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by Isti Hidayah

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The role of mathematics in designing laboratory activity

The role of mathematics in designing laboratory activity

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Abstract. The purpose of this study is to analyze terminal velocity of a spherical ball in the liquid viscosity experiment. The analysis was performed using a simple calculus approach. Using the limit of $v(t)$ as t approaches infinity ($t \rightarrow \infty$) and the asymptotic value ($v(t) = \infty$) as well as the differential and integral operations, the terminal velocity can be estimated. The time to reach the terminal velocity and the distance traveled by a spherical ball can also be determined. Therefore, the factors that affect the terminal velocity, the time to reach that velocity, and the distance traveled can be analyzed. Such information may be used to improve the design of the experiment.

1. Introduction

Liquid viscosity experiment is a type of physics experiment that is usually done both in physics subjects in high school and basic physics courses for freshman in undergraduate program. In this experiment, measurement of distance and time is usually done to determine the speed of the ball that moves vertically downward in a liquid. After the ball velocity is obtained, the activity is usually continued to determine the viscosity coefficient of the liquid.

Figure 1 shows a ball that is moving in a liquid. There are three forces acting on the ball, namely gravity (**w**) or attraction by the earth, frictional force by the liquid (**f**), and the upward force of the displaced liquid (**F**). The magnitude of gravity is expressed as $w = mg$ with m and g respectively are mass of the ball and the magnitude of the acceleration of gravity of the earth. The magnitude of upward force by the liquid being displaced, according to Archimides, is $F = V\rho g$ with V is the volume of the ball that is submerged, ρ is the density of the liquid, and g is the magnitude of the acceleration of the earth's gravity. The magnitude of friction force by a liquid, according to Stokes , is $f = kv$, with k is the coefficient whose value depends on the nature of the resistive medium [1]. For spherical objects whose radius (r) is small and the magnitude of velocity (v) is also small, then $k = 6\pi\eta r$, with η is the viscosity of the liquid [2,3]. For larger bubbles, any other formalism in more complicated force fields should be employed [4,5].

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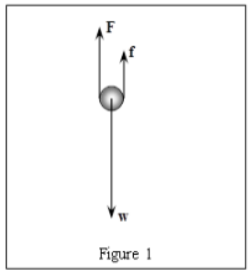

Figure 1. Ball in liquid

A ball is dropped without the initial velocity from the surface of a liquid, at first the ball will move straight downward at an increasing velocity. Because on the ball works the frictional force that is resisting the falling motion of the ball, the magnitude of the frictional force is directly proportional to the velocity of the ball, then finally the velocity of the sphere will reach the maximum magnitude or final velocity, which also called terminal velocity [6,7,8]. After reaching the terminal velocity, the force acting on the ball satisfies Newton's first law, that is, the resultant force is equal to zero or $w = f + F$, so that the ball moves straight at a constant velocity. When the ball is moving straight at a constant velocity, an experiment to determine the terminal viscosity of the liquid is carried out, but usually the determination of this condition (terminal velocity) is done only based on estimates.

The interesting thing about this experiment, so long as it seems never questioned: how long does the ball move, measured from the time the ball is released from the surface of the liquid, before the ball reaches its maximum speed? Or, at what distance from the surface of the liquid (where the ball is released at zero velocity), the ball reaches its terminal velocity?

Therefore, this paper will analyze the determination of the terminal velocity, the time to reach this velocity, and the distance from the surface of the liquid. The determination of the magnitude of the terminal velocity will be conducted using a simple calculus approach. The choice of this approach is based on the success of Alavarez et al. [1] in determining the terminal velocity in magnetic motion in eddy current induced columns.

2. Materials and Methods

The first step is to determine the magnitude of the terminal velocity using the calculus term, which uses the limit of $v(t)$ as t approaches infinity ($t \to \infty$). By using the definition of limits at $t \to \infty$ and asymptotic lines and differential and integral operations, the magnitude of the terminal velocity in the falling ball viscosity experiment can be estimated.

The next step, based on the terminal velocity equation that has been obtained, and by using differential and integral operations, the time to reach the terminal velocity and the distance of the path traveled by the ball can be determined. Finally, factors that affect the magnitude of the terminal velocity, time and distance traveled by the ball to reach that velocity can be analyzed. The results of this analysis are important; especially they can be used to design the experiment.

3. Result and Discussion

At the beginning of the motion, that is, shortly after the ball is dropped on the surface of the liquid, the ball moves straight down with changing velocity, or moves with acceleration. According to Newton's second law, the motion's equation of an object with a mass of m with the velocity v can be expressed as follows

$$F_{net} = m\frac{dv}{dt}$$

Because there are three forces acting on the ball, namely m, F, and f, then

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$$w - F - f = m\frac{dv}{dt}$$
$$mg - V\rho g - kv = m\frac{dv}{dt}$$
$$\frac{m}{mg - V\rho g - kv}dv = dt \qquad (1)$$

If the two segments of equation (1) are integrated, it will be obtained

$$\int \frac{m}{mg - V\rho g - kv}dv = \int dt$$

$$\ln(mg - V\rho g - kv) = -\frac{k}{m}t + c$$

If at $t_o = 0$, the magnitude of velocity is $v = 0$, or $v(t_o = 0) = 0$, then

$$\ln(mg - V\rho g) = c$$

So the equation becomes

$$\ln(mg - V\rho g - kv) = -\frac{k}{m}t + \ln(mg - V\rho g)$$
$$\ln\frac{mg - V\rho g - kv}{mg - V\rho g} = -\frac{k}{m}t$$

or

$$\frac{mg - V\rho g - kv}{mg - V\rho g} = e^{-\frac{k}{m}t}$$

Based on this equation, the function of velocity with respect of time, $v(t)$, can be determined, that is

$$v(t) = \frac{mg - V\rho g}{k} - \frac{mg - V\rho g}{k}e^{-\frac{k}{m}t} \qquad (2)$$

The graph of $v(t)$ versus t of equation (2) is shown in Figure 2.

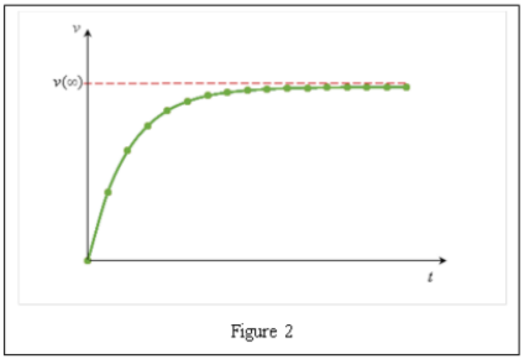

Figure 2. The graph of $v(t)$ versus t of Equation (2)

The asymptotic value of the $v(t)$ curve in Figure 1 is obtained by substituting $t = \infty$ in equation (2), so that it is obtained

$$v(\infty) \approx \frac{mg - V\rho g}{k} \tag{3}$$

In Figure 2, this asymptotic value is illustrated by a horizontal dashed line. Furthermore, the terminal velocity is obtained if $t \to \infty$, or $v(t \to \infty)$. Figure 2 shows that $v(t \to \infty)$ is slightly smaller than $v(\infty)$. Alvarez et al. [1] assumed that the experimental terminal velocity (v) is achieved when it reaches an arbitrary value, e.g. 99.9% of the asymptotic value, therefore

$$v \approx 0.999 v(\infty) \approx 0.999 \frac{mg - V\rho g}{k} \tag{4}$$

Equation (4) shows that the magnitude of the terminal velocity is directly proportional to the mass of the ball, the volume of the ball, the density of the liquid, the magnitude of the gravity acceleration, and inversely proportional to the coefficient k which describes the nature of the resistive fluid. The experimental results showed that for an iron ball with diameter 8 mm, the magnitude of the terminal velocity in oil and in glycerin liquid respectively was 30 cm/s and 10 cm/s [9]. Then using equation (1), the time to reach the terminal velocity can also be determined, that is

$$t = \int dt = \int_0^{0.999 v(\infty)} \frac{m}{mg - V\rho g - kv} dv$$

$$t = \int_0^{0.999 \frac{mg - V\rho g}{k}} \frac{m}{mg - V\rho g - kv} dv$$

$$t = \frac{m}{k} \ln \frac{1}{1 - 0.999}$$

$$t \approx \frac{m}{k} \ln 1000 \tag{5}$$

and, with the length of the path taken by the ball until it reaches the terminal velocity can also be determined, as follows

$$\frac{dx}{dt} = v$$
$$dx = v dt$$
$$x = \int dx$$

$$x = \int v dt$$

$$x = \int \left(\frac{mg - V\rho g}{k} - \frac{mg - V\rho g}{k} e^{-\frac{k}{m}t} \right) dt$$

$$x = \frac{mg - V\rho g}{k}t + \frac{m^2}{k^2}ge^{-\frac{k}{m}t} - \frac{V\rho m}{k^2}ge^{-\frac{k}{m}t} + c$$

If at $t_o = 0$ the displacement is $x = 0$, or $x(t_o = 0) = 0$, and by using equation (5), it can be obtained

$$x(t) \approx 5.9087\left(1 - \frac{V\rho}{m}\right)\frac{m^2}{k^2}g \quad (6)$$

If the magnitude of gravity acceleration (g) is $9.8\ \mathrm{m/s^2}$, then

$$x(t) \approx 57.906\left(1 - \frac{V\rho}{m}\right)\frac{m^2}{k^2} \quad (7)$$

Equations (5) and (7) respectively show the time interval and distance of the path of a ball that is dropped precisely on the surface of the fluid with an initial velocity of zero. If the data about m, k, V, and ρ is known, then, when and where the ball reaches the terminal velocity can be determined. This information is very useful for designing the experiment of falling ball viscosity, so that the experiment can be carried out on the condition that the ball has been moving in a straight line with a constant velocity.

4. Conclusion

Using simple calculus, the limit $v(t)$ at $t \to \infty$, and the asymptotic value $(t = \infty)$, the magnitude of the terminal velocity of a ball dropped precisely from the fluid surface can be determined. Furthermore, when and where the ball reaches the terminal velocity can also be determined. Thus, the area where the ball has moved in a constant velocity can also be determined. This information is important, especially for designing an experiment of the viscosity of a liquid with a falling object method.

The application of this calculus concept in terms of learning aspects has three advantages. First, the application of calculus shows the role of mathematics as the basis for designing a physics experiment, in this case the experiment of falling ball viscosity. Second, the application of this calculus can provide an overview of the concept of calculus, especially in this case about the concept of limits that are often difficult to imagine by students. Third, the application of calculus in physics experiments is an example of integrated material in physics learning.

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