarantee the validity of the data, the interview was conducted. The results of interview recording (video) were observed, transcribed, and presented in the form of narration.

Data validity was done based on the group of respondents. The data were resulted from the respondents at level-3 and level-4 because each of them only consisted of one student. Then the validity of the data is through triangulation methods by comparing the results of assignments and interview results. While the validity of data from level-2 and level-1 respondents was by using source triangulation by matching the students' work between student A and B. This is because there were two respondents in the level-1 and level-2 who were able to communicate and were willing to be interviewed.

Findings / Results

After the data were analyzed, the results of this study are presented. The results of this study were classified into two, i.e.: the SSE-SSP profiles built by the students, and the extent to which the SSE-SSP patterns built to be a level of structure in understanding group concepts.

1. *The SSE-SSP profiles built by the students*

     If it is seen from the distribution of answers in table 3 below, it shows the same difficulties as in the first assignment. The following is presented the results of coding answers based on the framework of Novotna's (2008) structure sense obtained in table 3 with "+" (fulfilling criteria) and "-' (not fulfilling criteria).

Table 3. The Distribution Coding Answers based on Novotna's Structure Sense Criteria

The Profile of Answers	SSE-1	SSE-2	SSE-3	SSP-1	SSP-2	SSP-3	SSP-4	The number of students who answered correctly
P-1	-	+	-	-	-	-	-	9
P-2	+	-	+	+	+	+	-	1
P-3	+	-	+	-	-	-	-	7
P-4	+	-	+	+	+	-	-	0
P-5	-	+	+	-	-	-	-	10
P-6	-	+	-	-	+	+	-	1

From the distribution of students' answers above, an answer pattern can be drawn from the structure Sense from Novotna (2008) with the answers from each student adjusting the type of structure sense with the order formed into 6 students' answer patterns and is added by one pattern from Novotna, so that there are 7 patterns as follows.

Pattern-1: SSE-2 (9 students)

Pattern-2: SSE-1 SSE-3 SSP-1 SSP-2 SSP-3 (1 student)

Pattern-3: SSE-1 SSE-3 (7 students)

Pattern-4: SSE-1 SSE-3 SSP-1 SSP-2 (0 students)

Pattern-5: SSE-2 SSE-3 (10 students)

Pattern-6: SSE-2 SSP-2 SSP-3 (1 student)

Pattern-7: SSE-1 SSE-3 SSP-1 SSP-2 SSP-3 SSP-4 (the pattern of Novotna's (2008) structure sense)

2. *The level of SSE-SSP patterns built in understanding group property*

From the seven answer patterns above, it is then constructed by grouping through the sequence of answers that lead to Novotna's structure sense, so that four levels are obtained as follows.

1) LEVEL-5: Pattern-7 is a very high level which is the most complete level of the seven stages of Novotna's structure sense.
2) LEVEL-4: Pattern-2 which is a high level which capable in stages: SSE-1, SSE-3, SSP-1, SSP-2, SSP-3
3) LEVEL-3: Pattern-4 is a medium level which capable in stages: SSP-1, SSE-3, SSP-1, SSP-2
4) LEVEL-2: Pattern-3 is a low level which capable in stages: SSE-1, SSE-3
5) LEVEL-1: Pattern-1, Pattern-5, and Pattern-6, which is the lowest level with structural stages starting from SSE-2, which is to recognize binary operations in unknown structures.
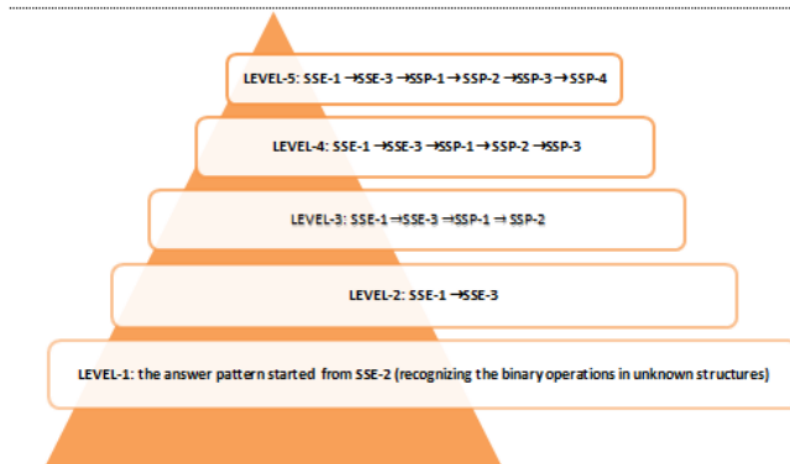

Figure 1. Structure Sense Levels on Set Elements, Binary Operation Notations, and Binary Operation Properties

From the four levels, it can be interpreted that the level of structure built from the answers of students at the moderate level (level-3) as many as one student, at level 2 as many as seven students, and at level-1 as many as 20 students. So students who reach level 4 are only one student who was chosen as the respondent.

Furthermore, the answers of the second assignment were interpreted from one respondent only. The results of the interview presentation are presented as follows.

The following will be conveyed the results of students' answers who became respondents presented in figures 2, 3, 4, and 5 as follows.
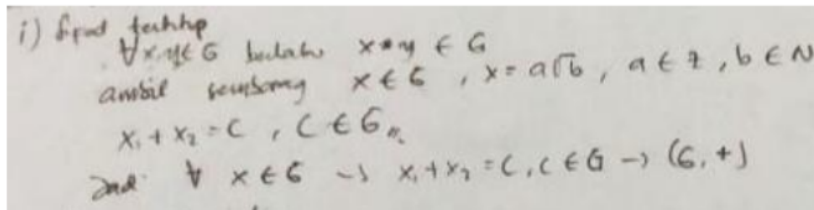

Figure 2. The Answer of Closure Property

From the presentation of the respondent's work, the structure of the selected element is in accordance with the SSE-1 criteria (because it is able to recognize binary operations in known structures), by showing in the form of "take any $x \in G, x = a\sqrt{b}, a \in Z, b \in N, x_1 + x_2 = c, c \in G$", then it matches the SSE-3 criteria because it is able to see elements from the set as objects to be manipulated, and understand the closed property as shown: "take any $x \in G, x = a\sqrt{b}, a \in Z, b \in N, x_1 + x_2 = c, c \in G$, to be $\forall x \in G, x_1 + x_2 = c, c \in G$". But there is a small note at the beginning of the respondent's answer written "$\forall x, y \in G$," applies $x + y \in G$" if it is observed that the expression in the initial appearance is not consistent with the evidence below even though the respondent's answer is only for definition. After being clarified with interviews, the respondent said that they were not aware that the writing was not the same (inconsistent) with the one chosen at the beginning as the symbol.

The following are the interview excerpts:

R: *"Why did you choose the element written at the beginning of x and y?"*

S: *" I took it from the element of G, just to write the definition"*

R: *"Then how about the x, x₁ and x₂ elements which suddenly appeared during the verification and operation?"*

S: *"I mean like this ma'am, by choosing one element, then I operate two elements which can automatically represent another set"*

R: *"Oh I see, but, is it consistent for a proof?"*

S: *"Oh yeah, ma'am, it means that the selected element is not the same, well, I understand"*

Figure 3. The Answer for Associative Property

For the answer of associative property, it will be categorized based on Novotna's structure sense into SSP-3. The figure 3 shows an understanding of the structure of the set of elements to be manipulated in binary operations that have shown correct and appropriate with the associative property through the commutative property, although the initial retrieval of elements x, y, z is not shown to be arbitrary. This result was clarified by the answer of the interview. The respondent acknowledged that the lack of elements taken was not written because he was not aware that this was important. The following are the interview excerpts:

P: *"Why was the taking of the initial element using a sample, is it enough to represent the other elements of the set?"*

S: *"Oh yeah ma'am, I am not aware that such a thing has not represented"*

Figure 4: The Answers of Identity Element

Furthermore, the classification of structure sense displayed for identity elements is included in the SSP-1 criteria. Figure 4 shows the collection of any set of elements becomes an initial error that should have found the identity element that applies to all set elements. So from the explanation of the answers of the respondent, it is known that he did not understand the element of identity in terms of definition, so they did not meet the SSP-1 criteria. After being clarified through an interview, it was revealed that the respondent did not realize that the writing must be started with "$\exists i \in G$" then "apply $x + i = i + x, \forall x \in G$", it was revealed that the respondent understood the definition. In this case, the researcher categorizes that the respondent is quite aware of the definition of the element of identity after the interview so that the respondent is the only one who answers close to the correct answer which meets the SSP-1 criteria. The following is the interview excerpt:

R: *"Why did you write at the beginning 'take any of x∈ G?"*

S: *"I think, it is in order to meet the G set"*

R: *"Do you know the definition of fulfilling the element of identity?"*

S: *"Yes, ma'am, there is an identity element in G so that it is applied $x + i = i + x$ for all G"*

R: *"Okay, well you just understood"*

Figure 5. The Answer of Inverse Element

From the answer in figure 5, it shows that the relationship between identity and inverse element with the form "$x^{-1} + x = x + x^{-1} = e$" even though the identity element "i" is replaced with "e", after being clarified through the interview, the respondent thought that the writing was mistaken. So this answer can be categorized into the SSP-2 criteria, which was initially categorized by the researcher in table 3, the respondent did not arrive at SSP-2. The following are the interview excerpts:

R: *"Why did you write the e element?"*

S: *"'e' is an element of identity, ma'am"*

R: *"What about the element i in your previous job?"*

S: *"Oh yeah, ma'am, I was mistaken"*

Furthermore, from the respondents' answers, they were able to use one property as a support tool to show the use of commutative properties for elements of identity. It can be seen in figure 8. Commutative for inverse elements can be seen in figure 9, and commutative for associative can be seen in Figure 7, therefore, the respondent can be categorized into SSP-3. There is a match between the results of the clarification through the interview that the respondent understood the three properties shown in the interview excerpts above.

The last category is SSP-4 if the respondent is able to maintain the quality and quantifier sequence. It was revealed that from the explanation of the answers in Figures 6, 7, 8 and 9, the quality of the respondent's answers have not been able to maintain the quality of truth expressively in each of the properties written, because there are still small mistakes that the respondent acknowledged through interview, sometimes he said "unconscious" or sometimes he said "mistyping" and there is inconsistency in the use of symbols, this causes the researcher to classify these errors as violations and are categorized as SSP-4.

So the specific conclusions for the structure sense that the respondents were able to build initially were grouped only to the stages: SSE-1, SSE-3, SSP-1, and SSP-3, apparently after the answers were clarified through interviews, the respondents met the categories: SSE-1, SSE-3, SSP-1, SSP-2, SSP-3. Thus the level of respondents has been able to reach level 4.

From the results of this study, the majority of students showed a lack of understanding of structure, this result is the same as the structure of Hoch and Dreyfus (2005). Most of those who do not use structure sense well so that they make miscalculations or fail to cancel undefined solutions. Those who use structure sense get answers quickly and accurately (Hoch & Dreyfus, 2005). From the results of this study, the students have not been able to reach such an arrangement in Novotna's (2008) stage, which is at level-5.

Discussion and Conclusion

Various reasons can be discussed in understanding the structure of set elements, operating structures, the structure of group properties which contain closure property, associative property, identity elements, inverse elements that have a related element and commutative properties in each axiom. The second understanding is related to the structure sense which becomes a must-have part in understanding structures in abstract algebra.

The inability to recognize the structure can be initiated from unsuccessfulness in dismantling symbolism to express meaning and structure (Novotna & Hoch, 2008). Then if the symbols related to the set of numbers as found by students in school mathematics course can be considered as one structure (Hoffman, 2017) which he thinks is correct and sufficient to show evidence. Furthermore, understanding symbols is a part that can help understanding the structure of elements of the set through leaving the understanding of arithmetic in high school algebra to become university algebra by struggling to transition from arithmetic to algebra (Goldenberg et al. as cited in Hoffman, 2017, p. 2) become an abstraction process.

Whereas individual knowledge about other concepts in the group must include an understanding of various mathematical properties and independent construction of certain examples, including groups consisting of undefined

elements and binary operations that fulfill axioms (Dubinsky et al. as cited in Oktac, 2016. P. 299) becomes a part of the understanding structure and giving structure sense. If someone starts with a very concrete group, transitions from 'group' to one of the quotients changes the property of the concrete elements to elements (Oktac, 2016) which is abstract. In addition, the genetic decomposition of the group concept is given in coordinating three schemes: set, binary operations and axioms (Oktac, 2016), but in reality, sets and operations are coordinated with axioms (Oktac, 2016) which are mutually integrated and interrelated.

The results of this study indicate that some students do not use precise definitions in completing assignments (Oktac, 2016) which can be a cause of disability in the structure sense. Whereas in their study Edwards and Ward (as cited in Octac, 2016, p. 302) state that the importance of the role of definition as an activity of thinking about mathematics, in terms of making definitions shows authentic participation in mathematical experiences. In order to make such kinds of activities work well, the students should be allowed to work in their own way until they find the logical consequences of the definitions they formulate, including "unintended consequences".

Other causes, such as in the studies of Iannone and Nardi (as cited in Oktac, 2016, p. 299), they observed that students interpret group axioms as the property of group elements rather than binary operations. After this phase is completed, students must compare their definitions with the definition used (Edwards & Ward as cited in Oktac, 2016, p. 303), in a proof. The definition also functions as the first statement used in the chain of deduction and may not contribute much to understanding the meaning of the fact being proven (Oktac, 2016), the structure is limited and is continuous between structural components, structural aspects that will be the basis of abstraction. The structure sense can be described well depending on the level of study involved in Middle school or at a university. There are two main stages for developing structure senses (Oktac, 2006), i.e. through SSE (Sense as Applied to Elements of Sets and Notion of Binary Operations) and SSP (Sense as Applied to Properties of Binary Operations).

The steps as shifts that work through examples to think abstractly about the mathematical structure identified by Simpson and Stehlikova (2006) are (1) looking at the elements in the set as objects in which operations act (by involving shifting process-objects); (2) showing the relationship between elements in the set which are the consequences of operations; (3) looking at the signs used by the teacher in defining abstract structures as object abstractions and operations, and looking at the names of relationships between signs as names for relations between objects and operations; (4) looking at other sets and operations as examples of general structures and as prototypical general structures; (5) using the system of formal symbols and property definitions to get the consequences and looking that the properties attached to the theorem are properties of all examples.

Novotna et al. (2006) identified at least three pathways for understanding structure:

1. The first path is extracting the known structures to form the basis of definitions, from which abstract concepts are constructed in a general context.
2. The second path is extracting properties from the known structures that leads to generalizations and then to definitions.
3. The third path is the construction of concepts through logical deduction from their definitions.

According to Oktac (2016), the difficulty in understanding the structure because of the shift that is difficult to do spontaneously requires a learning strategy to help students improve their learning through the steps needed for structural understanding in Abstract Algebra. So beside the role of group examples, the good meaning of definitions and the process of abstraction of set elements associated with binary operations is a part that can help transition shift to have the structure sense in group axioms.

Understanding the structure of abstract algebra learning is an important concern that supports the understanding of the concept as a whole. There are six patterns of structure senses of SSE-SSP Profiles built by students which consist of two patterns that are close to the ability to understand the structure of Novotna and Hoch (2008). From the seven patterns of structure sense forming five levels that can be classified as a reference level of structure sense, only one student was able to reach level 4 (high level). The highest level of structure sense uses Novotna's (2008) level, while the lowest level is the level that does not have the right answer. The results of this study show that there are six patterns of answer structure senses and five levels of structure senses built by students, and there is one student who is able to reach level 4 (high level). Thus the tendency of the inability of students is because the structure of the set elements is not in the form of numbers so that it is difficult to do numerically and the structure cannot be recognized. The inability to recognize the structure of the set elements, operating notations, and binary operating properties is one of the causes of constraints in constructing connection of mathematic concepts.

This study has some strengths i.e., 1) recognizing the structure sense of set elements, binary operations, and the identity-inverse property can foster the ability to connect mathematical concepts; 2) recognizing the structure sense of set elements, binary operation notation, and the nature of binary operations can establish connections in the group axioms. The limited variation of questions become obstacles in growing the level of structure sense of the respondents.

Suggestions

The results of this study can be recommended for further research. The recommendations are as follows.

1. The structure sense is built from the assignments given from more varied addition of sets types and binary operations; and

2. The five levels constructed in this study can still develop to be more proportional through the pattern of answers formed depending on more varied cognitive abilities.

Acknowledgements

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